FinQuiz Formula Sheet CFA Program Level I

Quantitative Methods

Learning Module 1: Rates and Returns

1. Interest Rate r

r = Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

Nominal risk-free rate = Real risk-free rate + Inflation premium

2. Holding Period Return (HPR)

$$R = \frac{(P_1 - P_0) + I_1}{P_0}$$

where

P_0 =price at the beginning of period

P_1 = price at the end of period

I =income

3. Arithmetic mean (AM)

$$\bar{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i,T-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

4. Geometric Mean Return

$$\bar{R}_{Gi} = \sqrt[T]{(1 + R_{i1}) \ldots \times (1 + R_{iT})} - 1$$

where,

R_{it} = return in period t

T = total number of periods

5. Harmonic Mean

$$\bar{X}_H = n/\sum_{i=1}^{n}\left(\frac{1}{X_i}\right)$$

with $X_i > 0$ for i = 1,2, …, n.

6. Money-weighted rate of return (MWR)

$$IRR = \sum_{t=0}^{T} \frac{CF_t}{(1 + IRR)^t} = 0$$

where,

IRR = internal rate of return

T = number of periods

CF_t = cash flow at time t

7. Time-weighted Returns (TWR)

$$r_{twr} = [(1 + rt,1) \times (1 + rt,2) \times \ldots \times (1 + rt,n)]^{1/N} - 1$$

8. Non-annual Compounding

PV (for more than one Compounding per year)

$$PV = FV_N\left(1 + \frac{r_s}{m}\right)^{-m\times N}$$

where r_s = stated ann i − rate

9. Annualized Return

$$r_{annual} = (1 + r_{period})^c - 1$$

where,

c = number of periods in a year

$$r_{weekly} = (1 + r_{daily})^5 - 1;$$

$$r_{weekly} = (1 + r_{annualy})^{1/32} - 1$$

10. Continuously Compounded Return CCR

CCR associated with a HPR (t to t + 1)

$r_{t,\,t+1}$= ln(1 + holding period return) or

$r_{t,\,t+1}$ = ln(price relative) = ln (P_{t+1}/P_t) = ln $(1 + R_{t,t+1})$

CCR associated with a HPR (0 to T)

$R_{0,T}$= ln (P_T/P_0) or

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

11. Gross Return

Gross return = Return – trading expenses–Other expense directly related to the generation of returns

12. Net Return

Net Return = Gross Return – all managerial and administrative expenses

13. After-Tax Nominal Return

After-tax nominal return = Total return – any allowance for taxes on dividends, interest & realized gains

14. Real Returns

$(1 + r) = (1 + r_{rF}) \times (1 + \pi) \times (1 + RP)$

$(1 + r_{real}) = (1 + r_{rF}) \times (1 + RP)$ or

$(1 + r_{real}) = (1 + r) \div (1 + \pi)$

where,

r = Nominal return

r_{rF} = Real risk-free return

π = Inflation

RP = Risk premium

Learning Module 2: The Time Value of Money in Finance

1. Present Value (PV) and Future Value (FV) Relation

$FV_N = PV(1 + r)^N$

$FV_N = PVe^{r_s \times N}$ *(for continuous compounding)*

$PV = FV_t(1 + r)^{-t}$ or $PV = \frac{FV_t}{(1+r)^t}$

$PV = FV\ e^{-rt}$ *(for continuous compounding)*

PV for Fixed Income

2. Discount Instrument:

$PV = \frac{FV_t}{(1+r)^t}$

3. Coupon Instrument:

$$P = \frac{PMT_1}{(1 + r)^1} + \frac{PMT_2}{(1 + r)^2} \cdots \frac{PMT_N + FV_N}{(1 + r)^1}$$

4. Perpetual Bond

PV = PMT/r

5. Annuity Instruments

$$A = \frac{r(PV)}{1 - (1 + r)^{-t}}$$

A= periodic cash flow.

r = market interest rate per period.

PV = initial value/principal of the loan or bond.

t = total no. of payment periods.

PV for Equity

6. Constant Dividends

$$PV_t = \frac{D_t}{r}$$

7. Constant Dividend Growth Rate

$D_{t+1} = D_t(1 + g)$

$$PV_t = \frac{D_t(1 + g)}{(r - g)}$$

assuming r – g > 0

8. Changing Dividend Growth Rate

$$PV_t = \sum_{i=1}^{n} \frac{D_t(1 + g_s)^i}{(1 + r)^i} + \frac{E(S_{t+n})}{(1 + r)^n}$$

where $E(S_{t+n})$ = stock value in n period

$$E(S_{t+n}) = \frac{D_{t+n+1}}{r - g_l}$$

9. Implied Return for Fixed-Income

$$\text{Implied return: } r = \left(\frac{FV_t}{PV}\right)^{1/t} - 1$$

PV (Coupon Bond) = $\sum_{i=1}^{N} \frac{PMT_i}{(1+r)_i}$

10. Implied Return and Implied Growth for Equity

Implied Return: $r = \frac{D_t(1+g)}{PV_t} + g$

Implied Growth: $g = r - \frac{D_{t+1}}{PV_t}$

where $D_t(1 + g) = D_{t+1}$

11. Price-to-Earnings Ratio (P/E):

$$\frac{PV_t}{E} = \frac{D_t}{E_t} \times \frac{(1 + g)}{r - g}$$

where

$$PV_t = \frac{D_t(1 + g)}{r - g}$$

12. Forward P/E Ratio

$$\frac{PV_t}{E_{t+1}} = \frac{\frac{D_{t+1}}{E_{t+1}}}{r - g}$$

13. Cash Flow Additivity

Two-Year Bond Future Value:

$FV_{2\,yrs} = 1(1 + r_2)^2$

14. Forward Exchange Rates

$$F = S_0 \times \frac{(1 + r_d)}{(1 + r_f)}$$

where

d = domestic currency

f = foreign currency

Learning Module 3
Statistical Measures of Asset Returns

Measures of Central Tendency

1. Arithmetic Mean: AM

$AM = \frac{\text{Sum of obvs in database}}{\text{No.of obvs in the database}}$

2. Sample Mean $\overline{X}$

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

where,

X_i = ith observation

N = no. of observations in the sample

3. Median = Middle Value

- For Even no of obvs locate median at $\frac{n}{2}$
- For Odd no. of obvs locate median at mean of $\frac{n}{2}$ *and* $\frac{(n+1)}{2}$

4. Mode

Observation that occurs most frequently in the distribution

5. Weighted Mean: X_w

$\overline{X_w} = \sum_{i=1}^{n} w_i X_i$ = (w_1X_1+ w_2X_2+....+ w_nX_n)

where,

X_1, X_2,...,X_n = observed values

w_1, w_2,...,w_3 = Corresponding weights, sum to 1.

6. Geometric Mean: GM

GM = $\sqrt[n]{X_1 X_2 \ldots X_n}$ with $X_i \geq 0$ for i = 1,2,...n.

or

$$\text{In } G = \frac{1}{n}\text{ In}(X_1X_2X_3 \ldots X_n)$$

or

$$\text{In } G = \frac{\sum_{i=1}^{n} \text{In}X_n}{n}$$

$G = e^{\text{InG}}$

7. Harmonic Mean: H.M

$$\text{H.M} = \overline{X_H} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{X_i}\right)}$$

with $X_i > 0$ for i = 1,2, ..., n.

Measures of Location

8. Four Measures called Quantiles (collectively)

- Quartiles = $\frac{Distribution}{4}$
- Quintiles = $\frac{Distribution}{5}$
- Deciles = $\frac{Distribution}{10}$,
- Percentiles = $L_y = (n+1)\frac{y}{100}$
- Interquartile range (IQR) = Q3 – Q1

Measures of Location

9. Range = Max. value – Min value

10. Mean Absolute Deviation: MAD

$$MAD = \frac{\sum_{i=1}^{n} |X_t - \bar{X}|}{n}$$

where,

$\bar{X}$=Sample mean

n=No. of observations in the sample

11. Sample Var: s^2

$$s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

12. Sample Standard Deviation: S.D

$$\text{Sample S.D} = s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}}$$

13. Geometric vs. Arithmetic:

$$GM \approx AM - \frac{Variance\ of\ R}{2}$$

14. Semi-deviation (Semi S.D)

$$\text{Semi S.D} = \sqrt{semivariance} = \sqrt{\sum_{For\ all\ X_i \leq \bar{X}} \frac{(X_i - \bar{X})^2}{n-1}}$$

15. Target Semi Var

$$\text{Target Semi-var} = \sum_{For\ all\ X_i \leq B} \frac{(X_i - B)^2}{n-1}$$

where B = Target Value

16. Target Semi-Deviation

$$Target\ S.D = \sqrt{target\ semivariance}$$
$$= \sqrt{\sum_{For\ all\ X_i \le B} \frac{(X_i - B)^2}{n-1}}$$

17. Coefficient of Variation CV

$$CV = \left(\frac{s}{\bar{X}}\right)$$

where s= sample S.D and $\bar{X}$ = sample mean

18. Excess Kurtosis = Kurtosis – 3

Correlation Between Two Variables

19. Sample Covariance

$$s_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

where,
n = sample size
X_i = ith observation on variable X
$\bar{X}$ = mean of the variable X observations
Y_i = ith observation on variable Y
$\bar{Y}$ = mean of the variable Y observations

20. Correlation coefficient: r

$$r_{XY} = \frac{\text{covariance of X and Y}}{\left(\text{sample S.D of X}\right)\left(\text{sample S.D of Y}\right)}$$
$$r = \frac{cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}}$$

Learning Module 4
Probability Trees and Conditional Expectations

1. Expected Value of Random Variable E(X)

$E(w_iX_i)$ = Probability-weighted average of the possible outcomes

2. Variance of a random variable σ² (X)

$\sigma^2(X) = E\{[X - E(X)]^2\}$

3. Standard Deviation S.D

$S.D = \sqrt{Variance}$

4. Conditional Expected Value: E(X|S)

of a random variable X given a scenario S.

$E(X|S) = P(X_1|S)X_1 + P(X_2|S)X_2 \ldots + P(X_n|S)X_n$

5. Total Probability Rule

$E(X) = E(X|S)P(S) + E(X|S^C)P(S^C)$
$E(X) = E(X|S_1)P(S_1) + E(X|S_2)P(S_2) + \ldots + E(X|S_n)P(S_n)$

where,
$E(X|S_i)$ = Expected value of X given Scenario i
$P(S_i)$ = Probability of Scenario i
$S_1, S_2 \ldots S_n$ are mutually exclusive and exhaustive scenarios.

6. Bayes' formula

$$P(Event|New\ Information) = \frac{P(New\ Information|Event)}{P(New\ Information)} \times P(Prior\ prob.\ of\ Event)$$

Learning Module 5
Portfolio Mathematics

1. Expected Value of Weighted Sum of random Variables

$E(w_iR_i) = w_i E(R_i)$

where,
wi = weight of variable i
Ri = random variable i

2. Expected Return on the Portfolio

$E(R_p) = E(w_1R_1 + w_2R_2 + \ldots + w_nR_n)$
$= w_1E(R_1) + w_2E(R_2) + \ldots + w_nE(R_n)$

3. Covariance between R_i and R_j

$Cov(R_i, R_f) = \Sigma_{i=1}^{n}[p(R_i - ER_i)(R_j - ER_f)]$

4. Portfolio variance

$$\sigma^2(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\omega_j Cov(R_i, R_j)$$

For three assets

$\sigma^2(R_p) = w_1^2\sigma^2(R_1) + w_2^2\sigma^2(R_2) + w_3^2\sigma^2(R_3) + 2w_1w_2Cov(R_1,R_2) + 2w_1w_3Cov(R_1,R_3) + 2w_2w_3Cov(R_2,R_3)$

where,

σ^2 = Corresponding variance of each asset in the portfolio

5. **Correlation: $\rho(R_i R_j)$**
(b/w two random variables R_i, R_j)
$$\rho(R_i R_j) = \frac{Cov\,(R_i R_j)}{\sigma R_i \times \sigma R_j}$$

6. **Safety-first Ratio: SFRatio**
$SFRatio = [E(R_P) - R_L]/\sigma_P$

7. **Sharpe Ratio:**
= [E (Rp) – R_f]/σ_p

Learning Module 6
Common Probability Distributions

For lognormal random variable

1. **Mean: μ_L**
$\mu_L = \exp(\mu + 0.50\sigma^2)$

2. **Variance: σ_L^2**
$\sigma_L^2 = \exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$.

3. **Log Normal Price**
$S_T = S_0 \exp(r_{0,T})$

where,
exp = e and $r_{0,t}$ = Continuously compounded return from 0 to T

4. **Price Relative**
= End price/Beg price
= $S_{t+1}/S_t = 1 + R_{t,t+1}$

where,
$R_{t,t+1}$ = holding period return on the stock from t to t + 1.

5. **Continuously compounded return**
(associated with a holding period from t to t + 1)

$r_{t,t+1}$ = ln(1 + holding period return)
or
$r_{t,t+1}$ = ln(price relative) = ln (S_{t+1}/S_t) = ln (1 + $R_{t,t+1}$)

6. **Continuously compounded return**
(associated with a holding period from 0 to T)
$R_{0,T}$ = ln (S_T/S_0) or $r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$

where,
$r_{T-l,T}$ = One-period continuously compounded returns

7. **When one-period continuously compounded returns are random variables.**
$$E(r_{0,T}) = E(r_{T-1,T}) + E(r_{T-2,T-1}) + \cdots + E(r_{0,1}) = \mu T$$

$$Variance = \sigma^2(r_{0,T}) = \sigma^2 T$$

S.D. = $\sigma(r_{0,T}) = \sigma\sqrt{T}$

8. **Annualized volatility**
= sample S.D. of one period continuously compounded returns × $\sqrt{T}$

Learning Module 7
Estimation and Inference

For Sample Mean

1. **Var of the distribution = $\frac{\sigma^2}{n}$**
2. **S.D of the distribution = $\sqrt{\frac{\sigma^2}{n}}$**
3. **Standard Error of the sample mean:**
   - *When the population S.D (σ) is known = $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$*
   - *When the population S.D (σ) is unknown = $s_{\bar{X}} = \frac{s}{\sqrt{n}}$*

where s = sample S.D
estimate of s = $\sqrt{sample\ variance} = \sqrt{s^2}$
$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Learning Module 8
Hypothesis Testing

1. **Standard Error of Sample Mean $\sigma_{\bar{X}}$**
When Population S.D/variance is known
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

- Test statistic is **Z-distributed**

$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

2. **Power of Test**

= 1 - Prob of Type II Error

3. **Test Statistic for a Test of Difference between Two Population Means**

Normally Distributed Populations, Variances Unknown but Assumed Equal) based on Independent samples

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}\right)}}$$

where,

S_p^2 = *Pooled estimator of the common variance.*

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

where $df = n_1 + n_2 - 2$.

4. **Test Statistic for a test of mean differences**

Normally distributed populations, unknown population variances

- $t = \frac{\bar{d} - \mu_{d0}}{s\bar{d}}$
- sample mean difference = $\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$
- sample variance = $S_d^2 = \frac{\sum_{i=0}^{n}(d_1 - \bar{d})^2}{n-1}$
- sample S.D = $\sqrt{S_d^2}$
- sample error of the sample mean difference = $s\bar{d} = \frac{S_d}{\sqrt{n}}$

5. **Chi Square Test Statistic**

For test concerning the value of a normal population variance

$$X^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

where $(n-1) = df$ *and* $S^2 =$ *sample variance* $= \frac{\sum_{i=0}^{n}(X_i - \bar{X})^2}{n-1}$

6. **Chi Square Confidence Interval for variance**

Lower limit = L = $\frac{(n-1)S^2}{X_{\alpha/2}^2}$

Upper limit = U = = $\frac{(n-1)S^2}{X_{1-\alpha/2}^2}$

7. **Test Statistic for a Test of Mean Differences**

Normally Distributed Populations, Unknown Population Variances

$$t = \frac{\bar{d} - \mu_{d0}}{S_{\bar{d}}}$$

where,

- $Sample\ mean\ difference = \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$
- $Sample\ variance = S_d^2 = \frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}$
- *Sample S.D.* = $\sqrt{s^2{}_d}$
- *n = number of pairs of observations*
- $Standard\ error\ of\ sample\ mean\ difference = s\bar{d} = \frac{S_d}{\sqrt{n}}$

8. **F-test**

Test concerning differences between variances of two normally distributed populations.

F = $\frac{S_1^2}{S_2^2}$

where

$S_1^2 = 1st\ sample\ var\ with\ n_1\ obs$ $S_1^2 = 2nd\ sample\ var\ with\ n_2\ obs$

$df_1 = n_1 - 1\ numerator\ df$

$df_2 = n_2 - 1\ denominator\ df$

9. **Relation between Chi Square and F-distribution**

$$F = \frac{X_1^2/m}{X_2^2/n}$$

where

- X_1^2 *is one chi square random variable with one m* df.
- X_2^2 *is another chi square random variable with one n* df.

Learning Module 9
Hypothesis Testing

Parametric Test of a Correlation

1. **Consider two variables X & Y**

$r_{XY} = \frac{s_{XY}}{s_X s_Y}$

s_{XY} = sample covariance between X & Y.

s_X & s_Y= S.D of X and Y respectively

2. **Sample Correlation: r**

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

3. **Spearman Rank Correlation: r_s**

$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_1^2}{n(n^2-1)}$$

- For small samples use table to find rejection points.
- For large sample size (n>30) use t-test as below:

$$t = \frac{(n-2)^{1/2} r_s}{(1-r_s^2)^{1/2}}$$

4. **Chi-Square Statistic: $\chi^2{}_s$**

Test of Independence

$$\chi^2 = \sum_{i=1}^{m} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where

- *Σ sum of all cells,*
- O_{ij} *is observed frequency*
- E_{ij} *is expected frequency*
- E_{ij} *= The expected frequencies*
- $E_{ij} = \frac{(Total\ row\ i)\times(Total\ row\ j)}{Overall\ Total}$
- *m = no. of cells, calculated by multiplying the no. of groups in the rows by the no. of groups in the columns.*

Learning Module 10
Simple Linear Regression

1. **Simple Linear Regression Y_i**

$Y_i = b_0 + b_1X_i + \varepsilon_i$,

where,
Y = dependent variable
X = independent variable
b_0 *= intercept*
b_1 *= slope coefficient*
ε = error term = Y_i - $\hat{Y}_i$
b_0 *and* b_1 *are called regression coefficients*

2. **Sum of Squares Error SSE**

$$(SSE) = \sum_{i=1}^{n} (y_i - \hat{y})^2$$

as $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$ therefore

SSE = $\sum_{i=1}^{n}(Y_i - (\hat{b}_0 + \hat{b}_1 X_i))^2$

3. **Slope Coefficient**

$$\widehat{b_1} = \frac{\text{cov(x,y)}}{\text{var(x)}} = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sum(X_i-\bar{X})^2}$$

4. **Intercept b_0**

$$\widehat{b_0} = \bar{Y} - \widehat{b_1}\bar{X}$$

where

$\bar{Y}$ *and* $\bar{X}$ are mean values

5. **Sample correlation: r**

$$r = \frac{Cov\ of\ Y\ and\ X}{(S.D\ of\ Y)((S.D\ of\ X))}$$

6. **Covariance of X and Y: Cov_{XY}**

$$Cov_{XY} = \frac{\sum_{i=1}^{n}(Y_i-\bar{Y})((X_i-\bar{X}))}{n-1}$$

7. **Standard deviation of Y: S_Y**

$$S_Y = \sqrt{\frac{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}{n-1}}$$

8. **Squared residuals $E(\varepsilon_i^2)$**

$$E(\varepsilon_i^2) = \sigma_i^2 \quad, i = 1, \ldots n$$

9. **Sum of Squared Regression: SSR**

$$SSR = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2$$

10. **Coefficient of Determination: R^2**

$$R^2 = \frac{\text{Sum of square regression}}{\text{Sum of square total}}$$

$$= \frac{\sum_{i=1}^{n}(\hat{Y}-\bar{Y})^2}{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}$$

(for single independent variable $R^2 = r^2$)

11. **Mean square regression: MSR**

$$MSR = \frac{Sum\ of\ square\ regression}{k}$$

$$= \frac{\sum_{i=1}^{n}\left(\hat{Y} - \bar{Y}\right)^2}{1}$$

12. **Mean Square Error: MSE**

$$MSR = \frac{sum\ of\ squares\ error}{n-k-1}$$

13. **F-Statistic or F-Test**

$$F = \frac{MSR}{MSE} = \frac{\left(\frac{Sum\ of\ square\ regression}{k}\right)}{\left(\frac{Sum\ of\ squares\ error}{n-k-1}\right)}$$

(df numerator = k = 1)
(df denominator = n – k – 1 = n – 2)

14. **ANOVA**

ANOVA	SS	MSS	F
Regression df = 1	SSR $= \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$	$\frac{SSR}{k}$	$\frac{SSR/k}{SSE/(n-k-1)}$
Error df = n-2	SSE $= \sum_{i=1}^{n}(y_i - \hat{y})^2$	$\frac{SSE}{n-k-1}$	
Total df = n-1	SST $= \sum_{i=1}^{n}(y_i - \bar{y})^2$		

15. **Test statistic**

$$t = \frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$$

16. **Standard error of slope coefficient: $s_{\hat{b}_1}$**

$$s_{\hat{b}_1} = \frac{s_e}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

17. **Standard error of the intercept: $s_{\hat{b}_1}$**

$$s_{\hat{b}_1} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

18. **Forecasted value of dependent variable: $\hat{Y}f$**

$$\hat{Y}f = \hat{b}_0 + \hat{b}_1 X_f$$

19. **Standard error of the intercept: s_f**

$$s_f = s_e\sqrt{1 + \frac{1}{n} + \frac{\left(X_f - \bar{X}\right)^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

20. **Log-lin Model: $\ln Y_i$**

$$\ln Y_i = b_0 + b_1 X_i$$

21. **Lin-log Model Y_i**

$$Y_i = b_0 + \ln X_i$$

22. **Log-log Model $\ln Y_i$**

$$\ln Y_i = b_0 + b_1 \ln X_i$$

ECONOMICS

Learning Module 1
Firms and Market Structures

1. **Break-Even Price**

P = ATC
P = AR = MR = ATC
where TR = TC.
P = price, ATC = Avg, Total Cost, MR = Marginal Revenue, AR = Avg. Revenue, TR = Total revenue

2. **Concentration Ratio: CR**

CR = Sum of sales values of the largest 10 firms / Total market sales

3. **Herfindahl-Hirschman index: HHI**

$HHI = \sum X_i^2$

where,
X_i^2 is squared market share of the i^{th} firm.

HHI = 1 for monopoly.
HHI ≈ 0 for a perfectly competitive industry.

Learning Module 2
Understanding Business Cycles