FinQuiz Formula Sheet CFA Program Level II

Reading 1: Introduction to Linear Regression

1. Linear Regression = $Y_i = b_0 + b_1X_i + \varepsilon_i$,

2. $Sum\ of\ square\ error\ (SSE) = \sum_{i=1}^{n}(y_i - \hat{y})^2$

3. $\widehat{b_1} = \frac{\text{cov(x,y)}}{\text{var(x)}} = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sum(X_i-\bar{X})^2}$

4. $\widehat{b_0} = \bar{Y} - \widehat{b_1}\,\bar{X}$

5. Sample correlation r = $\frac{Covariance\ of\ Y\ and\ X}{(standard\ deviation\ of\ Y)((standard\ deviation\ of\ X))}$

6. $Cov_{XY} = \frac{\sum_{i=1}^{n}(Y_i-\bar{Y})((X_i-\bar{X}))}{n-1}$

7. Standard deviation of Y= $S_Y = \sqrt{\frac{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}{n-1}}$

8. Squared residuals $E(\varepsilon_i^2) = \sigma_i^2\ , i = 1, \dots n$

9. $Sum\ of\ square\ regression\ (SSR) = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$

10. Coefficient of Determination (R^2) $= \frac{\text{Sum of square regression}}{\text{Sum of square total}} = \frac{\sum_{i=1}^{n}(\hat{Y} - \bar{Y})^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$

    (for single independent variable $R^2 = r^2$)

11. Mean square regression MSR = $\frac{Sum\ of\ square\ regression}{k} = \frac{\sum_{i=1}^{n}(\hat{Y}-\bar{Y})^2}{1}$

12. Mean square error MSE = $\frac{sum\ of\ squares\ error}{n-k-1}$

13. F-Statistic or F-Test = $\frac{MSR}{MSE} = \frac{\left(\frac{Sum\ of\ square\ regression}{k}\right)}{\left(\frac{Sum\ of\ squares\ error}{n-k-1}\right)}$

    (df numerator = k = 1)

    (df denominator = n – k – 1 = n – 2)

14. ANOVA

ANOVA	SS	MSS	F
Regression df = 1	$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$	$\frac{SSR}{k}$	$\frac{SSR/k}{SSE/(n-k-1)}$
Error df = n-2	$SSE = \sum_{i=1}^{n}(y_i - \hat{y})^2$	$\frac{SSE}{n-k-1}$	
Total df = n-1	$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$		

15. Test statistic t= $\frac{\hat{b}_1 - B_1}{s_{\hat{b}_1}}$

16. Standard error of slope coefficient = $s_{\hat{b}_1} = \frac{s_e}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2}}$

17. Standard error of the intercept = $s_{\hat{b}_1} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}}$

18. Forecasted value of dependent variable = $\hat{Y}f = \hat{b}_0 + \hat{b}_1 X_f$

19. Standard error of the intercept = s_f = $s_e\sqrt{1+\frac{1}{n}+\frac{(X_f-\bar{X})^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}}$

20. Log-lin Model = $\ln Y_i = b_0 + b_1 X_i$

21. Lin-log Model = $Y_i = b_0 + \ln X_i$

22. Log-log Model = = $\ln Y_i = b_0 + b_1 \ln X_i$

Reading 2: Multiple Regression

1. $Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \ldots + b_kX_{ki} + \varepsilon_i, i = 1, 2, \ldots n$

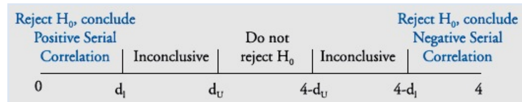
2. Prediction equation = $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_{1i} + \hat{b}_2 X_{2i} + \ldots + \hat{b}_k X_{ki} + \varepsilon_i, i$

3. Adjusted $R^2 = \bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1-R^2)$

4. Breusch–Pagan test
   - H_0 = No conditional Heteroskedasticity exists
   - H_A = Conditional Heteroskedasticity exists
   - Test statistic = n × $R^2_{residuals}$

5. Durbin-Waston Test = $DW = \frac{\sum_{t=2}^{T}(\hat{\varepsilon}_t-\hat{\varepsilon}_{t-1})^2}{\sum_{t=1}^{T}\hat{\varepsilon}_t{}^2}$

- For Large Sample size DW Statistic (d) = d ≈ 2 (1 – r)



Reading 3: Time Series Analysis

1. Linear Trend Models = $y_t = b_0 + b_1 t + \varepsilon_t$
   - Predicted/fitted value of y_t in period (T + 1) = $\hat{y}_{t+1} = \hat{b}_0 + \hat{b}_1(T+1)$

2. Log-Linear Trend Models = $y_t = e^{b_0+b_1 t}$

3. Autoregressive Time-Series Models:
   - First order autoregressive AR (1) = $x_t = b_0 + b_1 x_{t-1} + \varepsilon_t$
   - pth-order autoregressive AR (p) = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-2} + \ldots + b_p x_{t-p} + \varepsilon_t$

4. Mean reverting level of $x_t = \frac{b_0}{1-b_1}$

5. Chain Rule of Forecasting:
   - One-period ahead forecast = $\hat{x}_{t+1} = \hat{b}_0 + \hat{b}_1 x_t$
   - Two-period ahead forecast= $\hat{x}_{t+2} = \hat{b}_0 + \hat{b}_1 x_{t+1}$

6. Random Walks and Unit Roots:
   - Random Walk without drift = $x_t = x_{t-1} + \varepsilon t$ where, $b_0 = 0$ and $b_1 = 1$.
   - Correcting Random Walk = $y_t = x_t - x_{t-1}$
   - Random walk with a drift = $x_t = b_0 + x_{t-1} + \varepsilon_t$ where, $b_0 \neq 0$ and $b_1 = 1$
   - By taking first difference $y_t = x_t - x_{t-1} = b_0 + \varepsilon_t$

7. Using Dickey-Fuller Test = $x_t - x_{t-1} = b_0 + (b_1 - 1)x_{t-1} + \varepsilon_t$

8. Smoothing Past Values with n-Period Moving Average = $\frac{x_t + x_{t-1} + x_{t-2} + \ldots + x_{t-(n-1)}}{n}$

9. Correcting Seasonality in Time Series Models:
   - For quarterly data = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-4} + \varepsilon_t$
   - For monthly data = $x_t = b_0 + b_1 x_{t-1} + b_2 x_{t-12} + \varepsilon_t$

10. ARCH model = $\hat{\varepsilon}^2{}_t = \alpha_0 + \alpha_1 \hat{\varepsilon}^2{}_{t-1} + \mu_t$ where μ_t is an error term
    - Predicting variance of errors in period t+1 = $\hat{\sigma}^2_{t+1} = \hat{\alpha}_0 + \alpha_1 \hat{\varepsilon}^2_t$

Reading 4: Machine Learning

LASSO:

1. Penalty term (when λ > 0) = $\lambda \sum_{k=1}^{K} |\bar{b}_k|$

2. $\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 + \lambda\sum_{k=1}^{K}|\hat{b}_K|$

3. When λ = 0, LASSO penalized regression = OLS regression

Reading 5: Big Data Projects

1. $X_{i(normalized)} = \frac{X_i - X_{min}}{X_{max} - X_{min}}$

where X_i = value of observation

Performance Metrics:

2. Accuracy = (TP + TN)/(TP + FP + TN + FN)

   F1 score = (2*P*R)/(P + R)

3. Receiver Operating Characteristic (ROC):

   False positive rate (FPR) = FP/(TN + FP) and

   True positive rate (TPR) = TP/(TP + FN), which is same as recall

4. Root Mean Square Error (RMSE):

$$\sum_{i=1}^{n}\frac{(Predicted_i - Actual_i)^2}{n}$$

Reading 6: Currency Exchange Rates

1. Bid-offer Spread = Offer price – Bid price
2. Fwd rate = Spot Exchange rate + $\frac{\text{Forward points}}{10{,}000}$
3. Forward premium/discount (in %) = $\frac{\text{spot exchange rate} - (\text{forward points}/10{,}000)}{\text{spot exchange rate}} - 1$
4. To convert spot rate into forward quote:
   - Spot exchange rate × (1 + % premium)
   - Spot exchange rate × (1 - % discount)
5. Covered interest rate parity:
   - $(1 + i_d) = S_{f/d}(1 + i_f)\left(\frac{1}{F_{f/d}}\right)$
   - $F_{f/d} = S_{f/d}\left(\frac{1+i_f}{(1+i_d)}\right)$
   - Using day count convention:

$$\left(1 + i_d\left[\frac{Actual}{360}\right]\right) = S_{f/d}\left(1 + i_f\left[\frac{Actual}{360}\right]\right)\left(\frac{1}{F_{f/d}}\right)$$

   - $F_{f/d} = S_{f/d}\left(\frac{1 + i_f\left[\frac{Actual}{360}\right]}{1 + i_d\left[\frac{Actual}{360}\right]}\right)$

6. Uncovered Interest Rate Parity :
   - $i_f - \%\Delta S^e{}_{f/d} = i_d$
   - $\%\Delta S^e{}_{f/d} = i_f - i_d$
   - Forward premium or discount:
   - For one year horizon =

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{i_f - i_d}{1 + i_d}\right) \cong S_{f/d}(i_f - i_d)$$

   - Using day count convention:

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{\left[\frac{Actual}{360}\right]}{1 + i_d\left[\frac{Actual}{360}\right]}\right)(i_f - i_d)$$

7. Forward discount or premium as % of spot rate:

$$\frac{F_{f/d} - S_{f/d}}{S_{f/d}} \cong (i_f - i_d)$$

   If uncovered interest rate parity holds

   - $= \frac{F_{f/d} - S_{f/d}}{S_{f/d}} = \%\Delta S^e_{f/d} \cong (i_f - i_d)$

8. Purchasing Power parity (PPP)
   - $P_f = S_{f/d} \times P_d$
   - $S_{f/d} = P_f / P_d$
9. Relative version of PPP = $\%\Delta S_{f/d} = \pi_f - \pi_d$
10. Ex ante version of PPP = $\%\Delta S^e_{f/d} = \pi^e_f - \pi^e_d$
11. Real Exchange Rate

$$q_{f/d} = \left(\frac{S_{f/d}P_d}{P_f}\right) = S_{f/d}\left(\frac{P_d}{P_f}\right)$$

or

$$q_{f/d} = S_{f/d}\left(\frac{CPI_d}{CPI_f}\right)$$

12. Fisher effect:
    - $i_d = r_d + \pi^e_d$
    - $i_f = r_f + \pi^e_f$

