Reading 14: Linking Pension Liabilities to Assets

1. Value of liability = $V_L = \sum_t \frac{B_t}{(1+r_t)^t}$
   where, B_t = Benefit payments at time t
2. Value of an asset = $V_B = \sum_t \frac{CF_t}{(1+r_t)^t}$
3. Intrinsic value of Future wage liability =
   $$V_{L-FW} = \frac{B}{r-g} \times \frac{((1+g)^s - 1) \times ((1+r)^{d-s} - 1)}{(1+r)^d}$$
   where, s = yrs till retirement
   d = yrs till demise and subsequent termination of the obligation

Reading 15: Capital Market Expectations

1. Precision of the estimate of the population mean ≈ 1 / $\sqrt{\text{no of obvs}}$
2. Multiple-regression analysis: A = $\beta_0 + \beta_1$ B + β_2 C + ε
3. Time series analysis: A = $\beta_0 + \beta_1$ Lagged values of A + β_2 Lagged values of B + β_2 Lagged values of C + ε
4. Shrinkage Estimator = (Wt of historical estimate × Historical parameter estimate) + (Wt of Target parameter estimate × Target parameter estimate)
5. Shrinkage estimator of Cov matrix = (Wt of historical Cov × Historical Cov) + (Wt of Target Cov × Target Cov)
6. Vol in Period t = $\sigma^2_t = \beta\sigma^2_{t-1} + (1-\beta)\,\varepsilon^2_t$
7. Multifactor Model: R on Asset i = $R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{iK}F_K + \varepsilon_i$
8. Value of asset at time t_0
   $= \sum_{t=1}^{\infty} \frac{CF\ at\ time\ t}{(1+Discount\ rate)^t}$
9. Expected RoR on Equity =
   $\frac{\text{Div per share at time 0 (1+LT g rate)}}{\text{Current share price}}$ + LT g rate
   = Div Yield + Capital Gains Yield
10. Nominal GDP = Real g rate in GDP + Expected long-run Inf rate
11. Earnings g rate = Nominal GDP g rate + Excess Corp g (for the index companies)
12. Expected RoR on Equity $\approx \frac{D}{P} - \Delta S + i + g + \Delta PE$

-ΔS = Positive repurchase yield
+ΔS = Negative repurchase yield
ΔPE = Expected Repricing Return

13. Labor supply g = Pop g rate + Labor force participation g rate
14. Expected income R = D/P - ΔS
15. Expected nominal earnings g R = i + g
16. Expected Capital gains R = Expected nominal earnings grate + Expected repricing R
17. Asset's expected return E (R_i) = Rf + $(RP)_1 + (RP)_2 + \ldots + (RP)_K$
18. Expected bond R [E (R_b)] = Real Rf + Inf premium + Default RP + Illiquidity P + Maturity P+ Tax P
19. Inf P = AvgInf rate expected over the maturity of the debt + P (or discount) for the prob attached to higher Inf than expected (or greater disinflation)
20. Inf P = Yield of conventional Govt. bonds (at a given maturity) – Yield on Inf-indexed bonds of the same maturity
21. Default RP = Expected default loss in yield terms + P for the non-diversifiable risk of default
22. Maturity P = Interest rate on longer-maturity, liquid Treasury debt - Interest rate on short-term Treasury debt
23. Equity RP = Expected ROE (e.g. expected return on the S&P 500) – YTM on a long-term Govt. bond (e.g. 10-year U.S. Treasury bond R)
24. Expected ROE using Bond-yield-plus-RP method = YTM on a LT Govt bond + Equity RP

25. Expected ROA $E(R_i)$ = Domestic Rf R + (β_i) × [Expected R on the world market portfolio – Domestic Rf rate of R]

Where, β_i = The asset's sensitivity to R on the world mktportf = Cov (R_i, R_M) / Var (R_M)

26. Asset class RP_i = Sharpe ratio of the world market portfolio × Asset's own volatility (σ_i) × Asset class's correlation with the world mktportf $(\rho_{i,M})$

$$RP_i = (RP_M / \sigma_M) \times \sigma_i \times \rho_{i,M}$$

Where, Sharpe Ratio of the world market portfolio = Expected excess R / S.D of the world mktportf→ represents systematic or non-diversifiable risk = RP_M / σ_M

27. RP for a completely segmented market (RP_i) = Asset's own volatility (σ_i) × Sharpe ratio of the world mktportf

28. RP of the asset class, assuming partial segmentation = (Degree of integration × RP under perfectly integrated markets) + ({1 - Degree of integration} × RP under completely segmented markets)

29. Illiquidity P = Required RoR on an illiquid asset at which its Sharpe ratio = mkt's Sharpe ratio – ICAPM required RoR

30. Cov b/w any two assets = Asset 1 beta × Asset 2 beta × Var of the mkt

31. Beta of asset 1 = $\left(\dfrac{\sigma_1 \times \rho(1,m)}{\sigma_m}\right)$

32. Beta of asset 2 = $\left(\dfrac{\sigma_2 \times \rho(2,m)}{\sigma_m}\right)$

33. GDP (using expenditure approach) = Consumption + Invst + Δ in Inventories + Govt spending + (Expo- Impo)

34. Output Gap = Potential value of GDP – Actual value of GDP

35. Neutral Level of Interest Rate = Target Inf Rate + Eco g

36. Taylor rule equation: $R_{optimal} = R_{neutral} + [0.5 \times (GDPg_{forecast} - GDPg_{trend})] + [0.5 \times (I_{forecast} - I_{target})]$

37. Trend g in GDP = g from labor inputs + g from Δ in labor productivity

38. g from labor inputs = g in potential labor force size + g in actual labor force participation

39. g from Δ in labor productivity = g from capital inputs + TFP g*
    - TFP g = g associated with increased efficiency in using capital inputs.

40. GDP g = α + β_1Consumer spending g + β_2Investment g

41. Consumer spending g = α + β_1Lagged consumer income g + β_2Interest rate

42. Investment g = α + β_1Lagged GDP g+ β_2Interest rate

43. Consumer Income g = Consumer spending growth lagged one period

Reading 16: Equity Market Valuation

1. Cobb-Douglas Production Function Y = $A \times K^{\alpha} \times L^{\beta}$

Where, Y = Total real economic output
A = Total factor productivity (TFP)
K = capital stock
α = Output elasticity of K
L = Labor input
β = Output elasticity of L

2. Cobb-Douglas Production Function Y (assuming constant R to Scale) = ln (Y) = ln (A) + αln (K) + (1 – α) ln (L)

Or

$$\frac{\Delta Y}{Y} \approx \frac{\Delta A}{A} + \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$$

3. Solow Residual = %ΔTFP = %ΔY – α (%ΔK) – (1 – α) %ΔL

4. H-Model: Value per share at time 0 = $\frac{D0}{\text{Discount rate} - \text{LT sustainible Div g rate}} \times \Big[(1 + \text{LT sustainable Div g rate}) + \frac{\text{Super normal g period}}{2} \times (\text{ST higher Div g rate} - \text{LT sustainable Div g rate})\Big]$

5. Gordon g Div discount model: Value per share at time 0 = $\frac{(D_0)\times(1+g)}{r-g}$

6. Forward justified P/E = $\frac{\text{Intrinsic value}}{\text{Yr ahead expected Earnings}}$

7. Fed Model: $\frac{\text{Fwd Operating Earnings (E1)}}{\text{Index Level (P0)}}$ = Long-term US Treasury securities

8. Yardeni Model: $= \frac{E_1}{P_0} = y_B - d \times LTEG$

Where, E_1/P_0 = Justified (forward) earnings yield on equities
y_B = Moody's A-rated corporate bond yield
LTEG = Consensus 5-yr earnings g forecast for the S&P 500
d = Discount or Weighting factor that represents the weight assigned by the market to the earnings projections

9. Yardeni estimated fair value of P/E ratio = $\frac{P_0}{E_1} = \frac{1}{y_B - d \times LTEG}$

10. Fair value of equity mkt under Yardeni Model (P_0) = $P_0 = \frac{E_1}{y_B - d \times LTEG}$

11. Discount/weighting factor (d) = $d = \frac{y_B - \frac{E_1}{P_0}}{LTEG}$

12. 10-year Moving Average Price/Earnings [P / 10-year MA (E)] = $\frac{\text{Real (or Inf-adjusted*) S\&P 500 } \textit{Price Index}}{\text{Moving Avg of preceding 10 yrs of Real or Inf adj Earnings}}$

*The stock index and reported earnings are adjusted for Inflation using the CPI

13. Real Stock Price Index $_t$ = (Nominal SPI_t × CPI $_{\text{base yr}}$) / CPI $_t$

14. Real Earnings $_t$ = (Nominal Earnings $_t$ × CPI $_{\text{base year}}$) / CPI $_{t+1}$

15. Tobin's q = $\frac{\text{MVof debt+MV of equity}}{\text{Replacement cost of assets}}$

Equity q = $\frac{\text{Equity MKt Cap}}{\text{Net Worth}} = \frac{\text{Price per share} \times \text{No of Shares O/S}}{\text{Replacment cost of assets} - \text{MV of liabilities}}$

Reading 17: Asset Allocation

1. Req R = [(1 + Spending rate) × (1 + Expected Inf %) × (1 + Cost of earning Invst R)] – 1

2. Risk-adj Expected R = Expected return for mix 'm'* – (0.005 × Investor's risk aversion × Var of R for mix 'm'*)

3. Risk Penalty = 0.005 × Investor's risk aversion × Var of R for mix 'm'*
   **expressed as % rather than as decimals*

4. Safety First Ratio = $\frac{\text{Expected Portfolio R} - \text{Threshold level}}{\text{Portfolio S.D.}}$

5. Include asset in the portfolio when: $\frac{E(R_{new})-R_F}{\sigma_{new}} > \left[\frac{E(R_{new})-R_F}{\sigma_{new}}\right] Corr\left(R_{new}, R_p\right)$

6. Contribution of Currency risk = Vol of asset R in domestic ¢ – Vol of asset R in local ¢
   Where Vol = volatility

7. Funding Ratio = $\frac{\text{Market Value of Pension Assets}}{\text{Present Value of Pension Liabilities}}$

8. $U_m^{ALM} = E(SR_m) - 0.005 R_A \sigma^2(SR_m)$
   - U_m^{ALM} = Surplus objective function's expected value for a particular asset mix m, for a particular investor with the specified risk aversion.

- E (SR_m)= Expected surplus return for asset mix 'm' = $\frac{\Delta \text{ in asset value} - \Delta \text{ in liability value}}{\text{Initial Asset Value}}$
- σ^2 (SR_m) =Varof the surplus R for the asset mix m in %.
- R_A=Risk-aversion level

9. Human Capital (t) $=\sum_{j=t}^{T} \frac{\text{Expected Earnings at age j}}{(1+\text{discount rate})^{j-t}}$
   t = current age T = life expectancy

Reading 18: Currency Management: An Introduction

1. Bid Fwd rate = Bid Spot exchange (X) rate $+ \frac{\text{Bid Fwd points}}{10{,}000}$
2. Offer Fwd rate = Offer Spot X rate + $\frac{\text{Offer Fwdd points}}{10{,}000}$
3. FwdPrem/Disc % = $\frac{\text{spot X rate} - (\frac{\text{fwd pnts}}{10{,}000})}{\text{spot X rate}} - 1$
4. To convert spot rate into a forward quote when points are represented as %,
   Spot X rate × (1 + % prem)
   Spot X rate × (1 - % disct)
5. Mark-to-MV on dealer's position = $\frac{\text{Settlement day CF}}{1+\text{Disct rate} * \left(\frac{t}{T}\right)}$
6. CF at settlement = Original contract size × (All-in-fwd rate for new, offsetting fwd position – Original fwd rate)
7. Hedge Ratio = $\frac{\text{Nominal Value of derivatives contract}}{\text{MV of the hedged asset}}$
8. R_{DC} =(1 + R_{FC})(1 + R_{FX})–1
9. R_{DC} (for multiple foreign assets) = $$\sum_{i=1}^{n} \omega_i \left(1+R_{FC,i}\right)\left(1+R_{FX,i}\right)-1$$
10. Total risk of DC returns = $$= \sigma^2(R_{DC}) \approx \sqrt{\sigma^2(R_{FC}) + \sigma^2(R_{FX}) + [2\sigma(R_{FC})\sigma(R_{FX})\rho(R_{FC}, R_{FX})]}$$
11. % Δ in spot X rate ($\%\Delta S_{H/L}$) = Interest rate on high-yield currency (i_H) – Interest rate on low-yield currency (i_L)
12. Forward Rate Bias = $\frac{F_{P/B} - S_{P/B}}{S_{P/B}} = \frac{(i_P - i_B)\left(\frac{t}{360}\right)}{1+i_B\left(\frac{t}{360}\right)}$
13. Net delta of the combined position = Option delta + Delta hedge
14. Size of Delta hedge (that would set net delta of the overall position to 0) = Option's delta × Nominal size of the contract
15. Long Straddle = Long atm put opt (with delta of -0.5) + Long atm call opt (with delta of +0.5)
16. Short Straddle = Short ATM put opt (with delta of -0.5) + Short ATM call opt (with delta of +0.5)
    ATM = at the money
    opt = option
17. Long Strangle: Long OTM put option + Long OTM call opt
    OTM = out of the money
18. Long Risk reversal = Long Call opt + Short Put opt
19. Short Risk reversal = Long Put opt + Short Call opt
20. Short seagull position = Long protective (ATM) put + Short deep OTM Call opt + Short deep OTM Put opt
21. Long seagull position = Short ATM call + Long deep-OTM Call opt + Long deep-OTM Put opt
22. Hedge ratio = $\frac{\text{Principal face value of the derivatives contract used as a hedge}}{\text{Principal face of the hedged asset}}$

23. Min or Optimal hedge ratio = ρ (R_{DC}; R_{FX}) $\times \left[\frac{S.D\ (R_{DC})}{S.D\ (R_{FX})}\right]$

Reading 19: Market Indexes and Benchmarks

1. Periodic R (Factor model based) = Rp = $a_p + b_1F_1 + b_2F_2 + \ldots + b_KF_K + \varepsilon_p$

2. For one factor model $Rp = a_p + \beta_p R_I + \varepsilon_p$
   Where, R_I = periodic R on mktindex
   a_p = "zero factor"
   β_p = beta = sensitivity
   ε_p = residual return

3. MV of stock = No of Shares Outstanding × Current Stock Mkt Price

4. Stock wgt(float-weighted index) = Mkt-cap wght × Free-float adjustment factor

5. Price-weighted index (PWI) = $(P_1+P_2+\ldots+P_n)/n$

Reading 20: Fixed-Income Portfolio Management – Part I

1. Steps to calculate PVdistribution (PVD) of CFs:

a) Wght of Index's total MV attributable to CFs in each period = $\frac{\text{PV of CFs from B index for specific period}}{\text{PV of Total CFs from B}}$
   where B = Benchmark

b) Contribution of each period's CFs to portfolio D = D of each period × Wght of index CFs in specific period

c) Benchmark's PVD = $\frac{\text{Cont of each period's CFs to portfolio D}}{\text{sum of all the periods' D cont}}$

2. Active R = Portfolio's R – B Index's R

3. Tracking Risk = S.D of Active R = $\left(\frac{\sum(\text{Active R} - \text{Mean Active R})^2}{n-1}\right)^{\frac{1}{2}}$

4. Semi-annual Total R = $\left(\frac{\text{Total Future Dollars}}{\text{Full Price of the Bond}}\right)^{\frac{1}{n}} - 1$

5. Dollar D = D × Portfolio Value × 0.01

6. Portfolio's Dollar D = Sum of dollar D of securities in portfolio

7. Rebalancing Ratio = $\frac{\text{Original Dollar D}}{\text{New Dollar D}}$

8. Cash required for rebalancing = (Rebalancing ratio – 1) × (total new MV of portfolio)

9. Controlling Position = Target Dollar D – Current Dollar D

10. Contribution of bond/sector to the portfolio D = $\left(\frac{\text{MV of bond or sector in the Portfolio}}{\text{Total Portfolio Value}}\right) \times$ Effective D of bond or sector

11. Spread D of a Portfolio = Market wgtdavg of the sector spread D of the individual securities

12. Net safety rate of return (Cushion Spread) = Immunized Rate – Min acceptable R

13. Dollar safety margin = Current bond portfolio value - PV of the required terminal value at new interest rate

14. Economic Surplus = MV of assets – PV of liabilities

15. Confidence Interval =Target Return +/- (k) × (S.D of Target R)

where, k = number of S.D around the expected target R

Reading 22: Fixed-Income Portfolio Management – Part II

1. D of Equity = $\frac{(\text{D of Assets} \times \text{Assets}) - (\text{D of Liab} \times \text{Liab})}{\text{Equity}}$

2. R_p = Portfolio RoR = $\frac{\text{Profit on borrowed funds + Profit on Equity}}{\text{Amount of Equity}}$ =
   = $[B \times (r_F - k) + E \times r_F] / E$
   $= r_F + [\frac{B}{E} \times (r_F - k)]$

3. Dollar interest = $\frac{\text{Amount borrowed} \times \text{Repo rate} \times \text{Repo term}}{360}$

4. New bond MV = $\frac{\text{Dollar D of Old Bond}}{\text{Duration of New Bond}} \times 100$

5. New bond Par value = $\frac{\text{Dollar D of Old Bond}}{\text{New Dollar D per Bond}} \times 100$

6. Shortfall risk = $\frac{\text{No of obs below the Target R}}{\text{Total No of Observations}}$

7. Target dollar D = Current dollar D without futures + Dollar D of futures position

8. No of Futures Contracts = $\frac{\text{Target \$ D} - \text{Current \$ D without futures}}{\text{\$ D per futures contract}}$

9. Dollar duration of futures contract = $\frac{\text{\$ D of Cheapest to}}{\text{Deliver issue}} \times \text{CF for CTD Issue}$

10. Hedge Ratio = $\frac{\text{Factor exposure of the bond (portfolio) to be hedged}}{\text{Factor exposure of Hedging instrument}}$
    or
    Hedge Ratio = $\left[\frac{\text{Duration of the bond to be hedged} \times \text{Price of the bond to be hedged}}{\text{Duration of the CTD bond} \times \text{Price of the CTD}}\right] \times$ (Conversion factor for CTD bond)

11. Basis = Cash (spot) price – Futures price

12. Yield on bond to be hedged = a + (Yield Beta × yield on CTD Issue) + Error

13. Hedge ratio = $\frac{D_H - P_H}{D_{CTD} P_{CTD}}$ × Conversion factor for CTD Issue × Yield Beta

14. Interest rate Swap (fixed-rate receiver/floating rate payer) = Long a fixed-rate bond + Short a floating-rate bond

15. \$ D of a swap for a fixed-rate receiver (floating rate payer) = \$ D of a fixed-rate bond − \$ D of a floating-rate bond
    OR
    \$ D of a swap for a fixed-rate receiver ≈ \$ D of a fixed-rate bond

16. Interest Rate Swap (*fixed-rate payer*/floating rate receiver) = Long a floating-rate bond + short a fixed-rate bond

17. \$ D of a swap for a fixed-rate payer = \$ D of a floating-rate bond − \$ D of a fixed-rate bond
    OR
    \$ D of a swap for a fixed-rate payer ≈ −\$ D of a fixed-rate bond

18. \$ D of a portfolio that includes a swap = \$ D of assets − \$ D of liabilities + \$ D of a swap position

19. D for an Option = Delta of Option × D of Underlying Instrument × (Price of underlying) / (price of Opt instrument) where Opt = Option

20. Payout to Opt Buyer or Opt value = MAX [(Strike value – Value at maturity), 0]

21. Credit spread call Opt value/Payoff = Max [(Spread at the opt maturity – Strike spread) × NP × Risk factor, 0]

22. Credit Forward Payoff = (Credit spread at the forward contract at maturity – Contracted credit spread) × NP× Risk factor

23. Change in Foreign bond Value (In terms of change in foreign yield only) = Duration × Δ Foreign yield × 100

24. Change in Foreign bond Value (when domestic rates change) = Duration × Yield beta × Δ Domestic yield × 100

25. $\Delta \text{Yield}_{\text{Foreign}} = \alpha$ + Yield beta or country beta (β) $(\Delta \text{ yield}_{\text{Domestic}}) + \varepsilon$

26. Estimated % $\Delta \text{ Value}_{\text{Foreign}}$ = Yield beta × Δ Domestic yield

27. D Cont of Domestic Bond = Wght of domestic bond in Portfolio × D of Domestic Bond

28. D Cont of Foreign Bond = Wght of foreign bond in Portfolio × D of Foreign Bond × Country beta