

1. INTRODUCTION

Derivatives derive their value from the economic performance of the underlying. Derivatives are of two types:

i) Over-the-counter: tailored to the counterparties' specific needs but have potentially lower level of liquidity.

ii) Exchange traded: Do not match investors' specific needs but mitigate counterparty risk.

Options:

Represent a contingent claim derivative that provide their owner with the right but not an obligation to a payoff determined by the future price of the underlying. Options have nonlinear payoffs.

2. POSITION EQUIVALENCIES

Put-call parity:

$S_0 + p_0 = c_0 + X/(1+r)^T$ where:

S_0 = price of underlying

p_0 = price (i.e. premium) of put option

c_0 = price (i.e. premium) of call option

$X/(1+r)^T$ = present value of the risk-free bond

Put-call-forward parity: $F_0(T)/(1+r)^T + p_0 = c_0 + X/(1+r)^T$

Note: A synthetic short forward position can achieve the same outcome as selling forwards or futures contracts. Refer to Exhibit 3, Reading 8.

2.2 Synthetic Put and Call

The symmetric payoff of a short and long stock, forward, and futures positions can be altered by implementing synthetic options positions.

Transforming a short stock position into a synthetic long put position by buying a call will convert the symmetrical payoff into asymmetrical payoff.

Synthetic long put option = Short stock + long call

Refer to Exhibit 4, Reading 8 for an illustration of the payoffs of a synthetic long put position.

2.1 Synthetic Forward Position

Synthetic long forward position = long call + short put

Note: A synthetic long forward position can achieve the same outcome as buying forwards or futures contracts. Refer to Exhibit 2, Reading 8.

Practice: Example 1, Volume 5, Reading 8.



A trader who wants to short the stock can:

- Borrow the stock from the market and short sell the borrowed shares or
- Create a synthetic short forward position: short call + long put

Practice: Example 2, Volume 5, Reading 8.



Long call position payoff can be achieved by:

- 1) A protective put strategy which will transform a long stock position into that of a long call position.
- 2) Adding a long put to a long forward or futures position

Refer to Exhibit 5, Reading 8 for an illustration of the payoffs of a synthetic long call position

3. COVERED CALLS AND PROTECTIVE PUTS

Covered call = Short call position + Long stock position
 Protective put = Long put position + Long stock position

3.1 Investment Objectives of Covered Calls

Summary of Greeks:

- Delta (Δ) \approx Change in value of option/Change in value of underlying
 - change in the price of an option in response to a change in the price of the underlying. Is a good approximation for small changes in the underlying's price.
 - Delta for long calls is positive; delta for long puts is negative
- Gamma (Γ) \approx Change in delta/Change in value of underlying
 - Always positive for long calls and long puts
- Vega (v) \approx Change in value of option/Change in volatility of underlying
 - Always positive for long calls and long puts
- Theta (θ):
 - Daily change in an option's price
 - Measures the sensitivity of option's price to the passage of time (time decay)
 - Theta for long calls and long puts is negative

3.1.1) Market Participant # 1: Yield Enhancement

A common motivation is cash generation in anticipation of limited upside moves in the underlying.

Covered call writer believes volatility of underlying asset will be less than option's implied volatility.

Call option writer gives up capital gains above strike price.

Example: Consider 3 scenarios (A and B) for a covered call position which includes:

1. A long stock position
2. A short call position with a November expiry and exercise price (X) of 17 (NOV 17 call)
3. A call premium (c_0) of 1.44

Scenario A: Stock price (S) drops to 5

- Investor earns call premium of 1.44
- Call option will be out-of-the-money drop to 0
- Portfolio value = $S - [\text{Max}(0, S - X)] + c_0 = 5 - 0 + 1.44 = 6.44$

Scenario B: S rises to 30

- Investor earns call premium of 1.44
- Call option value will be in-the-money and rise to $30 - 17 = 13$
- Portfolio value = $S - [\text{Max}(0, S - X)] + c_0 = 30 - 13 + 1.44 = 18.44$

3.1.2) Market Participant #2: Reducing a Position at a Favorable Price

Consider the following example:

A portfolio manager wants to reduce allocation to energy stocks by selling call options. Following are relevant details:

- Current stock price = 16.00
- Exercise price of call option = 15.00
- Call option premium = 1.25
- She expects the stock price to remain stable over the next month

Scenario A: Stock price remains stable at 16.00

- Call option is in the money as stock price, $16.00 >$ exercise price, 15.00
- She receives 15.00 when options are exercised
- She receives upfront premium of 1.25 when options are written
- Compared to selling the stock and receiving 16.00, writing options resulted in a price improvement of 0.25 ($[15 + 1.25] - 16.00$) or 1.56% ($0.25/16.00$) over a month's time.

Scenario B: Stock price falls to 10 over next month

- Call option will be out-of-the-money
- Manager will only realize $10 + 1.25 = 11.25$ on the covered call position

American option call premium = Time value + Intrinsic/Exercise Value = Time value + $\text{Max}(0, S - X)$

3.1.3) Market Participant # 3: Target Price Realization

This strategy involves writing calls with an exercise price near the target stock price. For example:

A bank holds a portfolio of stocks which are currently priced slightly lower than the target price of 16. If bank manager writes near-term call options with a premium of 0.97 and exercise price of 16:

- If stock price > 16, calls will be exercised, and stocks will be called away at 16 with option premium adding an additional 6% [0.97/16] positive return to the stock
- If stock price doesn't rise beyond 16, the manager may write a new call with the same objective

Limitations:

- Opportunity loss relative to the outright sale of stock for the investor if individual stock or market suffers a decline
- Opportunity loss if stock price rises sharply above exercise price and stock is called away at a lower-than-market price

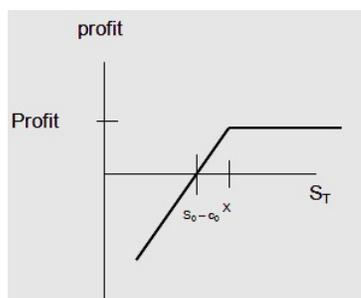
Other considerations:

- A short call position will reduce the delta of the portfolio
- Lower portfolio delta reduces upside opportunity
- Shares have a gamma of 0; short positions have a negative gamma
- Gamma is greatest for ATM options and becomes smaller as option moves in or out of the money
- Gamma can increase dramatically as time to expiration approaches or volatility increases

3.1.4) Profit and Loss at Expiration

To summarize:

- Value at expiration = Value of the underlying + Value of the short call = $V_T = S_T - \max(0, S_T - X)$
- Profit = Profit from buying the underlying + Profit from selling the call = $V_T - S_0 + C_0$
- Maximum Profit = $X - S_0 + C_0$
- Max loss would occur when $S_T = 0$. Thus, Maximum Loss = $S_0 - C_0$
- Breakeven = $S_T^* = S_0 - C_0$



Note: Most equity covered call writing occurs with exchange-traded options which allows the call writer to buy back the option if stock price declines any time before expiration.

Practice: Example 3, Volume 5, Reading 8.



Important consideration: If strike price exceeds stock price, call option value is entirely explained by time value as intrinsic value is 0 ($\max(0, [S - X])$).

3.2 Investment Objective of Protective Puts

Protective Put = Long stock position + Long Put position

This provides protection against a decline in value.

It is similar to "insurance" i.e. buying insurance in the form of the put, paying a premium to the seller of the insurance, the put writer.

Put v/s Insurance:

The exercise price of the put is like the insurance deductible because the magnitude of the exercise price reflects the risk assumed by the party who owns the underlying. A higher exercise price of the put option is equivalent to a lower insurance deductible.

- The higher the exercise price, the higher the option premium and the less risk assumed by the holder of the underlying and the more risk assumed by the put seller.
- In insurance, the higher the deductible, the more risk assumed by the insured party and the less risk assumed by the insurer.

3.2.1) Loss Protection/UpSide Preservation

- Higher strike puts are more expensive and will provide greater protection against price declines.
- Longer-term American puts are more expensive than their equivalent (same strike price) shorter-maturity puts.
- The put buyer must be assured that the put will not expire before an anticipated price decline occurs
- Insurance cost can be reduced by increasing size of deductible (current stock price – put exercise price)
- Theta or erosion of option value due to the passage of time will be greater for the cheaper, near-term expiry date option compared to the more-expensive, later expiry date option (assuming volatility is held constant).
- A price shock to the underlying asset may increase market's expectation of future volatility and increase put premium.

Important consideration: If stock price exceeds strike price, put option value is entirely explained by time value (theta) as intrinsic value is 0 ($\text{Max } 0, [X - S]$).

To summarize:

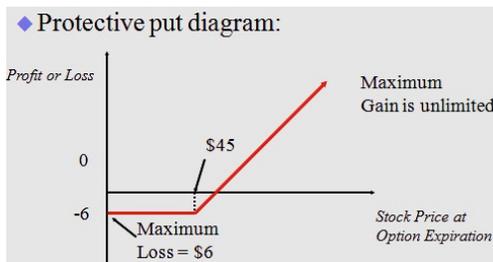
- a) Value at expiration: $V_T = S_T + \max(0, X - S_T)$
- b) Profit = $V_T - S_0 - p_0$
- c) Maximum Profit = ∞
- d) The maximum loss would occur when underlying asset is sold at exercise price. Thus, Maximum Loss = $S_0 + p_0 - X$
- e) In order to breakeven, the underlying must be at least as high as the amount paid up front to establish the position. Thus, Breakeven = $S_T^* = S_0 + p_0$

Example:

Strike price = $X = \$45$

Option cost = $p_0 = \$6$.

- The maximum possible loss is \$6
- The potential gain is unlimited



The profit and loss diagram for a protective put has a shape similar to a long call position. The position, Long asset + long put, is equivalent to the position, Long call + long risk-free bond.

Practice: Example 4, Volume 5, Reading 8.



3.3 Equivalence to Long Asset/Short Forward Position

Call Delta	Put Delta
Ranges from 0 to 1	Ranges from 0 to - 1
When share price close to strike price, long ATM option delta = 0.5	When share price close to strike price: long ATM option delta = - 0.5
Long positions in assets have a delta of 1	
Short positions in options & assets have a delta of - 1	
Call delta - put delta = 1 for options on same underlying with same BSM model inputs	
Deltas of futures and forwards on non-dividend paying stocks is 1 for long position and - 1 for short positions	

Position delta is overall or portfolio delta. For example, delta of a portfolio comprising 100 shares of stocks and 1 short ATM call option is 50:

Delta of share position = $100 \times +1 = + 100$
 Delta of short call position = $100 \times -0.5 = - 50$
 Portfolio delta = $100 - 50 = 50$

3.4 Writing Puts

Writing a cash secured put (or fiduciary put):

Strategy: Writing a put option and simultaneously depositing an amount of money equal to the exercise price in an account.
 Investment view: Someone who is bullish on the stock or wants to acquire shares at a particular price

Note: Similar to the long underlying stock position covering the short call position in a covered call strategy, the short put position is covered by cash in the account in a cash secured put strategy.

Naked put: Writing a put option without escrowing the exercise price (or setting aside cash to buy the underlying if the put is exercised).

Example: Consider a long-put position:

- Strike price = 15
- Price (p_0) = 1.65
- Underlying stock price (S_0) = 15.84
- Expiry = September
- Gamma (Γ) = 0.136
- Delta (Δ) = - 0.335
- Theta (Θ) = - 0.015
- Vega (v) = 0.017

Change A: Stock price rises by 0.10; from 15.84 to 15.94:

1. Δ (Changes at the rate of Γ), so:
 $\Delta_1 \approx \Delta_0 + (\Gamma \times \Delta S) \approx - 0.335 + (0.136 \times 0.10) \approx - 0.321$
2. Γ changes slightly to 0.133
3. $p_1 \approx p_0 + (\Delta_0 \times \Delta S) \approx 0.65 + (- 0.335 \times 0.10) \approx 0.617$

Change B: Time to expiration changes from 30 days to 29 days

$p_1 \approx p_0 + (\Theta \times \Delta t) \approx 0.65 + (- 0.015 \times 1) \approx 0.635$

Change C: Implied volatility increases by 1 percentage point from 58.44% to 59.44%

$p_1 \approx p_0 + (v \times \Delta \text{Vol}) \approx 0.65 + (0.017 \times 1) \approx 0.667$

Note: In above example, if stock price at expiration is above 15, put writer will keep premium and can have shares put to her. Effective purchase price would be $15.00 - 0.65 = 14.35$.

Higher strike price for put option will increase delta & theta but make gamma more negative. In this scenario, option writer is in a more bullish position compared to writing a put option with a lower strike price. In summary, for put options:

Maximum gain = option premium received
 Breakeven price = Exercise price – Option premium
 Maximum loss = Strike price – option premium

Writing a covered call and writing a put are similar in terms of risk and return characteristics and shape of graphical function.

3.5 Risk Reduction Using Covered Calls and Protective Puts

Covered calls and protective puts may be considered risk reduction or hedging strategies.

3.5.1) Covered Calls

Covered call position = Long stock + Short call

Position eliminates uncertainty for price increases.

Maximum loss = $S_0 - c_0$ (Even if stock price reduces to 0, loss is reduced by the amount of premium received).

Maximum gain = $X - S_0 + c_0$

3.5.2) Protective Puts

Protective put position = Long stock + long put

Price uncertainty is eliminated for price decreases. Continually purchasing puts to protect against price decline will reduce volatility of portfolio but is an expensive strategy.

3.5.3) Buying Calls and Writing Puts on a Short Position

Investor who is the short the underlying asset can hedge the risk of a price increase by purchasing a call. New portfolio will appear as (Call – Stock).

The long call will offset portfolio losses when the share price increases.

In addition, the investor may sell put options to generate premium income:

- When share price declines, the short stock position increases in value but portfolio gains will be reduced by the short put.
- When share price increases, the short stock position suffers a loss, the put option will expire worthless, and the investor will keep premium income.

Short Stock Seller Position Resulting from Writing Puts or Buying Calls

	Short Stock + Long Call	Short Stock + Short Put
When S > X	Maximum loss = $X - S_0 + c_0$	Maximum loss = Unlimited on stock position and only cushioned by p_0
When S < X	<ul style="list-style-type: none"> • Investor loses c_0 • Short position profit = $S - S_0$ 	Maximum gain = $S_0 - X + p_0$
Vega (due to option position)	Positive	Negative
Theta	Negative	Positive

Practice: Example 5, Volume 5, Reading 8.



4. SPREADS AND COMBINATIONS

Money spread: Position comprising two options with the same underlying but different exercise price

4.1 Bull Spreads and Bear Spreads

Bull spread: becomes more valuable when the price of the underlying asset rises

Bear spread: become more valuable when the price of the underlying asset declines

Debit spreads are effectively long because the long option value exceeds the short option value.

Credit spreads are effectively short because the short option value exceeds the long option value.

4.1.1) Bull Spread

Bull Call Spread: This strategy involves a combination of a long position in a call with a lower exercise price and a short position in a call with a higher exercise price i.e.

- Buy a call (X_1) with option cost c_1 and sell a call (X_2) with option cost c_2 , where $X_1 < X_2$ and $c_1 > c_2$.

Note that the lower the exercise price of a call option, the more expensive it is.

Rationale to use Bull Call Spread: Bull call spread is used when investor expects that the **stock price** or underlying asset price will **increase** in the near future.

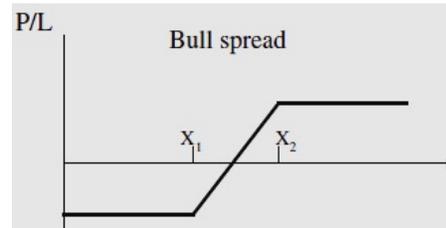
Characteristics:

- This strategy gains when stock price rises/ market goes up.

To summarize:

- a) The initial value of the Bull call spread = $V_0 = c_1 - c_2$
- b) Value at expiration: $V_T =$ value of long call – Value of short call = $\max(0, S_T - X_1) - \max(0, S_T - X_2)$
- c) Profit = Profit from long call + profit from short call.
Thus,
$$\text{Profit} = V_T - c_1 + c_2$$
- d) Maximum Profit = $X_2 - X_1 - c_1 + c_2$
- e) Maximum Loss = $c_1 - c_2$
- f) Breakeven = $S_T^* = X_1 + c_1 - c_2$

Note from 'a' that the price of the call option written, X_2 , is less expensive than the call option purchased, X_1 . Therefore, call bull spread involves an initial cash outflow (a debit spread).



A. Bull Put spread: In bull put spread, investor buys a put with a lower exercise price and sells a otherwise identical put with a higher strike price.

- Buy a put (X_1) and sell a put (X_2), with $X_1 < X_2$.
- Since put with a higher exercise price (X_2) is expensive than a put with a lower exercise price (X_1), bull put spread generates cash inflow at initiation of the position.
- Profit occurs when both put options expire out-of-the-money i.e. investor will earn net premium (i.e. price of underlying rises above X_1 and X_2).

4.1.2) Bear Spread

Bear Put Spread: This strategy involves a combination of a long position in a put with a higher exercise price and a short position in a put with a lower exercise price i.e.

- Buy a put (X_2) with option cost p_2 and sell a put (X_1) with option cost p_1 , where $X_1 < X_2$ and $p_1 < p_2$.

Note that the higher the exercise price of a put option, the more expensive it is.

Rationale to use Bear Put Spread: Bear Put spread is used when investor expects that the **stock price** or underlying asset price will **decrease** in the future.

To summarize:

- a) The initial value of the bear put spread = $V_0 = p_2 - p_1$
- b) Value at expiration: $V_T =$ value of long put – Value of short put = $\max(0, X_2 - S_T) - \max(0, X_1 - S_T)$
- c) Profit = Profit from long put + profit from short put.
Thus,

Profit = $V_T - p_2 + p_1$

d) Maximum Profit occurs when both puts expire in-the-money i.e. when underlying price \leq short put exercise price ($S_T \leq X_1$),

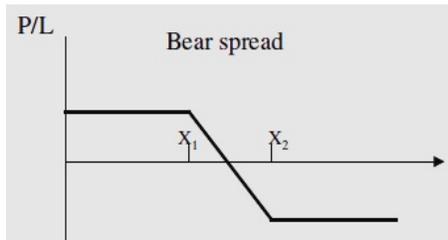
- Short put is exercised and investor will buy an asset at X_1 and
- This asset is sold at X_2 when long put is exercised. Thus,

Maximum Profit = $X_2 - X_1 - p_2 + p_1$

e) Maximum Loss occurs when both puts expire out-of-the-money and investor loses net premium i.e. when $S_T > X_2$. Thus,

Maximum Loss = $p_2 - p_1$

f) Breakeven = $S_T^* = X_2 - p_2 + p_1$



4.1.3) Refining Spreads

4.1.3.1) Adding a Short Leg to a Long Position

Example:

Suppose a trader has purchased a Nov 40 call in September by paying a premium (c_1) of 1.50 when the underlying stock was selling for 37. One month later (October), the stock rises in price to 48 and is believed to stabilize from now onwards. Following are premiums for one-month options:

Strike (X_2)	Premium (c_2)
40	8.30
45	4.20
50	1.91

If trader creates a bull spread by writing a 50-strike call option, the following outcomes result:

- When stock price (S_T) = 50 or higher (say, 52):
 - Exercise value: Long call + Short call = $\text{Max}(0, 52 - 40) + - \text{Max}(0, 52 - 50) = 10$
 - Profit = Exercise value + $c_2 - c_1 = 10.00 + 1.91 - 1.50 = 10.41$. This is maximum profit
- When S_T is 40 or lower, say 30:
 - Exercise value = $0 + 0 = 0$
 - Profit = $0 + 1.91 - 1.50 = 0.41$
- Between the two strike prices, exercise value of spread rises steadily with increase in stock price.
- If stock price remains unchanged at 48, Profit = $8.00 + 0.41 = 8.41$
- Therefore, minimum profit = $c_2 - c_1$ & maximum profit = $X_2 - X_1 + c_2 - c_1$

- **Note:** when S_T declines, position can lose 96% = $[(X_2 - X_1)/\text{Maximum profit}] = (10/10.41)$ of its maximum profit.

4.1.3.2) Spreads and Delta

Increasing portfolio delta will result in greater profits (losses) when the underlying asset increases (decreases).

See the illustration in the section 4.1.3.2 for a detailed explanation of how a trader can profit by adjusting his or her position delta in response to changes in the underlying asset price.

Practice: Example 6, Volume 5, Reading 8.



4.2 Straddles

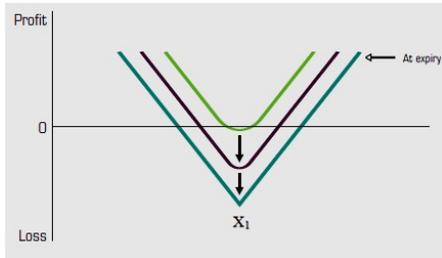
Long straddle: It involves buying a put and a call with same strike price on the same underlying with the same expiration; both options are at-the-money.

- In this strategy, an investor can make profit from upside or downside movement of the underlying price.
- Due to call option, the gain on upside is unlimited and due to put option, downside gain is quite large but limited.
- Straddle is a strategy that is based on the volatility of the underlying. It benefits from high volatility.
- Straddle is a costly strategy.
- The long straddle will only be profitable if price increase/decrease > total options premium cost
- When exercise price is close to stock prices, delta of position = 0
- If price increases (decreases) significantly, delta of call will approach + 1 (0) and delta of put will approach 0 (- 1), resulting in position delta of + 1 (- 1).

Rationale to use Straddle: Straddle is to be used only when the investor expects that volatility of the underlying will be relatively higher than what market expects but is not certain regarding the direction of the movement of the underlying price.

To summarize:

- a) Value at expiration: $V_T = \text{max}(0, S_T - X) + \text{max}(0, X - S_T)$
- b) Profit = $V_T - p_0 - c_0$
- c) Maximum Profit = ∞
- d) Maximum Loss occurs when both call and put options expire at-the money and investor loses premiums on both options i.e. Maximum Loss = $p_0 + c_0$
- e) Breakeven = $S_T^* = X \pm (p_0 + c_0)$



Short Straddle: It involves selling a put and a call with same strike price on the same underlying with the same expiration; both options are at-the-money.

- This strategy is preferably used when investor has neutral view of the volatility or when investor expects a decrease in volatility.
- This strategy gains when both the options expire at-the money i.e. investor earns call and put premium.
- This strategy has unlimited loss potential.
- This strategy has a positive theta and will benefit from erosion of time value of short call and put positions unlike the long straddle which has a negative theta.

Strangle: Combination of long put and call positions on underlying with the different exercise prices but the same expiration date.

4.3

Collars

Collar: A position combining long shares of stock and a long put with an exercise price below the current stock price and a short call with an exercise price above the current stock price.

- When call option premium is equal to put option premium, no net premium is required up front. This strategy is known as a Zero-Cost Collar*.
- This strategy provides downside protection at the expense of giving up upside potential. Therefore, zero-cost only refers to the fact the no cash is required to be paid up front.
- In Zero-cost collar, first of all investor selects exercise price of the put option. Then, the call exercise price is set such that the call premium offsets the put premium so that there is no initial outlay for the options.
- Typically,
 - Put exercise price (e.g. X_1) < current value of the underlying.
 - Call exercise price (e.g. X_2) must be > current value of the underlying.
- When price < X_1 , put provides protection against loss.
- When price > X_2 , short call reduces gains.
- When price lies between X_1 and X_2 , both put and call are out-of-the-money.

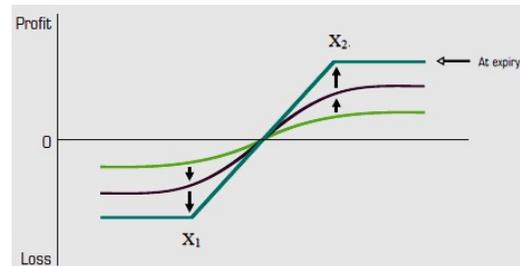
4.3.1) Collars on an Existing Holding

A zero-cost collar involves the purchase of a put and sale of a call with the same premiums.

To summarize:

(For zero-cost collar)

- Initial value of the position = value of the underlying asset = $V_0 = S_0$
- Value at expiration: $V_T =$ Value of underlying $S_T +$ Value of the put option + Value of the short call option = $S_T + \max(0, X_1 - S_T) - \max(0, S_T - X_2)$
- Profit = $V_T - V_0 = V_T - S_0$
- Maximum Profit = $X_2 - S_0$
- Maximum Loss = $S_0 - X_1$
- Breakeven = $S_1^* = S_0$



4.3.2) The Risk of a Collar

A collar sacrifices upside price appreciation (by writing calls) in exchange for downside protection (by purchasing puts). Collar narrows distribution of possible investment outcomes (risk reducing) and limits return potential.

Because call strike (X_2) is > put strike (X_1), call option delta will be more negative* compared to put option delta. Therefore, overall delta of collar position will be negative and will reduce the delta of the long stock position.

* Short call position has a negative delta.

At expiration:

If underlying price > call strike, short call delta approaches -1, put option delta approaches 0, and underlying portfolio delta approaches 1; overall portfolio delta becomes 0.

If underlying price < put strike, short call delta approaches 0, put option delta approaches -1, and underlying portfolio delta approaches 1; overall portfolio delta becomes 0.

To summarize:

- As put and call prices move further away from underlying asset price, collar position

replicates the gain/loss pattern of long position in the underlying security

- As put and call prices move toward each other, the expected returns and volatility become less equity-like and converge to a risk-free fixed-income return
- Collars are an intermediate between a pure equity & fixed-income exposure

4.3.3) The Risk of Spreads

Collar v/s Bull Spread: The collar is quite similar to a bull spread i.e. both have a cap on the gain and a floor on the loss. However, bull spread does not involve actually holding the underlying.

4.4 Calendar Spread

Calendar position: Buying one option and selling the same option but with different expiration dates, on the same underlying with the same strike price.

A. Long calendar spread: Buying the more distant call/put and selling the near-term call/put

Primary objective of a calendar spread is to take advantage of option time decay by writing shorter-term option (with greater time decay) and buying longer-term option.

B. Short calendar spread: Buying the near-term call/put and selling the more distant call/put.

- Theta for in-the-money calls may provide a motivation for a short calendar spread.
- A big move in the underlying or a decrease in implied volatility will help a short calendar spread.
- An increase in implied volatility will help a long calendar spread.
- Calendar spreads are sensitive to movements in underlying & changes in implied volatility.

5. IMPLIED VOLATILITY AND VOLATILITY SKEW

Implied volatility:

- Implied volatility is not a directly observable variable but derived from the option pricing model such as the Black-Scholes-Merton option pricing model.
- Implied volatility may differ according to option strike prices, for example: implied volatilities of out-of-the-money (OTM) puts is greater than that of ATM and OTM calls.

Realized volatility (historical volatility):

- Measures the range of past price outcomes for the underlying asset
- Is the square root of the realized variance of returns
- Steps for calculating realized volatility over, say, the past month of the S&P 500 index (assuming 1 month = 21 trading days):
 - Step 1: Calculate daily percentage change for each day's index closing price = $(P_t - P_{t-1}) / P_{t-1}$
 - Step 2: Calculate standard deviation for the period using Step 1 figure

The following formula can be applied to both implied and realized volatility:

Converting monthly volatility to annual volatility:

$$\sigma_{\text{Monthly}}(\%) = \sigma_{\text{Annual}}(\%) / \sqrt{\frac{252}{21}}$$

Limitation of BSM model:

- Volatility is assumed to remain constant but in reality, option prices display volatility skew or volatility smile.
- Volatility smile: Implied volatility curve (implied volatility on y-axis; strike prices on x-axis) is U-shaped as implied volatilities priced into both OTM puts and calls trade at a premium to implied volatilities of ATM options.
- Volatility skew: Implied volatility increases for OTM puts and decreases for OTM calls, as the strike price moves away from the current price.
- Volatility skew is prevalent across asset classes and over time because investors have less interest in OTM calls and more interest in OTM puts as portfolio insurance

Extent of volatility skew depends on following factors:

- Investor sentiment
- Relative supply/demand for puts and calls:
 - Bearish market sentiment indicator: Implied volatility is higher (relative to historical levels) for puts with strike prices below the underlying asset's price & demand for put options increases as well as skewness is high.
 - Bullish market indicator: Implied volatility is higher (relative to historical levels) for calls with strike prices above the underlying asset's price & demand for call options increases

Volatility skew:

- Level of volatility skew varies over time for most asset classes
- Trading strategies which attempt to profit from a volatility skew and changes in its shape over time include:
 - A: Risk reversal: Long (short) calls + Long (short) puts on the same underlying with the same expiration date is a long (short) risk reversal. Risk reversals are delta-hedged
 - Rationale for long risk reversal: When trader believes that put implied volatility > call implied volatility. Underlying asset is sold to create a delta hedged position. In addition, trader expects call to increase more (or decrease less) in implied volatility relative to put.

- A. Contango (most commonly observed): occurs when implied volatilities for longer-term options > shorter-term options
- B. Inverted: occurs when markets are in stressed and de-risking sentiment is high and demand for short-term options increases

Implied volatility surface

Can be thought of as a three-dimensional plot, which shows the volatility skew and volatility smile, for put and call options with the following features in common:

- underlying asset
- days to expiration (x-axis)
- option strike prices (y-axis) and
- implied volatilities (z-axis)

Implied volatility surface is not flat as implied volatility varies across different option maturities and displays skew and can be used to infer changes in market expectations.

Note: Shape of volatility skew reflects market participants' fear about future market stress.

Term structure of volatility: Graphical representation of implied volatility across time. Shapes of the term structure include:

6. INVESTMENT OBJECTIVES AND STRATEGY SELECTION

6.1 The Necessity of Setting an Objective

When setting investment objectives for an options strategy, investors need to consider the following factors:

- Outlook of the market for the asset underlying the derivative
- Volatility and other investors' perception of volatility of the underlying
- Impact of Greeks (delta, gamma, theta, and vega) on profit/loss of options position
- Option premium paid (received) for long (short) option positions
- Why volatility-based derivatives are used – to benefit from rising volatility (long volatility) or falling volatility (short volatility), or protect against downside risk

Advantages of derivatives compared to investing in the underlying:

- Allow two parties with differing needs and market views to adjust quickly without entering into costly trades for the underlying
- Leverage: Allows taking a large exposure to the underlying by putting up a fraction of the cost

- Liquidity: For example: the ability to buy/sell credit protection using index credit derivatives (CDS)

6.2 Criteria for Identifying Appropriate Option Strategies

Examples of profitable option strategies given the expected change in implied volatility and outlook for the direction of movement of the underlying asset:

Outlook on the Trend of Underlying Asset

		Bearish	Trading range/neutral view	Bullish
Expected move in implied volatility	Decrease	Write calls	Write straddle	Write puts
	Remain unchanged	Write calls and buy puts	Calendar spread	Buy calls and write puts
	Increase	Buy puts	Buy straddle	Buy calls

Investors use collars to hedge risk of a long stock position and smooth volatility.

When investor maintains a bullish market view and implied volatility is expected to decrease:

- writing puts is a profitable strategy as the put moves further out of the money and decreases in price and
- a long risk reversal (long calls, short puts) can be used to implement the bullish market view.

When investors maintain a bearish market view with expectation of increased implied volatility:

- Underlying stock can be purchased by selling puts when investors expect price to decline below option strike price or target price
- puts will be more expensive and effective cost of purchasing stock will be less than target price by the size of put premium received.

Purchasing call/put spreads:

- Purchase of call bull (put bear) spread is most profitable when:
 - investor has a bullish (bearish) view and
 - underlying market is not clearly trending upward or downward
- If implied volatility is skewed & implied volatility of OTM puts > than in-the-money or ATM puts, cost of bearish spread is lower because:
 - OTM (more expensive) puts are sold and
 - ATM or in-the-money puts (cheaper) are purchased.

If view of the trend in underlying asset is neutral and expectation is that implied volatility will increase, profitable strategy is a long straddle position as strategy takes advantage of increase in long call/put prices resulting from increase in volatility, vega, and gamma.

If view is that market will trade in range and implied volatility will decrease, investors should write the straddle.

Calendar spreads:

Long calendar spread:

- Combines a long-term bullish outlook on the underlying asset with a near-term neutral outlook
- A long volatility strategy; longer-term option has a higher vega than the shorter-term option.
- Maximum profit point is when short expires worthless and a surge in implied volatility increases price of longer-term option.

Dangers of calendar spreads: Gamma is not zero for a long calendar spread which may result in underlying stock prices moving too far away from strike prices. Therefore, calendar spreads are implemented in markets characterized by low implied volatility when the underlying stock is expected to remain in a trading range.

7. USES OF OPTIONS IN PORTFOLIO MANAGEMENT

Sections 7.1-7.7 illustrate mini cases in which market participants use derivative products to alter a risk exposure or solve a problem.

Practice: Example 7, Volume 5, Reading 8.



Practice: CFA Institute's Curriculum end of Chapter Questions + FinQuiz Questions & Item-sets

