Reading 6: Time Value of Money

1. Interest Rate (i)
   - \( i = \text{RF} + \text{Inf P} + \text{Default Risk}\)
   - \( i = \text{Real RF} + \text{Rate} + \text{Inf P}\)
   - \( i \text{ rate as a growth rate} = g = \left( \frac{FV}{PV} \right)^{\frac{1}{N}} - 1 \)

2. PV and FV of CF =
   - \( PV = \frac{FV}{(1+r)^N} \)
   - \( PV \text{ of Perpetuity} = \frac{PMT}{r} \)
   - \( PV \text{ (for more than one Compounding per year)} = PV = FV_N \left( 1 + \frac{r_c}{m} \right)^{-m \times N} \)
   - \( PV \text{ (for Continuous Compounding)} = PV_N = PV \times e^{r_c \times N} \)
   - Solving for \( N \) = \( \frac{LN(PV)}{LN(1+r)} \) (where LN = natural log)

4. Stated & Effective Rates
   - Periodic \( i \) Rate = \( \frac{\text{Stated Ann i Rate}}{\text{No of Compounding Periods in One Year}} \)
   - Effective (or Equivalent) Ann Rate (\( EAR = EFF\% \)) = \( (1 + \text{Periodic i Rate})^N - 1 \)
   - \( EAR \text{ (with Continuous Compounding)} = EAR = e^{r_c} - 1 \)

5. PV & FV of Ordinary Annuity
   - \( PV_{OA} = \sum_{t=1}^{n} \frac{PMT}{(1+r)^t} = \frac{PMT}{r} \left( 1 - \frac{1}{(1+r)^N} \right) \)
   - \( FV_{OA} = \sum_{t=1}^{n} (PMT_t(1+r))^N-t = PMT \left( \frac{(1+r)^N-1}{r} \right) \)
   - \( \text{Size of Annuity Payment} = PMT = \frac{PV}{PV \text{ of Annuity Factor}} \)
   - \( PV \text{ of Annuity Factor} = \frac{1 - \frac{1}{(1+r)^m \times N}}{\frac{r}{m}} \)

6. PV & FV of Annuity Due
   - \( PV_{AD} = PMT \left( 1 - \frac{1}{(1+r)^N} \right) + PMT \text{ at } t = PV_{OA} + PMT \)
   - \( FV_{AD} = PMT \left( \frac{(1+r)^N-1}{r} \right) (1+r) = FV_{OA} \times (1+r) \)

Reading 7: Discounted Cash Flow Applications

1. NPV = \( \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - CF_0 \)
2. IRR (when project’s CFs are perpetuity) = \( \text{NPV} = -I_0 + \frac{CF}{IRR} = 0 \)
3. \( HPR = \left( \frac{P_f - P_o + D_t}{P_o} \right) \)

Formula Sheet

4. \( MMWR = \sum_{t=0}^{N} \frac{CF_t}{(1+IRR)^t} = 0 \) (IRR represents the MWR)
5. TWR:
   - TWR (when no external CF) = \( r_{TWR} = \frac{I_1 - I_0}{I_0} \)
   - TWR (for more than one periods) = \( r_{TWR} = [(1+R_1) \times (1+R_2) \times \ldots (1+R_n)] - 1 \)
   - Annualized TWR (when investment is for more than one year) = \( [(1 + R_1)(1 + R_2 \ldots + (1 + R_n))]^{\frac{1}{N}} - 1 \)
   - TWR (for the year) = \( r_{TWR} = [(1+R_1) \times (1+R_2) \times \ldots (1+R_{365})] - 1 \) where \( R_1 = \frac{MV_f - MV_0}{MV_0} \)
6. Bank Discount Yield = \( BDY = r_{BD} = \frac{360 \text{ Par-Price}}{n \text{ Par}} \) therefore Price = Par \( (1 - \frac{n \times r_{BD}}{360}) \)
7. Holding Period Yield = \( HPY = \frac{(P_f - P_o + D_t)}{P_o} \)
8. Effective Annual Yield = \( EAY = (1 + \frac{HPY}{365})^{365/t} - 1 \) (Rule: \( EAY > BDY \))
9. Money Market Yield (or CD equivalent Yield) \( r_{MM} \):
   - \( r_{MM} = HPY \times \left( \frac{360}{t} \right) \)
   - \( r_{MM} = (r_{BD}) \times \frac{\text{Face Value of the Treasury Bill}}{\text{Purchase Price}} \)
8. Bond Equivalent Yield = BDY = Semiannual Yield × 2

Reading 8: Statistical Concepts & Market Returns

1. Range = Max Value – Min Value

2. Class Interval = \( i \geq \frac{H - L}{k} \) where
   - \( i \) = class interval
   - \( H \) = highest value
   - \( L \) = lowest value, \( k \) = No. of classes.

3. Absolute Frequency = Actual No of Observations (obvs) in a given class interval

4. Relative Frequency = \( \frac{\text{Absolute Frequency}}{\text{Total No of Obvs}} \)

5. Cumulative Absolute Frequency = Add up the Absolute Frequencies

6. Cumulative Relative Frequency = Add up the Relative Frequencies

7. Arithmetic Mean = \( \frac{\text{Sum of obvs in database}}{\text{No. of obvs in the database}} \)

8. Median = Middle No (when observations are arranged in ascending/descending order)

9. Mode = obvs that occurs most frequently in the distribution

10. Weighted Mean = \( \bar{X}_w = \frac{\sum_{i=1}^{n} w_iX_i}{\sum_{i=1}^{n} w_i} \)

11. Geometric Mean = \( GM = \left( X_1 \times X_2 \times \ldots \times X_n \right)^{\frac{1}{n}} \)

12. Harmonic Mean = \( H.M = \bar{X}_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}} \)

13. Population Mean = \( \mu = \frac{\sum_{i=1}^{n} X_i}{n} \) with \( X_i > 0 \) for \( i = 1, 2, \ldots, n \)

14. Sample Mean = \( \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \) where \( n \) = number of observation in the sample

15. Measures of Location:
   - Quartiles = \( \frac{\text{Distribution}}{4} \)
   - Quintiles = \( \frac{\text{Distribution}}{5} \)
   - Deciles = \( \frac{\text{Distribution}}{10} \)
   - Percentiles = \( P_y = (n + 1) \times \frac{y}{100} \)

16. Mean Absolute Deviation = MAD = \( \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n} \)

17. Population Var = \( \sigma^2 = \frac{\sum_{i=1}^{n} (X_i - \mu)^2}{N} \)

18. Population S.D = \( \sigma = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \mu)^2}{N}} \)

19. Sample Var = \( s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1} \)

20. Sample S.D = \( s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}} \)

21. Semi-var = \( \sum_{\text{For all } X_i \leq \bar{X}} (X_i - \bar{X})^2 \)

22. Semi-deviation (Semi S.D) = \( \sqrt{\text{variance}} = \frac{1}{n-1} \sum_{\text{For all } X_i \leq \bar{X}} (X_i - \bar{X})^2 \)

23. Target Semi-var = \( \sum_{\text{For all } X_i \leq B} (X_i - B)^2 \)
   where \( B \) = Target Value

24. Target Semi-Deviation = \( \sqrt{\text{target variance}} = \sqrt{\sum_{\text{For all } X_i \leq B} (X_i - B)^2} \)

25. Coefficient of Variation = \( CV = \frac{s}{\bar{X}} \)
   where \( s \) = sample S.D and \( \bar{X} \) = sample mean

26. Sharpe Ratio = \( \frac{\text{Mean Portfolio R} - \text{Mean RF R}}{\text{S.D of Portfolio R}} \)

27. Excess Kurtosis = Kurtosis – 3
Reading 9: Probability Concepts

1. Empirical Prob of an event E = \( P(E) = \frac{\text{Prob of event } E}{\text{Total Prob}} \)

2. Odds for event E = \( \frac{\text{Prob of } E}{1-\text{Prob of } E} \)

3. Odds against event E = \( \frac{1-\text{Prob of } E}{\text{Prob of } E} \)

4. Conditional Prob of A given that B has occurred = \( P(A|B) = \frac{P(AB)}{P(B)} \) → P(B) ≠ 0.

5. Multiplication Rule (Joint probability that both events will happen):
   \[ P(A \text{ and } B) = P(AB) = P(A|B) \times P(B) \]
   \[ P(B \text{ and } A) = P(B|A) = P(B) \times P(A) \]

6. Addition Rule (Prob that event A or B will occur):
   \[ P(A \text{ or } B) = P(A) + P(B) - P(AB) \]
   \[ P(A \text{ or } B) = P(A) + P(B) \text{ (when events are mutually exclusive because } P(AB) = 0) \]

7. Independent Events:
   - Two events are independent if:
     \[ P(B|A) = P(B) \text{ or if } P(A|B) = P(A) \]
   - Multiplication Rule for two independent events = \( P(A \text{ and } B) = P(AB) = P(A) \times P(B) \)
   - Multiplication Rule for three independent events = \( P(A \text{ and } B \text{ and } C) = P(ABC) = P(A) \times P(B) \times P(C) \)

8. Complement Rule (for an event S) = \( P(S') = 1 \) (where S' is the event not S)

9. Total Probability Rule:
   \[ P(A) = P(AS) + P(AS') = P(A|S) \times P(S) + P(A|S') \times P(S') \]
   \[ P(A) = P(AS_1) + P(AS_2) + \ldots + P(AS_n) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + \ldots + P(A|S_n) \times P(S_n) \]
   (where \( S_1, S_2, \ldots, S_n \) are mutually exclusive and exhaustive scenarios)

10. Expected R = \( E(wR_i) = w_iE(R_i) \)

11. \( \text{Cov}(R_i, R_j) = \sum_{i=1}^{n} (P(R_i - E(R_i))(R_j - E(R_j)) \]
    \[ \text{Cov}(R_i, R_j) = \text{Cov}(R_j, R_i) \]
    \[ \text{Cov}(R_i, R_i) = \sigma^2(R_i) \]

12. Portfolio Var = \( \sigma^2(R_p) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_iw_j\text{Cov}(R_i, R_j) \]
    \[ \sigma^2(R_p) = w_1^2\sigma^2(R_1) + w_2^2\sigma^2(R_2) + w_3^2\sigma^2(R_3) + 2w_1w_2\text{Cov}(R_1, R_2) + 2w_1w_3\text{Cov}(R_1, R_3) + 2w_2w_3\text{Cov}(R_2, R_3) \]

Reading 10: Common Probability Distributions

1. Probability Function (for a binomial random variable) \( p(x) = p(X=x) = \binom{n}{x}p^x(1-p)^{n-x} = \frac{n!}{(n-x)!x!}p^x(1-p)^{n-x} \) (for \( x = 0, 1, 2, \ldots, n \))
• x = success out of n trials
• n-x = failures out of n trials
• p = probability of success
• 1-p = probability of failure
• n = no of trials.

2. Probability Density Function (pdf) = f(x) = \[ \begin{cases} \frac{1}{b-a} & for \ a \leq x \leq b \\ 0 & \end{cases} \]
   \[ F(x) = \frac{x-a}{b-a} \] for a < x < b

3. Normal Density Func = \[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right) \] for \(-\infty < x < +\infty\)

4. Estimations by using Normal Distribution:
   • Approximately 50% of all obsv fall in the interval \(\mu \pm \frac{2}{3}\sigma\)
   • Approx 68% of all obvs fall in the interval \(\mu \pm \sigma\)
   • Approx 95% of all obvs fall in the interval \(\mu \pm 2\sigma\)
   • Approx 99% of all obvs fall in the interval \(\mu \pm 3\sigma\)
   • More precise intervals for 95% of the obvs are \(\mu \pm 1.96\sigma\) and for 99% of the observations are \(\mu \pm 2.58\sigma\).

5. Z-Score (how many S.Ds away from the mean the point x lies) \(z = \frac{x-\mu}{\sigma}\) (when X is normally distributed)

6. Roy’s Safety-Frist Criterion = SF Ratio = \[ \frac{\left| E(R_p) - R_f \right|}{\sigma_p} \]

7. Sharpe Ratio = \[ \frac{\left| E(R_p) - R_f \right|}{\sigma_p} \]

8. Value at Risk = VAR = Minimum $ loss expected over a specified period at a specified prob level.

9. Mean (\(\mu_L\)) of a lognormal random variable = \(\exp(\mu + 0.50\sigma^2)\)

10. Variance (\(\sigma_L^2\)) of a lognormal random variable = \(\exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]\).

11. Log Normal Price = \(S_T = S_0 \exp(\mu_T\cdot T)\)

12. Price relative = End price / Beg price = \(S_{T+1} / S_1 = 1 + R_{T+1}\)

   \[ R_{t+1} = \text{holding period return on the stock from t to t + 1} \]

13. Continuously compounded return associated with a holding period from t to t + 1:

   \[ r_{t+1} = \text{ln}(1 + \text{holding period return}) \] or

   \[ r_{t+1} = \text{ln}(\text{price relative}) = \text{ln}(S_{t+1} / S_t) = \text{ln}(1 + R_{t+1}) \]

14. Continuously compounded return associated with a holding period from 0 to T:

   \[ R_{0,T} = \ln(\frac{S_T}{S_0}) \text{ or } R_{0,T} = R_{T-1,T} + R_{T-2,T-1} + \cdots + R_{0,1} \]

   Where,

   \[ R_{T-1,T} = \text{One-period continuously compounded returns} \]

15. When one-period continuously compounded returns (i.e. \(R_{0,1}\)) are IID random variables.

   \[ E(R_{0,T}) = E(R_{T-1,T}) + E(R_{T-2,T-1}) + \cdots + E(R_{0,1}) = \mu_T \text{ And} \]

   \[ \text{Variance} = \sigma^2(R_{0,T}) = \sigma^2 T \]

   \[ \text{S.D.} = \sigma(R_{0,1}) = \sigma \sqrt{T} \]

16. Annualized volatility = sample S.D. of one period continuously compounded returns \(\times \sqrt{T}\)

**Reading 11: Sampling and Estimation**

1. Var of the distribution of the sample mean = \[ \frac{\sigma^2}{n} \]

2. S.D of the distribution of the sample mean = \[ \sqrt{\frac{\sigma^2}{n}} \]
3. Standard Error of the sample mean:
   - When the population S.D (σ) is known
     \[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \]
   - When the population S.D (σ) is not known
     \[ s_{\bar{x}} = \frac{s}{\sqrt{n}} \]
     where \( s \) = sample S.D estimate of \( s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \)

4. Finite Population Correction Factor = fpc
   \[ \text{fpc} = \sqrt{\frac{N-n}{N-1}} \]
   where \( N \) = population

5. New Adjusted Estimate of Standard Error
   \[ \text{Adjusted Estimate of S.D} = \sqrt{\frac{S^2}{1 + \text{fpc}}} \]
   \( \text{fpc} = \sqrt{\frac{N-n}{N-1}} \)

6. Construction of Confidence Interval (CI) = Point estimate ± (Reliability factor × Standard error)
   - CI for normally distributed population with known variance = \( \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \)
   - CI for normally distributed population with unknown variance = \( \bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} \)
     where \( s \) = sample S.D.

7. Student’s t distribution
   \[ \mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} \]

8. Z-ratio
   \[ Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \]

9. t-ratio
   \[ t = \frac{\bar{x} - \mu}{s / \sqrt{n}} \]

Reading 12: Hypothesis Testing

1. Test Statistic =
   \[ \frac{\text{Sample Statistic} - \text{Hypothesized Value of pop parameter}}{\text{standard error of sample statistic}} \]

2. Power of Test = 1 - Prob of Type II Error

3. \( z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \) (when sample size is large or small but pop S.D is known)

4. \( z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \) (when sample size is large but pop S.D is unknown where \( s \) is sample S.D)

5. \( t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \) (when sample size is large or small and pop S.D is unknown and pop sampled is normally or approximately normally distributed)

6. Test Statistic for a test of diff b/w two pop means (normally distributed, pop var unknown but assumed equal)
   \[ t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p} \]
   \( s_p^2 = \frac{(s_1^2 + s_2^2)^{1/2}}{(n_1 + n_2)} \)

7. Test Statistic for a test of diff b/w two pop means (normally distributed, unequal and unknown pop var unknown)
   \[ t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p} \]
   \[ s_p^2 = \frac{(s_1^2 - s_2^2)^{1/2}}{(n_1 - 1) + (n_2 - 1)} \]

8. Test Statistic for a test of mean differences (normally distributed populations, unknown population variances)
   - \( t = \frac{\bar{d} - \mu_d}{S_d} \)
   - sample mean difference \( \bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i \)
   - sample variance \( S_d^2 = \frac{\sum_{i=1}^{n} (d_i - \bar{d})^2}{n-1} \)
   - sample S.D = \( S_d \)
8. Chi Square Test Statistic (for test concerning the value of a normal population variance) \( X^2 = \frac{(n-1)S^2}{\sigma_0^2} \) where \( n-1 \) = df and \( S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1} \)

9. Chi Square Confidence Interval for variance
   - Lower limit = \( L = \frac{(n-1)S^2}{X_{n/2}^2} \)
   - Upper limit = \( U = \frac{(n-1)S^2}{X_{1-n/2}^2} \)

10. F-test (test concerning differences between variances of two normally distributed populations) \( F = \frac{S_1^2}{S_2^2} \)
    - \( S_1^2 \) = 1st sample var with \( n_1 \) obs
    - \( S_2^2 \) = 2nd sample var with \( n_2 \) obs
    - \( df_1 = n_1 - 1 \) numerator df
    - \( df_2 = n_2 - 1 \) denominator df

11. Relation between Chi Square and F-distribution = \( F = \frac{X_1^2/m}{X_2^2/n} \) where:
    - \( X_1^2 \) is one chi square random variable with one m degrees of freedom
    - \( X_2^2 \) is another chi square random variable with one n degrees of freedom

12. Spearman Rank Correlation = \( r_s \)
    - Formula: \( t = \frac{(n-2)^{1/2}r_s}{(1-r^2)^{1/2}} \)

Reading 13: Technical Analysis

1. Relative Strength Analysis = \( \frac{\text{Price of asset}}{\text{Price of the Benchmark Asset}} \)

2. Price Target for the
   - Head and Shoulders = Neckline – (Head – Neckline)
   - Inverse Head and Shoulders = Neckline + (Neckline– Head)

3. Simple Moving Average = \( \frac{P_1+P_2+P_3+\ldots+P_n}{N} \)

4. Momentum Oscillator (or Rate of Change Oscillator ROC):
   - Momentum Oscillator Value = \( (V-V_s) \times 100 \)

5. Relative Strength Index = \( \text{RSI} = 100 - \frac{\text{H14} - \text{L14}}{\text{H14} - \text{L14}} \)

6. Stochastic Oscillator (composed of two lines %K and %D):
   - %K = \( 100 \left( \frac{C-L14}{H14-L14} \right) \)
   - %D = Average of the last three %K values calculated daily.

7. Put/Call Ratio (Type of Sentiment Indicators) = \( \frac{\text{Volume of Put Options Traded}}{\text{Volume of Call Options Traded}} \)

8. Short Interest Ratio (Type of Sentiment Indicators) = \( \frac{\text{Short Interest}}{\text{Average Daily Trading Volume}} \)

9. Arms Index TRIN i.e. Trading Index (Type of Flow of funds Indicator) = \( \frac{\text{No of Advan Issues} - \text{No of Declin Issues}}{\text{Volume of Advan Issues} - \text{Volume of Declin Issues}} \)
Reading 14: Topics in Demand & Supply Analysis

1. \( Q^d = f(P_x, I, P_y) \)
   
   Price Elasticity of Demand = \( E_{P_x}^d = \frac{\% \Delta \text{ in Quantity Demanded}}{\% \Delta \text{ in Price}} = \frac{\Delta Q^d}{Q^d} \times \frac{P_x}{\Delta P_x} \)

2. Income Elasticity of Demand = \( E_I^d = \frac{\% \Delta \text{ in Quantity Demanded}}{\% \Delta \text{ in Income}} = \frac{\Delta Q^d}{Q^d} \times \frac{I}{\Delta I} \)

3. Cross Elasticity = \( E_{P_y}^d = \frac{\% \Delta \text{ in Quantity Demanded of Good X}}{\% \Delta \text{ in Price of Good Y}} = \frac{\Delta Q^d}{Q^d} \times \frac{P_y}{\Delta P_y} \)

4. Total cost of production = \( TC = (w)(L) + (r)(K) \)

5. TR = \( (P)(Q) \)

6. MR = \( \frac{\Delta TR}{\Delta Q} \)

7. MR = \( \frac{(P)(\Delta Q)}{\Delta Q} + \frac{(Q)(\Delta P)}{\Delta Q} = P + Q \times \frac{\Delta P}{\Delta Q} \)

Reading 15: The firm & Market Structures

1. In perfect competition, Marginal revenue = Avg. Revenue = Price = Demand

2. Marginal Revenue = Price \times \left(1 - \frac{1}{\text{Price Elasticity of Demand}}\right)

3. Concentration Ratio = \( \frac{\text{Sum of sales values of the largest 10 firms}}{\text{Total Market Sales}} \)

4. Herfindahl-Hirschman Index = \( \text{Sum of the squares of the market shares of the top N companies in an industry} \)

Reading 16: Aggregate Output, Prices & Economic Growth

1. Nominal GDP = Prices in year \( t \) \times Quantity produced in year \( t \)

2. Real GDP = Prices in the base year \times Quantity produced in year \( t \)

3. Implicit price deflator for GDP or GDP deflator = \( \frac{\text{value of current yr output at current yr prices}}{\text{value of current yr output at base yr prices}} \times 100 \)

4. Real GDP = \([\text{Nominal GDP} / \text{GDP deflator}] \div 100\]

5. GDP deflator = \( \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100 \)

6. GDP = Consumer spending on final good & services + Gross private domestic invst + Govt. spending on final goods & services + Govt. gross fixed invst + Exp – Imp + Statistical discrepancy

7. Net Taxes = Taxes – Transfer payments

8. GDP = National income + Capital consumption allowance + Statistical discrepancy

9. National Income = Compensation of employees + Corp & Govt enterprise profits before taxes + Interest income + unincorporated business net income + rent + indirect business taxes less subsidies

10. Total Amount Earned by Capital = Profit + Capital Consumption Allowance

11. PI = National income – Indirect business taxes – Corp income taxes – Undistributed Corp profits + Transfer payments

12. Personal disposable income (PDI) = Personal income – Personal taxes OR GDP (Y) + Transfer payments (F) – (R/E + Depreciation) – direct and indirect taxes (R)

13. Business Saving = R/E + Depreciation

14. Household saving = PDI - Consumption expenditures - Interest paid by consumers to business - Personal transfer payments to foreigners

15. Business sector saving = Undistributed corporate profits + Capital consumption allowance