Reading 6: Time Value of Money

1. Interest Rate (i)
   - i = Rf + Inf P + Default Risk P + Liquidity P + Maturity P
   - Nominal Rf i rate = Real Rf i Rate + Inf P
   - i rate as a growth rate = g = $\left(\frac{FV_N}{PV}\right)^{\frac{1}{N}}$-1

2. PV and FV of CF =
   - PV = $\frac{FV}{(1+r)^N}$
   - PV of Perpetuity = $\frac{PMT}{r}$
   - PV (for more than one Compounding per year) = PV= $FV_N\left(1+\frac{r_s}{m}\right)^{-m\times N}$ $where\ r_s = stated\ ann\ i - rate$
   - $FV_N = PV(1+r)^N$
   - FV (for more than one Compounding per year) = $FV_N = \left(1+\frac{r_s}{m}\right)^{m\times N}$
   - FV (for Continuous Compounding) = $FV_N = PVe^{r_s\times N}$
   - Solving for N = $\frac{LN\left(\frac{FV}{PV}\right)}{LN(1+r)}$ (where LN = natural log)

4. Stated & Effective Rates
   - Periodic i Rate = $\frac{Stated\ Ann\ i\ Rate}{No\ of\ Compounding\ Periods\ in\ One\ Year}$
   - Effective (or Equivalent) Ann Rate (EAR = EFF%) = $(1+Periodic\ i\ Rate)^m - 1$
   - EAR (with Continuous Compounding) = EAR = $e^{r_s} - 1$

5. PV & FV of Ordinary Annuity
   - $PV_{OA} = \sum_{t=1}^{n}\frac{PMT}{(1+r)^t} = PMT\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$
   - $FV_{OA} = \sum_{t=1}^{n}\left(PMT_t(1+r)\right)^{N-t} = PMT\left[\frac{(1+r)^N-1}{r}\right]$
   - Size of Annuity Payment = PMT = $\frac{PV}{PV\ of\ Annuity\ Factor}$
   - PV of Annuity Factor = $\frac{1-\frac{1}{\left[1+\left(\frac{rs}{m}\right)\right]^{m\times N}}}{\frac{rs}{m}}$

6. PV & FV of Annuity Due
   - $PV_{AD} = PMT\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$ + PMT at t = PV_{OA} + PMT
   - $FV_{AD} = PMT\left[\frac{(1+r)^N-1}{r}\right](1+r)$ = $FV_{OA}\times(1+r)$

Reading 7: Discounted Cash Flow Applications

1. NPV = $\sum_{t=1}^{n}\frac{CF_t}{(1+r)^t} - cf_0$

2. IRR (when project's CFs are perpetuity) = NPV = - I_O + $\frac{\overline{CF}}{IRR}$ = 0

3. HPR = $\frac{(P_1 - P_0 + D_1)}{P_0}$

4. MMWR = $\sum_{i=0}^{T}\frac{CF}{(1+IRR)^t} = 0$ (IRR represents the MWR)

5. TWR:
   - TWR (when no external CF) = r_{TWR} = HPR = $r_t = \frac{MV_1 - MV_0}{MV_0}$
   - TWR (for more than one periods) = $r_{TWR} = [(1+r_{t,1})\times(1+r_{t,2})\times\ldots(1+r_{t,n})] - 1$
   - Annualized TWR (when investment is for more than one year) $= \left[(1+R_1)(1+R_2 \ldots + (1+R_n))\right]^{\frac{1}{n}} - 1$
   - TWR (for the year) = $r_{TWR} = [(1+R_1)\times(1+R_2)\times\ldots(1+R_{365})] - 1$ where $R_1 = \frac{MV_1 - MV_0}{MV_0}$

6. Bank Discount Yield = BDY = r_{BD} = $\frac{360}{n}\ \frac{Par-Price}{Par}$ therefore Price = Par $\left(1 - \frac{n\times r_{BD}}{360}\right)$

7. Holding Period Yield = HPY = $\frac{(P_1 - P_0 + D_1)}{P_0}$

8. Effective Annual Yield = EAY = $(1+HPY)^{365/t} - 1$ (Rule: EAY > BDY)

9. Money Market Yield (or CD equivalent Yield) r_{MM}:
   - r_{MM} = HPY × $\left(\frac{360}{t}\right)$
   - $r_{MM} = (r_{BD})\times\frac{Face\ Value\ of\ the\ Treasury\ Bill}{Purchase\ Price}$

- $r_{MM} = \frac{360\,(r_{BD})}{360-(t)(r_{BD})}$ (Rule: $r_{MM} > r_{BD}$)

10. Bond Equivalent Yield = BDY = Semiannual Yield × 2

Reading 8: Statistical Concepts & Market Returns

1. Range = Max Value – Min Value

2. Class Interval = $i \geq \frac{H-L}{k}$ where
   - i = class interval
   - H = highest value
   - L = lowest value, k = No. of classes.

3. Absolute Frequency = Actual No of Observations (obvs) in a given class interval

4. Relative Frequency = $\frac{Absolute\ Frequency}{Total\ No\ of\ Obvs}$

5. Cumulative Absolute Frequency = Add up the Absolute Frequencies

6. Cumulative Relative Frequency = Add up the Relative Frequencies

7. Arithmetic Mean = $\frac{Sum\ of\ obvs\ in\ database}{No.of\ obvs\ in\ the\ database}$

8. Median = Middle No (when observations are arranged in ascending/descending order)
   - For Even no of obvs locate median at $\frac{n}{2}$
   - For Odd no. of obvs locate median at $\frac{n+1}{2}$

9. Mode = obvs that occurs most frequently in the distribution

10. Weighted Mean = $\overline{X_w} = \sum_{i=1}^{n} w_i X_i$ = ($w_1X_1 + w_2X_2 + \ldots + w_nX_n$)

11. Geometric Mean = GM = $\sqrt[n]{X_1 X_2 \ldots X_n}$ with $X_i \geq 0$ for i = 1,2,…n.

12. Harmonic Mean = H.M = $\overline{X_H} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{X_i}\right)}$

13. Population Mean = $\mu = \frac{\sum_i^n X_i}{N}$ with $X_i > 0$ for i = 1,2,.,.,n.

14. Sample Mean = $\bar{X} = \frac{\sum_i^n X_i}{n}$ where n = number of observation in the sample

15. Measures of Location:
   - Quartiles = $\frac{Distribution}{4}$
   - Quintiles = $\frac{Distribution}{5}$
   - Deciles = $\frac{Distribution}{10}$,
   - Percentiles = $L_y = (n+1)\frac{y}{100}$

16. Mean Absolute Deviation = MAD = $\frac{\sum_{i=1}^{n}|X_t-\bar{X}|}{n}$

17. Population Var = $\sigma^2 = \frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$

18. Population S.D = $\sqrt{\sigma^2} = \sqrt{\frac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}}$

19. Sample Var = $s^2 = \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$

20. Sample S.D = s = $\sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$

21. Semi-var = $\sum_{For\ all\ X_i \leq \bar{X}} \frac{(X_i-\bar{X})^2}{n-1}$

22. Semi-deviation (Semi S.D) = $\sqrt{semivariance} = \sqrt{\sum_{For\ all\ X_i \leq \bar{X}} \frac{(X_i-\bar{X})^2}{n-1}}$

23. Target Semi-var = $\sum_{For\ all\ X_i \leq B} \frac{(X_i-B)^2}{n-1}$ where B = Target Value

24. Target Semi-Deviation = $\sqrt{target\ semivariance} = \sqrt{\sum_{For\ all\ X_i \leq B} \frac{(X_i-B)^2}{n-1}}$

25. Coefficient of Variation = CV = $\left(\frac{s}{\bar{X}}\right)$ where s= sample S.D and $\bar{X}$ = sample mean

26. Sharpe Ratio = $\frac{Mean\ Portfolio\ R - Mean\ Rf\ R}{S.D\ of\ Portfolio\ R}$

27. Excess Kurtosis = Kurtosis – 3

28. Geometric Mean R ≈ $Arithmetic\ Mean\ R - \frac{Variance\ of\ R}{2}$

Reading 9: Probability Concepts

1. Empirical Prob of an event E = P(E) = $\frac{Prob\ of\ event\ E}{Total\ Prob}$

2. Odds for event E = $\frac{Prob\ of\ E}{1-Prob\ of\ E}$

3. Odds against event E = $\frac{1-Prob\ of\ E}{Prob\ of\ E}$

4. Conditional Prob of A given that B has occurred = P(A|B) = $\frac{P(AB)}{P(B)}$ → P(B) ≠ 0.

5. Multiplication Rule (Joint probability that both events will happen):

   P(A and B) = P(AB) = P(A|B) × P(B)
   P(B and A) = P(BA) = P(B|A) × P(A)

6. Addition Rule (Prob that event A or B will occur):

   P(A or B) = P(A) + P(B) – P(AB)
   P(A or B) = P(A) + P(B) (when events are mutually exclusive because P(AB) = 0)

7. Independent Events:
   - Two events are independent if: P(B|A) = P(B) or if P(A|B) = P(A)
   - Multiplication Rule for two independent events = P(A & B) = P(AB) = P(A)× P(B)
   - Multiplication Rule for three independent events = P(A and B and C) = P(ABC) = P(A) × P(B) × P(C)

8. Complement Rule (for an event S) = P(S) + P(S^C) = 1 (where S^C is the event not S)

9. Total Probability Rule:
   P(A) = P(AS) + P(AS^C) = P(A|S)×P(S) + P(A|S^C)×P(S^C)
   P(A) = P(AS_1) + P(AS_2) +….+ P(AS_n) = P(A|S_1)×P(S_1) + P(A|S_2)×P(S_2)… P(A|S_n)×P(S_n)

   (where S_1, S_2, …,S_n are mutually exclusive and exhaustive scenarios)

10. Expected R = E(w_iR_i) = w_iE(R_i)

11. Cov (R_i R_j) = $\sum_{i=1}^{n}\left(p(R_i - ER_i)\right)\left(R_j - ER_j\right)$
    Cov (R_i R_j) = Cov (R_j R_i)
    Cov (R, R) = σ^2 (R)

12. Portfolio Var = σ^2 (R_p) = $\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov(R_i R_j)$
    σ^2 (R_p) = $w_1^2\sigma^2(R_1) + w_2^2\sigma^2(R_2) + w_3^2\sigma^2(R_3) + 2w_1w_2Cov(R_1,R_2) + 2w_1w_3Cov(R_1,R_3) + 2w_2w_3Cov(R_2,R_3)$

13. Standard Deviation (S.D) = $\sqrt{w_1^2R_i + w_2^2R_2 + w_3^2R_3}$

14. Correlation (b/w two random variables R_i, R_j) = $\rho(R_iR_j) = \frac{Cov\ (R_iR_j)}{\sigma R_i \times \sigma R_j}$

15. Bayes' Formula = $P(Event|New\ Information) = \frac{P(New\ Information|Event)}{P(New\ Information)} \times P(Prior\ prob.of\ Event)$

16. Multiplication Rule of Counting = n factorial = $n!$ = n (n-1)(n-2)(n-3)…1.

17. Multinomial Formula (General formula for labeling problem) = $\frac{n!}{n_1!n_2!\ldots n_k!}$

18. Combination Formula (Binomial Formula) $= {}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$

    where n = total no. of objects and r = no. of objects selected.

19. Permutation = ${}^nP_r = \frac{n!}{(n-r)!}$

Reading 10: Common Probability Distributions

1. Probability Function (for a binomial random variable) p(x) = p(X=x) = $\binom{n}{x}p^x(1-p)^{n-x} = = \frac{n!}{(n-x)!x!p^x}(1-p)^{n-x}$ (for x = 0,1,2….n)

- x = success out of n trials
- n-x = failures out of n trials
- p = probability of success
- 1-p = probability of failure
- n = no of trials.

2. Probability Density Function (pdf) = f(x) $= \begin{cases} \frac{1}{b-a} \; for \; a \leq x \leq b = \\ 0 \end{cases}$
   F(x) = $\frac{x-a}{b-a} for \; a < x < b$

3. Normal Density Funct = $f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$ for $-\infty < x < +\infty$

4. Estimations by using Normal Distribution:

   - Approximately 50% of all obsv fall in the interval $\mu \pm \frac{2}{3}\sigma$
   - Approx 68% of all obvs fall in the interval $\mu \pm \sigma$
   - Approx 95% of all obvs fall in the interval $\mu \pm 2\sigma$
   - Approx 99% of all obvs fall in the interval $\mu \pm 3\sigma$
   - More precise intervals for 95% of the obvs are $\mu \pm 1.96\sigma$ and for 99% of the observations are $\mu \pm 2.58\sigma$.

5. Z-Score (how many S.Ds away from the mean the point x lies) $z = standard \; normal \; random \; variable = \frac{X-\mu}{\sigma}$ (when X is normally distributed)

6. Roy's Safety-Frist Criterion = SF Ratio = $\frac{[E(R_P)-R_L]}{\sigma_P}$

7. Sharpe Ratio = = $\frac{[E(R_P)-R_f]}{\sigma_P}$

8. Value at Risk = VAR = Minimum $ loss expected over a specified period at a specified prob level.

9. Mean (μ_L) of a lognormal random variable = exp (μ + 0.50σ^2)

10. Variance (σ_L^2) of a lognormal random variable = exp ($2\mu + \sigma^2$) × [exp (σ^2) – 1].

11. Log Normal Price = $S_T = S_0 exp\,(r_{0,T})$
    Where, exp = e and $r_{0,t}$ = Continuously compounded return from 0 to T

12. Price relative = End price / Beg price = $S_{t+1}/ S_t = 1 + R_{t,\,t+1}$

    where,
    $R_{t,\,t+1}$ = holding period return on the stock from t to t + 1.

13. Continuously compounded return associated with a holding period from t to t + 1:

    $r_{t,\,t+1}$= ln(1 + holding period return) *or*
    $r_{t,\,t+1}$ = ln(price relative) = ln (S_{t+1} / S_t) = ln (1 + $R_{t,t+1}$)

14. Continuously compounded return associated with a holding period from 0 to T:

    $R_{0,T}$= ln (S_T / S_0) *or* $r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$

    Where,
    $r_{T-1,\,T}$ = One-period continuously compounded returns

15. When one-period continuously compounded returns (i.e. $r_{0,1}$) are IID random variables.

    $E(r_{0,T}) = E(r_{T-1,T}) + E(r_{T-2,T-1}) + \cdots + E(r_{0,1}) = \mu T \; And$

    $Variance = \sigma^2(r_{0,T}) = \sigma^2 T$

    S.D. = $\sigma\,(r_{0,T}) = \sigma\sqrt{T}$

16. Annualized volatility = sample S.D. of one period continuously compounded returns × $\sqrt{T}$

Reading 11: Sampling and Estimation

1. Var of the distribution of the sample mean $= \frac{\sigma^2}{n}$
2. S.D of the distribution of the sample mean $= \sqrt{\frac{\sigma^2}{n}}$