Reading 4: Introduction to Regression

1. Sample Cov (X,Y) = \( \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \)

2. Correlation Coefficient = \( r_{XY} = \frac{\text{cov}(X,Y)}{\text{var}(X)\text{var}(Y)} \) or \( r = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\text{var}(Y)}} \)

3. t-test (for normally distributed variables) = \( t = \frac{\bar{b}_1 - b_1}{s_{b_1}} \) t distribution with \( n - 2 \) deg. of freedom

4. Linear Regression = \( Y_i = b_0 + b_1X_i + e_i \)
   - Intercept (\( b_0 \)) = \( \bar{b}_0 = \bar{Y} - \bar{b}_1\bar{X} \)
   - Slope or regression coefficient = \( b_1 = \frac{\text{cov}(X,Y)}{\text{var}(X)} \) or \( b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} \)

5. Standard Error of Estimate SEE = \( \hat{S}_e = \sqrt{\frac{\text{SSE}}{n-1}} \)

6. Coefficient of Determination (\( R^2 \)) = \( \frac{\text{SST} - \text{SSE}}{\text{SST}} = \frac{\text{RSS}}{\text{SST}} \) where, \( 0 \leq R^2 \leq 1 \) (for single independent variable \( R^2 = r^2 \))

7. \( \text{SST} = \text{SSE} + \text{SSR} \) (or RSS)

8. Hypothesis Testing:
   - Null and Alternative hypotheses
   - \( H_0: b_1 = 0 \) (no linear relationship)
   - \( H_1: b_1 \neq 0 \) (linear relationship does exist)
   - Test statistic = \( t = \frac{\bar{b}_1 - b_1}{s_{b_1}} \)
   - Confidence Interval = \( b_1 \pm t_c s_{b_1} \)

9. ANOVA (Analysis of variance) =

<table>
<thead>
<tr>
<th>Source of Variability</th>
<th>DoF</th>
<th>Sum of Squares</th>
<th>Mean Sum of Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression (Explained)</td>
<td>1</td>
<td>RSS</td>
<td>MSR = RSS/1</td>
</tr>
<tr>
<td>Error (Unexplained)</td>
<td>n-2</td>
<td>SSE</td>
<td>MSE = SSE/(n-2)</td>
</tr>
<tr>
<td>Total</td>
<td>n-1</td>
<td>SST = RSS + SSE</td>
<td></td>
</tr>
</tbody>
</table>

10. \( F \)-Statistic or \( F \)-Test = \( \frac{\text{MSR}}{\text{MSE}} = \frac{\frac{\text{RSS}}{k}}{\frac{\text{SSE}}{(n-k-1)}} \) 
    (df numerator = k = 1) 
    (df denominator = n – k – 1 = n – 2)
11. Prediction Intervals = \( \hat{Y} \pm t_{\alpha/2} s_f \),

where \( s_f^2 = s^2 \left[ 1 + \frac{1}{n} + \frac{(X-\bar{X})^2}{(n-1)\bar{X}^2} \right] \) and

\[ s_f = \sqrt{s_f^2} \]

Reading 5: Multiple Regression & Issues in Regression Analysis

1. \( Y_i = b_0 + b_1 X_{i1} + b_2 X_{i2} + \ldots + b_k X_{ik} + \epsilon_i; i = 1, 2, \ldots, n \)

2. Prediction equation = \( \hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_{i1} + \hat{b}_2 X_{i2} + \ldots + \hat{b}_k X_{ik} + \epsilon_i \)

3. Adjusted \( R^2 = R^2 = 1 - \left( \frac{n - 1}{n - k - 1} \right) (1 - R^2) \)

4. Breusch–Pagan test
   - \( H_0: \) No conditional Heteroskedasticity exists
   - \( H_A: \) Conditional Heteroskedasticity exists
   - Test statistic = \( n \times R^2_{\text{residuals}} \)

5. Durbin-Watson Test = \( DW = \frac{\sum_{t=2}^{T} (\hat{e}_t - \hat{e}_{t-1})^2}{\sum_{t=1}^{T} \hat{e}_t^2} \)
   - For Large Sample size DW Statistic (d) = \( d = 2 (1 - r) \)

Reading 6: Time Series Analysis

1. Linear Trend Models = \( y_t = b_0 + b_1 t + \epsilon_t \)
   - Predicted/fitted value of \( y_t \) in period \( (T + 1) = \hat{y}_{t+1} = \hat{b}_0 + \hat{b}_1 (T + 1) \)

2. Log-Linear Trend Models = \( y_t = e^{b_0 + b_1 t} \)

3. Autoregressive Time-Series Models:
   - First order autoregressive AR (1) = \( x_t = b_0 + b_1 x_{t-1} + \epsilon_t \)
   - pth-order autoregressive AR (p) = \( x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \ldots + b_p x_{t-p} + \epsilon_t \)

4. Mean reverting level of \( x_t = \frac{b_0}{1 - b_1} \)

5. Chain Rule of Forecasting:
   - One-period ahead forecast = \( \hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1 x_t \)
   - Two-period ahead forecast = \( \hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1 x_{t+1} \)

6. Random Walks and Unit Roots:
   - Random Walk without drift = \( x_t = x_{t-1} + \epsilon_t \) where, \( b_0 = 0 \) and \( b_1 = 1 \).
   - Correcting Random Walk = \( y_t = x_t - x_{t-1} + \epsilon_t \) where, \( b_0 \neq 0 \) and \( b_1 = 1 \).
   - By taking first difference \( y_t = x_t - x_{t-1} = b_0 + \epsilon_t \)

Reading 8: Big Data Projects

7. Using Dickey-Fuller Test = \( x_t - x_{t-1} = b_0 + (b_1 - 1) x_{t-1} + \epsilon_t \)

8. Smoothing Past Values with n-Period Moving Average = \( x_t + x_{t-1} + x_{t-2} + \ldots + x_{t-(n-1)} \)

9. Correcting Seasonality in Time Series Models:
   - For quarterly data = \( x_t = b_0 + b_1 x_{t/4} + b_2 x_{t/4} + \epsilon_t \)
   - For monthly data = \( x_t = b_0 + b_1 x_{t/12} + b_2 x_{t/12} + \epsilon_t \)

10. ARCH model = \( \hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\epsilon}_{t-1}^2 + \mu_t \) where \( \mu_t \) is an error term

   - Predicting variance of errors in period \( t+1 = \hat{\mu}_{t+1} = \hat{\mu}_0 + \alpha_1 \epsilon_t^2 \)

Reading 7: Machine Learning

LASSO:
1. Penalty term (when \( \lambda > 0 \)) = \( \lambda \sum_{k=1}^{K} |\hat{\beta}_k| \)
2. \( \sum_{t=1}^{T} (Y_t - \hat{Y}_t)^2 + \lambda \sum_{k=1}^{K} |\hat{\beta}_k| \)
3. When \( \lambda = 0 \), LASSO penalized regression = OLS regression

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1. \( X_{i(normalized)} = \frac{X_i - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}} \)

where \( X_i \) = value of observation

Performance Metrics:

2. Accuracy = \( (\text{TP} + \text{TN})/(\text{TP} + \text{FP} + \text{TN} + \text{FN}) \)
   F1 score = \( (2*\text{P}*\text{R})/ (\text{P} + \text{R}) \)

3. Receiver Operating Characteristic (ROC):
   False positive rate (FPR) = \( \text{FP}/(\text{TN} + \text{FP}) \)
   True positive rate (TPR) = \( \text{TP}/(\text{TP} + \text{FN}) \)

4. Root Mean Square Error (RMSE):
\[
\sum_{i=1}^{n} \frac{(\text{Predicted}_i - \text{Actual}_i)^2}{n}
\]

5. Covered interest rate parity:
   \( (1 + i_d) = S_{f/d} \left( 1 + i_f \right) \left( \frac{1}{F_{f/d}} \right) \)
   \( F_{f/d} = S_{f/d} \left( 1 + i_f \left( \frac{\text{Actual}}{360} \right) \right) \)

6. Uncovered Interest Rate Parity:
   \( i_f - \%\Delta S_{f/d} = i_d \)
   \( \%\Delta S_{f/d} = i_f - i_d \)
   Forward premium or discount:
   For one year horizon = \( F_{f/d} - S_{f/d} \)
   \( S_{f/d} \left( 1 + i_d \left( \frac{\text{Actual}}{360} \right) \right) = S_{f/d} (i_f - i_d) \)

7. Forward discount or premium as % of spot rate:
\[
\frac{F_{f/d} - S_{f/d}}{S_{f/d}} \equiv (i_f - i_d)
\]

8. Purchasing Power parity (PPP)
   \( P_t = S_{\text{f/d}} \times P_d \)
   \( S_{\text{f/d}} = P_t / P_d \)

9. Relative version of PPP = \( \%\Delta S_{\text{f/d}} = \pi_t - \pi_d \)

10. Ex ante version of PPP = \( \%\Delta S_{\text{f/d}} = \pi_t - \pi_d \)

11. Real Exchange Rate
\[
q_{\text{f/d}} = \left( \frac{S_{f/d} P_d}{P_f} \right) = S_{f/d} \left( \frac{P_d}{P_f} \right)
\]
\[
q_{f/d} = S_{f/d} \left( \frac{\text{CPI}_d}{\text{CPI}_f} \right)
\]

12. Fisher effect:
   \( i_t = r_t + \pi_t \)
   \( i_d = r_d + \pi_d \)

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Formula Sheet

Level II 2020

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Reading 9: Excerpt from ‘Probabilistic Approaches, Scenario Analysis, Decision Tree & Simulations’

Reading 10: Currency Exchange Rates

1. Bid-offer Spread = Offer price – Bid price
2. Fwd rate = Spot Exchange rate + Forward points
3. Forward premium/discount (in %) = spot exchange rate – (forward points/10,000) – 1
4. To convert spot rate into forward quote:

\[
\text{Forward} = \text{Bid} = \text{Spot} \times \left( 1 + \%\text{discount} \right)
\]

\[
\text{Fwd} = \text{Bid} = \text{Spot} \times \left( 1 + \%\text{discount} \right)
\]

\[
\text{Bid} = \text{Spot} \times \left( 1 + \%\text{premium} \right)
\]

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1. Economic growth = Annual % Δ in real GDP or in real per capita GDP

2. \( P = GDP \left( \frac{E}{GDP} \right) \left( \frac{P}{E} \right) \)

3. Expressing in terms of logarithmic rates:
   - \( (1/T) \% \Delta P = (1/T) \% \Delta GDP + (1/T) \% \Delta (P / E) \)
   - % Δ in stock MV = % Δ in GDP + % Δ in share of earnings (profit) in GDP + % Δ in the P/E multiple

4. A two-factor aggregate production function: \( Y = AF (K, L) \)

5. Cobb-Douglas Production Function = \( F(K, L) = K^a L^{1-a} \)

6. Under Cobb-Douglas production function:
   - Marginal product of capital = MPK = \( \alpha AK \) \( L^{1-a} = \alpha Y / K \)
   - \( \alpha Y / K = r \Rightarrow \frac{r}{\alpha} = r (K) / Y = \text{Capital income} / \text{Output or GDP} \)

7. Output per worker or Average labor productivity \( (Y/L) \) or \( y \):
   - GDP/Labor input = TFP \times \text{capital-to-labor ratio} \times \text{share of capital in GDP}
   - \( Or \ y = Y/L = Ak^a \)

8. Contribution of Capital Deepening = Labor productivity growth rate – TFP

9. Contribution of Improvement in technology = Labor productivity growth rate – Capital Deepening

10. Growth Accounting based on Solow Approach = \( \Delta Y / Y = \Delta A / A + \alpha \Delta K / K + (1 - \alpha) \Delta L / L \)

11. Labor productivity growth accounting equation
   - Growth rate in potential GDP = \( LTg \) rate of labor force + \( LTg \) rate in labor productivity

12. Balanced or Steady State Rate of Growth in Neoclassical Growth Theory:
   - Growth in physical capital stock = \( \Delta K = sY - \delta K \)

13. In the steady state:
   - Growth rate of capital per worker = \( \Delta k / k = \Delta y / y = \Delta A / A + \alpha \Delta k / k = \text{TFP} \frac{1}{1-\alpha} \Rightarrow \text{Steady state growth rate of labor productivity} \)
   - Growth rate of Total output = \( \Delta Y / Y = \text{Growth rate of TFP scaled by labor force share} + \text{Growth rate in the labor force} = \frac{\theta}{1-\alpha} + n \)

14. During the transition to the steady state growth path:
   - Growth rates of output per capita = \( \Delta y / y = \left[ \left( \frac{\theta}{1-\alpha} \right) + \alpha s \left( \frac{Y}{K} - \Psi \right) \right] = \left( \frac{\theta}{1-\alpha} \right) + \alpha s \left( \frac{y}{k - \Psi} \right) \)
   - Capital-to-labor ratio = \( \Delta k / k = \left[ \left( \frac{\theta}{1-\alpha} \right) + \frac{s}{\alpha} \left( \frac{Y}{K} - \Psi \right) \right] = \left( \frac{\theta}{1-\alpha} \right) + \frac{s}{\alpha} \left( \frac{y}{k - \Psi} \right) \)

15. Proportional impact of the saving rate change on the capital-to-labor ratio and per capita income over time:
   - \( k_{\text{new}} = \left[ \left( \frac{Y}{K} \right)_{\text{new}} \right]^{\frac{1}{1-\alpha}} \)
   - \( k_{\text{old}} = \left[ \left( \frac{Y}{K} \right)_{\text{old}} \right]^{\frac{1}{1-\alpha}} \)
   - \( y_{\text{new}} = \left( \frac{k_{\text{new}}}{k_{\text{old}}} \right)^{\alpha} \)
   - \( y_{\text{old}} = \left( \frac{k_{\text{new}}}{k_{\text{old}}} \right) \)

16. Production function in the endogenous growth model = \( y_e = f(k_e) = ck_e \)