

FinQuiz Formula Sheet CFA Program Level II

QUANTITATIVE METHODS

Learning Module 1: Basics of Multiple Regression and Underlying Assumptions

1. Multiple Regression

$$Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \dots + b_kX_{ki} + \varepsilon_i, i = 1, 2, \dots, n.$$

2. Prediction Equation

$$\text{Prediction equation} = \hat{Y}_i = \hat{b}_0 + \hat{b}_1X_{1i} + \hat{b}_2X_{2i} + \dots + \hat{b}_kX_{ki} + \varepsilon_i, i$$

Learning Module 2: Evaluating Regression Model Fit and Interpreting Model Results

1. Coefficient of Determination: R^2

$$\begin{aligned} &= \frac{\text{Sum of square regression}}{\text{Sum of square total}} \\ &= \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \end{aligned}$$

2. Adjusted R^2

$$\bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1} \right) (1 - R^2)$$

3. F-Statistic or F-Test

$$= \frac{\text{MSR}}{\text{MSE}} = \frac{\left(\frac{\text{Sum of square regression}}{k} \right)}{\left(\frac{\text{Sum of squares error}}{n-k-1} \right)}$$

where,

$$\text{df numerator} = k = 1$$

$$\text{df denominator} = n - k - 1 = n - 2$$

4. ANOVA

ANOVA	SS	MSS	F
Regression df = 1	SSR $= \sum_{i=1}^n (\hat{y}_i - \bar{y})^2$	$\frac{\text{SSR}}{k}$	$\frac{\text{SSR}/k}{\text{SSE}/(n-k-1)}$
Error df = n-2	SSE $= \sum_{i=1}^n (y_i - \hat{y}_i)^2$	$\frac{\text{SSE}}{n-k-1}$	
Total df = n-1	SST $= \sum_{i=1}^n (y_i - \bar{y})^2$		

5. Akaike's information criterion (AIC)

$$\text{AIC} = n \ln \left(\frac{\text{SSE}}{n} \right) + 2(k+1)$$

6. Schwarz's Bayesian information criterion (BIC or SBC)

$$\text{BIC} = n \ln \left(\frac{\text{SSE}}{n} \right) + \ln(n)(k+1)$$

Learning Module 3: Model Misspecification

1. Breusch-Pagan (BP) test

$$\text{Test statistic} = n \times R^2_{\text{residuals}}$$

where,

$R^2_{\text{residuals}} = R^2$ from a second regression of the squared residuals from the first regression on the independent variables
 n = number of observations

2. Variance Inflation Factor VIF_j

$$VIF_j = \frac{1}{1 - R_j^2}$$

Learning Module 4:**Extensions of Multiple Regression****1. Studentized residual t_i^***

$$t_i^* = \frac{e_i^*}{S_{e^*}} = \frac{n}{\sqrt{SSE(1-h_{ii})-e_i^2}}$$

2. Cook's distance D_i

$$D_i = \frac{e_i^2}{k \times MSE} \left[\frac{h_{ii}}{(1-h_{ii})^2} \right]$$

3. Linear regression

with 3 independent Variables

$$Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + b_3X_{3i} + e_i$$

4. Logistic regression (logit)

$$= \ln \left(\frac{p}{1-p} \right)$$

$$= b_0 + b_1X_1 + b_2X_2 + b_3X_3 + \varepsilon$$

$$P = \frac{1}{1 + \exp[-(b_1X_1 + b_2X_2 + b_3X_3 + \varepsilon)]}$$

Learning Module 5:**Time Series Analysis****1. Linear Trend Models**

$$y_t = b_0 + b_1t + \varepsilon_t$$

Predicted/fitted value of y_t in period (T + 1)

$$= \hat{y}_{t+1} = \hat{b}_0 + \hat{b}_1(T + 1)$$

2. Log-Linear Trend Models

$$y_t = e^{b_0 + b_1t}$$

3. Autoregressive Time-Series Models

First order autoregressive AR (1):

$$x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$$

pth-order autoregressive AR (p):

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \dots + b_p x_{t-p} + \varepsilon_t$$

4. Mean reverting level

$$x_t = \frac{b_0}{1 - b_1}$$

5. Chain Rule of Forecasting:

One-period ahead forecast

$$\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1 x_t$$

Two-period ahead forecast=

$$\hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1 x_{t+1}$$

6. Random Walks and Unit Roots:*Random Walk without drift:*

$$x_t = x_{t-1} + \varepsilon_t \text{ where, } b_0 = 0 \text{ and } b_1 = 1.$$

Correcting Random Walk:

$$y_t = x_t - x_{t-1}$$

Random walk with a drift:

$$x_t = b_0 + x_{t-1} + \varepsilon_t \text{ where, } b_0 \neq 0 \text{ and } b_1 = 1$$

By taking first difference

$$y_t = x_t - x_{t-1} = b_0 + \varepsilon_t$$

7. Using Dickey-Fuller Test

$$x_t - x_{t-1} = b_0 + (b_1 - 1) x_{t-1} + \varepsilon_t$$

8. Smoothing Past Values with n-Period Moving Average

$$\frac{x_t + x_{t-1} + x_{t-2} + \dots + x_{t-(n-1)}}{n}$$

9. Correcting Seasonality in Time Series Models:*For quarterly data*

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-4} + \varepsilon_t$$

For monthly data

$$x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-12} + \varepsilon_t$$

10. ARCH model =

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + \mu_t$$

where

 μ_t is an error term

Predicting variance of errors in period

$$t+1 = \hat{\sigma}_{t+1}^2 = \hat{\alpha}_0 + \alpha_1 \hat{\varepsilon}_t^2$$

Learning Module 6:**Machine Learning****1. LASSO:**Penalty term (when $\lambda > 0$) = $\lambda \sum_{k=1}^K |\bar{b}_k|$

$$\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 + \lambda \sum_{k=1}^K |\hat{\beta}_k|$$

When $\lambda = 0$,

LASSO penalized regression = OLS regression

Learning Module 7:

Big Data Projects

1. Normalization

$$X_{i(\text{normalized})} = \frac{X_i - X_{\min}}{X_{\max} - X_{\min}}$$

where X_i = value of observation

Performance Metrics:

2. Accuracy

$$= \frac{TP+TN}{TP+FP+TN+FN}$$

$$F1 \text{ score} = (2*P*R)/(P + R)$$

where

T = true, F = false, P = positive,

N = negative

3. Receiver Operating Characteristic (ROC):

False positive rate: FPR

$$FPR = \frac{FP}{TN+FP}$$

True positive rate TPR:

$$TPR = \frac{TP}{TN+FP}$$

4. Root Mean Square Error: RMSE:

$$RMSE = \sqrt{\sum_{i=1}^n \frac{(\text{Predicted}_i - \text{Actual}_i)^2}{n}}$$

ECONOMICS

Learning Module 1

Currency Exchange Rates

1. Bid-offer Spread

Offer price – Bid price

2. Forward Rate:

$$\begin{aligned} \text{Fwd rate} &= \text{Spot Exchange rate} \\ &+ \frac{\text{Forward points}}{10,000} \end{aligned}$$

3. Forward Premium or Discount:

$$= \frac{\text{spot exchange rate} - \left(\frac{\text{forward points}}{10,000}\right)}{\text{spot exchange rate} - 1}$$

4. To convert spot rate into forward quote:

Spot exchange rate \times (1 + % premium)
Spot exchange rate \times (1 - % discount)

5. Covered interest rate parity:

$$(1 + i_d) = S_{f/d} (1 + i_f) \left(\frac{1}{F_{f/d}}\right)$$

$$F_{f/d} = S_{f/d} \left(\frac{1 + i_f}{(1 + i_d)}\right)$$

Using day count convention:

$$\left(1 + i_d \left[\frac{\text{Actual}}{360}\right]\right) =$$

$$S_{f/d} \left(1 + i_f \left[\frac{\text{Actual}}{360}\right]\right) \left(\frac{1}{F_{f/d}}\right)$$

$$F_{f/d} = S_{f/d} \left(\frac{1 + i_f \left[\frac{\text{Actual}}{360}\right]}{1 + i_d \left[\frac{\text{Actual}}{360}\right]}\right)$$

6. Uncovered Interest Rate Parity :

- $i_f - \% \Delta S_{f/d}^e = i_d$
- $\% \Delta S_{f/d}^e = i_f - i_d$

Forward premium or discount:

- For one year horizon =

$$F_{f/d} - S_{f/d} =$$

$$S_{f/d} \left(\frac{i_f - i_d}{1 + i_d}\right) \cong S_{f/d} (i_f - i_d)$$

- Using day count convention:

$$F_{f/d} - S_{f/d} = S_{f/d} \left(\frac{\left[\frac{\text{Actual}}{360}\right]}{1 + i_d \left[\frac{\text{Actual}}{360}\right]}\right) (i_f - i_d)$$

7. Forward discount or premium as % of spot rate:

$$\frac{F_{f/d} - S_{f/d}}{S_{f/d}} \cong (i_f - i_d)$$

If uncovered interest rate parity holds

- $$= \frac{F_{f/d} - S_{f/d}}{S_{f/d}} = \% \Delta S_{f/d}^e \cong (i_f - i_d)$$

8. Purchasing Power parity (PPP)

- $P_f = S_{f/d} \times P_d$
- $S_{f/d} = P_f / P_d$

9. Relative version of PPP

$$= \% \Delta S_{f/d} = \pi_f - \pi_d$$

10. Ex ante version of PPP

$$= \% \Delta S_{f/d}^e = \pi_f^e - \pi_d^e$$

11. Real Exchange Rate

$$q_{f/d} = \left(\frac{S_{f/d} P_d}{P_f} \right) = S_{f/d} \left(\frac{P_d}{P_f} \right)$$

or

$$q_{f/d} = S_{f/d} \left(\frac{CPI_d}{CPI_f} \right)$$

12. Fisher effect:

$$i_d = r_d + \pi_d^e$$

$$i_f = r_f + \pi_f^e$$

$$i_f - i_d = (r_f - r_d) + (\pi_f^e - \pi_d^e)$$

$$(r_f - r_d) = (i_f - i_d) - (\pi_f^e - \pi_d^e)$$

Learning Module 1

Economic Growth

1. Economic growth:

Annual % Δ in real GDP or in real per capita GDP

2. Relation between Stock Prices and Economic growth:

$$P = \text{GDP} \left(\frac{E}{\text{GDP}} \right) \left(\frac{P}{E} \right)$$

where,

P = Aggregate value (price) of equities

E = Aggregate corporate earnings

3. Expressing in terms of logarithmic rates:

- $(1/T) \% \Delta P = (1/T) \% \Delta \text{GDP} + (1/T) \% \Delta (E / \text{GDP}) + (1/T) \% \Delta (P / E)$
- $\% \Delta$ in stock MV = $\% \Delta$ in GDP + $\% \Delta$ in share of earnings (profit) in GDP + $\% \Delta$ in the P/E multiple

4. A two-factor aggregate production function:

$$Y = A F(K, L)$$

Y = Level of aggregate output in the economy

L = Quantity of labor or number of workers or hours in the economy

K = Stock of capital used to produce goods and services

A = Total Factor Productivity (TFP)

5. Cobb-Douglas Production Function

$$F(K, L) = K^\alpha L^{1-\alpha}$$

6. Under Cobb-Douglas production function:

Marginal product of capital = $MPK = \alpha$

$$AK^{\alpha-1} L^{1-\alpha} = \alpha Y/K$$

$\alpha Y/K = r \rightarrow \alpha = r(K) / Y = \text{Capital income} / \text{Output or GDP}$

7. Output per worker or Average labor productivity (Y/L or y):

$\text{GDP/Labor input} = \text{TFP} \times \text{capital-to-labor ratio} \times \text{share of capital in GDP}$

Or

$$y = Y/L = Ak^\alpha$$

8. Contribution of Capital Deepening

= Labor productivity growth rate - TFP

9. Contribution of Improvement in technology

Labor productivity growth rate - Capital Deepening

10. Growth Accounting based on Solow Approach

$$\Delta Y / Y = \Delta A / A + \alpha \Delta K / K + (1 - \alpha) \Delta L / L$$

11. Labor productivity growth accounting equation