Reading 7: Correlation & Regression

1. Sample Cov (X, Y) $= \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{n-1}$

2. Correlation Coefficient = $r_{XY} = \frac{cov_{XY}}{(s_X)(s_Y)}$ or
$r = \frac{cov(X,Y)}{\sqrt{var(X)}\sqrt{var(Y)}}$

3. t-test (for normally distributed variables) =
$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ t distribution with (n – 2) deg. of freedom

4. Linear Regression = $Y_i = b_0 + b_1X_i + \varepsilon_i$,
   - Intercept (b_0) = $\overline{b_0} = \bar{y} - b_1\bar{x}$ =
   - Slope or regression coefficient = $b_1 = \frac{cov(x,y)}{var(x)}$ or $= \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

5. Standard Error of Estimate SEE = $S_E = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum_{i=1}^{n}(y_i-\hat{y})^2}{n-k-1}}$

6. Coefficient of Determination (R^2) =
$= \frac{SST-SSE}{SST} = \frac{RSS}{SST}$ where, $0 \le R^2 \le 1$
(for single independent variable $R^2 = r^2$)

7. SST = SSE + SSR(or RSS)

8. Hypothesis Testing:
   - Null and Alternative hypotheses
   - H_0: $b_1 = 0$ (no linear relationship)
   - H_1: $b_1 \neq 0$ (linear relationship does exist)
   - Test statistic = $t = \frac{\hat{b}_1 - b_1}{s_{b_1}}$
   - Confidence Interval = $b_1 \pm t_c s_{b_1}$

9. ANOVA (Analysis of variance) =

ANOVA	SS	MSS	F
Regression df = k	$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$	$\frac{SSR}{k}$	$\frac{SSR/k}{SSE/(n-k-1)}$
Error df = n-k-1	$SSE = \sum_{i=1}^{n}(y_i - \hat{y})^2$	$\frac{SSE}{n-k-1}$	
Total df = n-1	$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$		

Source of Variability	DoF	Sum of Squares	Mean Sum of Squares
Regression (Explained)	1	RSS	MSR = RSS/1
Error (Unexplained)	n-2	SSE	MSE = SSE/n-2
Total	n-1	SST= RSS + SSE	

10. F-Statistic or F-Test = $\frac{MSR}{MSE} = \frac{(\frac{RSS}{k})}{(\frac{SSE}{n-k-1})}$
(df numerator = k = 1)
(df denominator = n – k – 1 = n – 2)

11. Prediction Intervals = $\hat{Y} \pm t_c s_f$
$where\ s_f^2 = s^2\left[1 + \frac{1}{n} + \frac{(X-\bar{X})^2}{(n-1)s_X^2}\right]$ and
$s_f = \sqrt{s_f^2}$

Reading 8: Multiple Regression & Issues in Regression Analysis

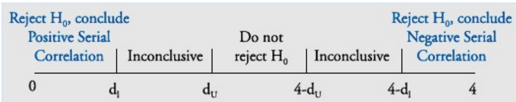
1. $Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \ldots + b_kX_{ki} + \varepsilon_i, i = 1, 2, \ldots n$

2. Prediction equation = $\hat{Y}_i = \hat{b}_0 + \hat{b}_1X_{1i} + \hat{b}_2X_{2i} + \ldots + \hat{b}_kX_{ki} + \varepsilon_i, i$

3. Adjusted $R^2 = \bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2)$

4. Breusch–Pagan test
   - H_0 = No conditional Heteroskedasticity exists
   - H_A = Conditional Heteroskedasticity exists
   - Test statistic = $n \times R^2_{residuals}$

5. Durbin-Waston Test = $DW = \frac{\sum_{t=2}^{T}(\hat{\varepsilon}_t - \hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^{T}\hat{\varepsilon}_t^2}$
   - For Large Sample size DW Statistic (d) = $d \approx 2(1 - r)$



Reading 9: Time Series Analysis

1. Linear Trend Models = $y_t = b_0 + b_1 t + \varepsilon_t$
   - Predicted/fitted value of y_t in period $(T + 1) = \hat{y}_{t+1} = \hat{b}_0 + \hat{b}_1(T+1)$
2. Log-Linear Trend Models = $y_t = e^{b_0 + b_1 t}$
3. Autoregressive Time-Series Models:
   - First order autoregressive AR (1) = $x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$
   - pth-order autoregressive AR (p) = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \ldots + b_p x_{t-p} + \varepsilon_t$
4. Mean reverting level of $x_t = \dfrac{b_0}{1 - b_1}$
5. Chain Rule of Forecasting:
   - One-period ahead forecast = $\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1 x_t$
   - Two-period ahead forecast= $\hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1 x_{t+1}$
6. Random Walks and Unit Roots:
   - Random Walk without drift = $x_t = x_{t-1} + \varepsilon_t$ where, $b_0 = 0$ and $b_1 = 1$.
   - Correcting Random Walk = $y_t = x_t - x_{t-1}$
   - Random walk with a drift = $x_t = b_0 + x_{t-1} + \varepsilon_t$ where, $b_0 \neq 0$ and $b_1 = 1$
   - By taking first difference $y_t = x_t - x_{t-1} = b_0 + \varepsilon_t$
7. Using Dickey-Fuller Test = $x_t - x_{t-1} = b_0 + (b_1 - 1)x_{t-1} + \varepsilon_t$
8. Smoothing Past Values with n-Period Moving Average =
$$\frac{x_t + x_{t-1} + x_{t-2} + \ldots + x_{t-(n-1)}}{n}$$
9. Correcting Seasonality in Time Series Models:
   - For quarterly data = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-4} + \varepsilon_t$
   - For monthly data = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-12} + \varepsilon_t$
10. ARCH model = $\hat{\varepsilon}^2_t = \alpha_0 + \alpha_1 \hat{\varepsilon}^2_{t-1} + \mu_t$ where μ_t is an error term
    - Predicting variance of errors in period t+1 = $\hat{\sigma}^2_{t+1} = \hat{\alpha}_0 + \alpha_1 \hat{\varepsilon}^2_t$

Reading 10: Excerpt from 'Probabilistic Approaches , Scenario Analysis, Decision Tree & Simulations'

Reading 11: Currency Exchange Rates

1. Bid-offer Spread = Offer price – Bid price
2. Fwd rate = Spot Exchange rate + $\dfrac{\text{Forward points}}{10{,}000}$
3. Forward premium/discount (in %) = $\dfrac{\text{spot exchange rate} - (\text{forward points}/10{,}000)}{\text{spot exchange rate}} - 1$
4. To convert spot rate into forward quote:
   - Spot exchange rate × (1 + % premium)
   - Spot exchange rate × (1 - % discount)
5. Covered interest rate parity:
   - $(1 + i_d) = S_{f/d}(1 + i_f)\left(\frac{1}{F_{f/d}}\right)$
   - $F_{f/d} = S_{f/d}\left(\frac{1+i_f}{(1+i_d)}\right)$
   - Using day count convention:
$$\left(1 + i_d\left[\frac{Actual}{360}\right]\right) = S_{f/d}\left(1 + i_f\left[\frac{Actual}{360}\right]\right)\left(\frac{1}{F_{f/d}}\right)$$
   - $F_{f/d} = S_{f/d}\left(\dfrac{1 + i_f\left[\frac{Actual}{360}\right]}{1 + i_d\left[\frac{Actual}{360}\right]}\right)$
6. Uncovered Interest Rate Parity :
   - $i_f - \%\Delta S^e_{f/d} = i_d$

- $$\%\Delta S^{e}_{f/d} = i_f - i_d$$
- Forward premium or discount:
- For one year horizon =

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{i_f - i_d}{1+i_d}\right) \cong S_{f/d}(i_f - i_d)$$

- Using day count convention:

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{\left[\frac{Actual}{360}\right]}{1+i_d\left[\frac{Actual}{360}\right]}\right)(i_f - i_d)$$

7. Forward discount or premium as % of spot rate:

$$\frac{F_{f/d} - S_{f/d}}{S_{f/d}} \cong (i_f - i_d)$$

If uncovered interest rate parity holds

- $$= \frac{F_{f/d} - S_{f/d}}{S_{f/d}} = \%\Delta S^{e}_{f/d} \cong (i_f - i_d)$$

8. Purchasing Power parity (PPP)
   - $P_f = S_{f/d} \times P_d$
   - $S_{f/d} = P_f / P_d$

9. Relative version of PPP = $\%\Delta S_{f/d} = \pi_f - \pi_d$

10. Ex ante version of PPP = $\%\Delta S^{e}_{f/d} = \pi^{e}_{f} - \pi^{e}_{d}$

11. Real Exchange Rate

$$q_{f/d} = \left(\frac{S_{f/d} P_d}{P_f}\right) = S_{f/d}\left(\frac{P_d}{P_f}\right)$$

or

$$q_{f/d} = S_{f/d}\left(\frac{CPI_d}{CPI_f}\right)$$

12. Fisher effect:
    - $i_d = r_d + \pi^{\varepsilon}_{d}$
    - $i_f = r_f + \pi^{\varepsilon}_{f}$
    - $i_f - i_d = (r_f - r_d) + (\pi^{\varepsilon}_{f} - \pi^{\varepsilon}_{d})$
    - $(r_f - r_d) = (i_f - i_d) - (\pi^{\varepsilon}_{f} - \pi^{\varepsilon}_{d})$

Reading 12: Economic Growth & The Investment Decision

1. Economic growth = Annual % Δ in real GDP or in real per capita GDP

2. $P = GDP\left(\frac{E}{GDP}\right)\left(\frac{P}{E}\right)$

3. Expressing in terms of logarithmic rates:
   - (1/T) % ΔP = (1/T) % ΔGDP + (1/T) %Δ (E / GDP) + (1/T) % Δ(P / E)
   - % Δ in stock MV = % Δ in GDP + % Δ in share of earnings (profit) in GDP + % Δ in the price-to-earnings multiple

4. A two-factor aggregate production function: Y = AF (K, L)

5. Cobb-Douglas Production Function = F (K, L) = $K^{\alpha} L^{1-\alpha}$

6. Under the Cobb-Douglas production function:
   - Marginal product of capital = MPK = $\alpha AK^{\alpha-1} L^{1-\alpha} = \alpha Y/K$
   - $\alpha Y/K = r$ ➔ $\alpha = r(K)/Y$ = Capital income / Output or GDP

7. Output per worker or Average labor productivity (Y/L or y):
   - GDP/Labor input = TFP × capital-to-labor ratio × share of capital in GDP
   - *Or* $y = Y/L = Ak^{\alpha}$

8. Contribution of Capital Deepening = Labor productivity growth rate – TFP

9. Contribution of Improvement in technology = Labor productivity growth rate – Capital Deepening

10. Growth Accounting based on Solow Approach = $\Delta Y/Y = \Delta A/A + \alpha\, \Delta K/K + (1-\alpha)\, \Delta L/L$

11. Labor productivity growth accounting equation
    - Growth rate in potential GDP = LT g rate of labor force + LT g rate in labor productivity

12. Balanced or Steady State Rate of Growth in Neoclassical Growth Theory:
    - Growth in physical capital stock = $\Delta K = sY - \delta K$

13. In the steady state:
    - Growth rate of capital per worker = $\Delta k / k = \Delta y / y = \Delta A / A + \alpha \Delta k / k = \frac{TFP}{1-\alpha}$ ➔ Steady state growth rate of labor productivity
    - Growth rate of Total output = $\Delta Y / Y$ = Growth rate of TFP scaled by labor force share + Growth rate in the labor force = $\frac{\theta}{1-\alpha} + n$
    - Steady state Output-to-capital ratio = $\frac{Y}{K} = \left(\frac{1}{s}\right)\left[\left(\frac{\theta}{1-\alpha}\right) + \delta + n\right] = \Psi$
    - Gross investment per worker = $\left[\left(\frac{\theta}{1-\alpha}\right) + \delta + n\right]k$
    - Slope of straight line = $[\delta + n + \theta / (1 - \alpha)]$

14. During the transition to the steady state growth path:
    - Growth rates of output per capita = $\Delta y / y = \left[\left(\frac{\theta}{1-\alpha}\right) + \alpha s\left(\frac{Y}{K} - \Psi\right)\right] = \left(\frac{\theta}{1-\alpha}\right) + \alpha s\,(y/k - \Psi)$
    - Capital-to-labor ratio = $\Delta k / k = \left[\left(\frac{\theta}{1-\alpha}\right) + s\left(\frac{Y}{K} - \Psi\right)\right] = \left(\frac{\theta}{1-\alpha}\right) + s\,(y/k - \Psi)$

15. Proportional impact of the saving rate change on the capital-to-labor ratio and per capita income over time:
    - $$\frac{k_{new}}{k_{old}} = \left[\frac{\left(\frac{Y}{K}\right)_{new}}{\left(\frac{Y}{K}\right)_{old}}\right]^{\frac{1}{\alpha-1}}$$
    - $$\frac{y_{new}}{y_{old}} = \left[\frac{k_{new}}{k_{old}}\right]^{\alpha}$$

16. Production function in the endogenous growth model = $y_e = f(k_e) = ck_e$
    - Growth rate of output per capita = $\Delta y_e/y_e = \Delta k_e/k_e = sc - \delta - n$

Reading 13: Economics of Regulation