2. INTEREST RATES AND TIME VALUE OF MONEY

The time value of money equates cash flows from different dates. Since cash now is preferred over future cash, we use an interest rate, r, to compare financial instruments with varied payment timings.

If USD 9,524 today equals USD 10,000 in a year, the difference, USD 476, is the compensation for the delay. This results in an interest rate of 5%.

Interest rates can be interpreted in **three** ways.

1) **Required rates of return:** It refers to the minimum rate of return that an investor must earn on his/her investment.

2) **Discount rates:** Interest rate can be interpreted as the rate at which the future value is discounted to estimate its value today.

3) **Opportunity cost:** Interest rate can be interpreted as the opportunity cost which represents the return forgone by an investor by spending money today rather than saving it. For example, an investor can earn 5% by investing $1000 today. If he/she decides to spend it today instead of investing it, he/she will forgo earning 5%.

Interest rate = r = Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

Determinants of Interest Rates

- **Real risk-free interest rate:** It reflects the single-period interest rate for a completely risk-free security when no inflation is expected.
- **Inflation premium:** It reflects the compensation for expected inflation.
  Nominal risk-free rate = *Real risk-free interest rate + Inflation premium*
  - E.g., interest rate on a 90-day U.S. Treasury bill (T-bill) refers to the nominal interest rate.
- **Default risk premium:** It reflects the compensation for default risk of the issuer.
- **Liquidity premium**: It reflects the compensation for the risk of loss associated with selling a security at a value less than its fair value due to high transaction costs.
- **Maturity premium**: It reflects the compensation for the high interest rate risk associated with long-term maturity.

Practice: Example 1 from the CFA Institute's Curriculum.

3. RATES OF RETURN

Risk and return are the two most important characteristics of an investment.

Return

Financial assets generate two types of return for investors.

1) Periodic income i.e. cash dividends or interest payments.
2) Capital gain or loss i.e. changes in the price of a financial asset.

Note:
Some assets provide return only through price movement or through periodic income (e.g., non-dividend paying stocks, retirement annuities etc.)

Stanley Notes™ 2025

Copyright © FinQuiz.com. All rights reserved.

1) Holding Period Return (HPR)

A holding period return refers to the return earned by an investor from holding an asset for a single specified period of time e.g. 1 day, 1 week, 1 month, 5 years etc.

$$Total\ return = Capital\ gain\ (or\ loss) yield + Dividend\ yield$$

$$R = \frac{(P_1 - P_0) + I_1}{P_0} - 1$$

where,

P_0 = *price at the beginning of period*

P_1 = *price at the end of period*

I = *income*

NOTE:

HPR for period longer than one year. For example, the holding period return for 3-year can be computed as follows:

Three year holding period return = $[(1 + R_1) \times (1 + R_2) \times (1 + R_3)] - 1$

where, R_i = annual return

2) Arithmetic or Mean Return

Arithmetic mean (AM) is the sum of all returns divided by the total number of observations.

$$\bar{R}_i = \frac{R_{i1} + R_{i2} + \cdots + R_{i.T-1} + R_{iT}}{T} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$$

- The AM is the ***average*** of the returns earned on a ***unit*** of investment at the ***beginning*** of each holding period.
- AM reflects a ***constant*** dollar investment at the beginning of each time period.

Advantages:

- The arithmetic mean return is easy to compute.
- It is the most commonly used value in statistics.
- It has known statistical properties i.e. standard deviation (S.D.).

S.D. reflects the dispersion of the observations around the mean. It can be used to determine whether the mean return is statistically different from zero.

3) Geometric Mean Return

$$\overline{R}_{Gi} = \sqrt[T]{(1 + R_{i1}) \times (1 + R_{i2}) \times (1 + R_{iT-1}) \times (1 + R_{iT})} - 1$$

where,

R_{it} = *return in period t*

T = *total number of periods*

- Geometric mean (GM) measures the average or compound growth rate over ***multiple*** periods.
- The GM return reflects compounding of returns and it is assumed that the investment amount is not reset at the beginning of each year. Thus, GM reflects a "buy-and-hold" strategy.

Advantage:

- GM return provides a more accurate measure of return that an investor will earn than an AM return.

Important to Note:

- If all returns (values) are identical ➔ the geometric mean = arithmetic average.
- If the return values are volatile ➔ the geometric mean < arithmetic average.
- When the actual holding period returns are not equal, ➔ AM is biased upwards.
- The greater the volatility of returns, the greater the difference between geometric mean and arithmetic average.

Geometric mean versus Arithmetic mean:

- The geometric mean return represents the growth rate or compound rate of return on an investment.
- The arithmetic mean return represents an average single-period return on an investment.
- The geometric mean is *always* ≤ arithmetic mean.

Copyright © FinQuiz.com. All rights reserved.

- When there is no variability in the observations (i.e. when all the observations in the series are the same), geometric mean = arithmetic mean
- The greater the variability of returns over time, the more the geometric mean will be lower than the arithmetic mean.
- The geometric mean return decreases with an increase in standard deviation (holding the arithmetic mean return constant).

In addition, the geometric mean ranks the two funds differently from that of an arithmetic mean.

Practice: Example 2,3,4,and 5 from the CFA Institute's Curriculum.

4) The Harmonic Mean

$$Harmonic\ Mean\ \bar{X}_H = n / \sum_{i=1}^{n} \left(\frac{1}{X_i}\right)$$

with $X_i > 0$ for i = 1,2, …, n.

- It is a special case of the weighted mean in which each observation's weight is inversely proportional to its magnitude.

Cost Averaging is an investment strategy involving periodic investments of fixed amount of money. Harmonic mean is appropriate when averaging the ratios, and the ratios are repeatedly applied to a fixed quantity to yield a variable number of units.

In cost averaging, the ratios to be averaged are prices per share at the date of the purchase, and then apply those prices to a constant amount of money to yield a variable number of shares.

Important to note:

- Harmonic mean formula cannot be used to compute average price paid when different amounts of money are invested at each date.
- When all the observations in the data set are the same, geometric mean = arithmetic mean = harmonic mean.
- When there is variability in the observations, harmonic mean < geometric mean < arithmetic mean.

Practice: Example 6 and 7 from the CFA Institute's Curriculum.

Besides arithmetic, geometric, and harmonic means, **trimmed** and **winsorized means** are used to reduce the influence of outliers in a dataset.

Outliers are extreme values (outliers) in a dataset may reflect a rare value in the population or an error.

5) Trimmed Mean

Trimmed Mean is the arithmetic mean of the distribution computed after excluding a stated small % of the lowest and highest values.

6) Winsorized Mean

In a winsorized mean, a stated % of the lowest values is assigned a specified low value and a stated % of the highest values is assigned a specified high value and then a mean is computed from the restated data.

E.g., in a 95% winsorized mean,

- The bottom 2.5 % of values are set = 2.5th percentile value.
- The upper 2.5% of values are set = 97.5th percentile value.

Practice: Exhibit 8 and Question-Set from the CFA Institute's Curriculum.

Copyright © FinQuiz.com. All rights reserved.

4. MONEY-WEIGHTED AND TIME-WEIGHTED RETURN

The money-weighted rate of return (MWR) measures the compound growth rate in the value of all funds invested in the account over the entire evaluation period. It represents an internal rate of return (IRR) of an investment. Like IRR,

- Amounts invested are cash *outflows* for the investor.
- Amounts returned or withdrawn by the investor are a cash *inflow* for the investor.
- The money that remains at the end of an investment cycle is a cash *inflow* for the investor.

The IRR is the discount rate at which the sum of present values of these cash flows = 0. IRR is computed as follows:

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

where,
T = number of periods
CF_t = cash flow at time t

Example: Assume,

- Amount invested in a mutual fund at the beginning of 1st year = $100
- Amount invested in a mutual fund at the beginning of 2nd year = $950
- Amount withdrawn at the end of 2nd year = $350
- Balance at the end of year 3 = +1270

CF_0 = –100
CF_1 = –950
CF_2 = +350
CF_3 = +1,270

$$\frac{CF_0}{(1+IRR)^0} + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \frac{CF_3}{(1+IRR)^3}$$
$$= \frac{-100}{1} + \frac{-950}{(1+IRR)^1} + \frac{+350}{(1+IRR)^2} + \frac{+1270}{(1+IRR)^3} = 0$$

IRR = 26.11%

Important to note:

- When funds are contributed to an account prior to a period of strong or positive (negative) performance, MWR> (<) AM and GM.
- When funds are withdrawn from an account prior to a period of strong or positive (negative) performance, MWR< (>) AM and GM.

Limitation of money weighted return: It cannot be used to compare return between different individuals or different investment opportunities.

Refer to: Example below Exhibit 11, CFA Institute's Curriculum.

Practice: Example 8 from the CFA Institute's Curriculum.

Time-Weighted Returns

The time-weighted rate of return (TWR) measures the compound rate of growth over a stated evaluation period of one unit of money initially invested in the account.

- In TWR, the account needs to be valued whenever an external cash flow occurs.
- TWR measures the actual rate of return earned by the portfolio manager.
- TWR is preferred to use to evaluate the performance of the portfolio manager *when the manager has no control over the deposits and withdrawals made by clients.*

Three steps to compute an exact TWR of return on a portfolio are as follows:

i) Compute portfolio value prior to any significant cash inflows/outflows.
ii) Compute holding period return for each sub-period.
iii) Link together sub-period returns to compute to compute the TWR for the entire evaluation period.

Copyright © FinQuiz.com. All rights reserved.

Note: For investment period >1year, geometric mean of annual return is used to calculate the TWR.

TWR requires determining a value for the account each time any cash flow occurs which can be costly.

One way to handle this issue is to calculate a reasonable approximation by valuing the portfolio at frequent interval. Higher the frequency, more accurate the approximation. However, marking to market an account on a daily basis is administratively more cumbersome

Time weighted return = $r_{twr} = [(1 + rt,1) \times (1 + rt,2) \times \ldots \times (1 + rt,n)]^{1/N} - 1$

Practice: Example 9 and 10 CFA Institute's Curriculum.

5. ANNUALIZED RETURN

A return for a period less than one year can be annualized by compounding it by the number of periods in a year.

$$r_{annual} = (1 + r_{period})^{c} - 1$$

where,
c = number of periods in a year
e.g. c = 4 for a quarter, c= 12 for a month.

Note

- A monthly return is compounded 12 times
- A weekly return is compounded 52 times
- A quarterly return is compounded 4 times
- A daily returns is compounded 365 times

$$r_{weekly} = (1 + r_{daily})^{5} - 1;$$
$$r_{weekly} = (1 + r_{annualy})^{1/52} - 1$$

Example:
Suppose, weekly return = 0.2%.Then the compound annual return is computed as follows:

$$r_{annual} = (1 + r_{weekly})^{52} - 1 = (1 + 0.2\%)^{52} - 1$$
$$= (1.002)^{52} - 1 = 0.1095 = 10.95\%$$

Suppose, return for 15 days = 0.4%, then the compound annual return is:

$$r_{annual} = (1 + r_{15})^{365/15} - 1 = (1 + 0.4\%)^{365/15} - 1$$
$$= (1.004)^{365/15} - 1 = 0.1020 = 10.20\%$$

Suppose, return for 18-month = 20%, then the compound annual return is:

$$r_{annual} = (1 + r_{18_{month}})^{2/3} - 1 = (1 + 0.20)^{2/3} - 1$$
$$= (0.1292) = 12.920\%$$

Benefit: By annualizing returns, we can compare different assets and over different time periods.

Limitation: When returns are annualized, it is assumed that money can be reinvested repeatedly and earn a similar return.

Continuously Compounded Rates of Return

It is important to note that when a stock's continuously compounded return is normally distributed, then future stock price is necessarily lognormally distributed.

$$S_T = S_0 \exp(r_{0,T})$$

Where,
exp = e
$r_{0,t}$ = Continuously compounded return from 0 to T

- Since S_T is proportional to the log of a normal random variable → S_T is lognormal.

Price relative = Ending price / Beginning price = $S_{t+1}/ S_t = 1 + R_{t, t+1}$

Copyright © FinQuiz.com. All rights reserved.

where,
$R_{t,\,t+1}$ *= holding period return on the stock from t to t + 1.*

Continuously compounded return associated with a holding period from t to t + 1:

$r_{t,\,t+1}$= ln(1 + holding period return)

Or

$r_{t,\,t+1}$ = ln(price relative) = ln (S_{t+1} / S_t) = ln $(1 + R_{t,t+1})$

NOTE: *The continuously compounded return < associated holding period return.*

Continuously compounded return associated with a holding period from 0 to T:

$R_{0,T}$= ln (S_T / S_0)

Or

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

Where,
$r_{T\text{-}l,\,T}$ = One-period continuously compounded returns

Example: Suppose, one-week holding period return = 0.04.

Equivalent continuously compounded return =
One-week continuously compounded return = ln (1.04)
= 0.039221

Practice: Example 11, 12 and 13 from the CFA Institute's Curriculum.

6. OTHER MAJOR RETURN MEASURES AND THEIR APPLICATIONS

Gross and Net Return

Gross return = Return – trading expenses (e.g. commissions) – any other expense directly related to the generation of returns

- Gross return is an appropriate measure to evaluate and compare the investment skill of asset managers because it excludes any fees related to the management and administration of an investment.
- Expenses that are not directly related to the generation of returns include management expenses, custodial fees, taxes etc.

Net Return = Gross Return – all managerial and administrative expenses

- Net return reflects the return that is actually earned by investors.

Pre-Tax and After-Tax Nominal Return

Capital gains and income return are usually taxed differently. Typically,

- Long-term capital gains receive preferential tax treatment.
- Interest income is taxed as ordinary income.
- Dividend income can be taxed as ordinary income, can have a lower tax rate, or can be exempt from taxes.

After-tax nominal return = Total return –any allowance for taxes on dividends, interest & realized gains

Tax liability can be minimized by:

- Selecting securities that receive favorable tax treatment.
- Reducing trading turnover.

Real Returns

$$(1 + r) = (1 + r_{rF}) \times (1 + \pi) \times (1 + RP)$$
$$(1 + r_{real}) = (1 + r_{rF}) \times (1 + RP) \; or$$
$$(1 + r_{real}) = (1 + r) \div (1 + \pi)$$

where,

r = *Nominal return*

r_{rF} = *Real risk-free return* ➔ *It reflects a compensation for postponing current consumption.*

π = *Inflation* ➔ *It reflects a compensation for loss of purchasing power.*

RP = *Risk premium* ➔ *It is a compensation for assuming risk.*

After-Tax Real Return: The after-tax real return represents the return that is received by an investor in return for postponing consumption and assuming risk after paying taxes on investment returns.

Benefits of using Real Returns:

- Real returns can be used to compare returns across time periods.
- Real returns can be used to compare returns among countries when returns are expressed in local currencies.
- After-tax real return can be used as a benchmark for making investment decisions. However, it is not commonly estimated because of the difficulty to estimate a general tax component applicable to all investors. Tax component depends on various factors i.e.
  - Investor's marginal tax rate.
  - Investment time horizon i.e. long-term v/s short-term
  - Type of account the asset is held in e.g. tax-exempt, tax-deferred, or normal.

Leveraged Return

There are **two** ways to leverage a return.

1) **Using futures contracts**. For example, if 5% of the notional value of the asset is invested, then the leveraged return i.e. the return on investor's own money (whether positive or negative) is 20 times the actual return of the underlying security.
2) **Borrowing money to purchase the asset**. For example, if 50% of the amount invested is borrowed, then the asset return (both gains and losses) to the investor is doubled.

Practice: Example 14 and 15 from the CFA Institute's Curriculum.

Practice: End of Chapter Practice Problems from the CFA Institute's Curriculum and Questions from FinQuiz Question-bank.

Copyright © FinQuiz.com. All rights reserved.

Stanley Notes™ 2025

1. INTRODUCTION

Individuals face financial choices involving savings, borrowing, and assessing costs. Investment analysis often requires evaluating transactions with both current and future cash flows.

- **Money** has a time value because a unit of money received today is worth more than a unit of money to be received tomorrow.

This means that a smaller amount of money today may be worth more than a larger amount of money in the future.

2. TIME VALUE OF MONEY IN FIXED INCOME AND EQUITY

- The timing of cash flows in financial instruments impacts their value, with earlier cash inflows being more valuable due to investment and interest earnings.
- The time value of money is a concept for comparing the current value of future cash flows.
- The discount rate r, reflecting risk and return, is crucial in determining the present value (PV) of future cash flows.
- Higher-risk investments have higher discount rates, reducing their present value.

The relationship between present value (PV) and future value (FV) can be represented by the equation:

$$FV_t = PV(1+r)^t$$

where,

PV	=	Present value of the investment
FV_t	=	Future value of the investment t periods from today
t	=	Total number of cash flows or the number of a specific period
r	=	Interest rate per period
$(1+r)^N$	=	FV factor

For continuous compounding, the relationship is represented by:

$$FV_t = PV\ e^{rt}$$

PV can also be expressed in terms of future values, using the following equation:

$$PV = FV_t(1+r)^{-t} \text{ or } PV = \frac{FV_t}{(1+r)^t}$$

and its continuous time equivalent is

$$PV = FV\ e^{-rt}$$

Fixed-Income Instruments and the Time Value of Money

Fixed-income instruments involve borrowing from investors with a promise to repay in the future. They have a discount rate (interest rate) and yield-to-maturity (YTM). Cash flows for these instruments follow three patterns:

1. **Discount Instrument:** Investor buys at PV and receives FV at maturity. Interest earned is the difference between FV and PV.

   $$\text{PV(Discount Bond)} = \frac{FV_t}{(1+r)^t}$$

   where, FV_t is the principal paid at time t and r is the market discount rate.

2. **Coupon Instrument:** Investor buys at PV and receives periodic interest payments (PMT) throughout the instrument's life. The last payment includes final interest and FV. The present value (PV) of a Coupon Bond is calculated using the formula:

$$PV(Coupon\ Bond) = \frac{PMT_1}{(1+r)^1} + \frac{PMT_2}{(1+r)^2} \cdots \frac{PMT_N + FV_N}{(1+r)^1}$$

Coupon Instruments represent a structure with periodic interest payments and principal repayment at maturity.

Payments are regular, usually semiannually but can be annually, quarterly or monthly.

Example: A 5-year corporate bond with a 6% coupon rate pays $60 per year in coupon payments ($1000 face value x 6%).

Perpetual Bonds are unique variation that have no maturity date and are often used for equity-like financing. Their PV for an indefinite period simplifies to:

PV(Perpetual Bond) = PMT/r.

Example: Suppose, a stock pays constant dividend of $10 per year, the required rate of return is 20%. Then the PV is calculated as follows.

PV = $10 / 0.20 = $50

3. **Annuity Instruments (level payments):** Investor purchases at PV and receives regular uniform payments (A) until maturity, covering both interest and principal repayment.

Annuity Instruments involve regular, uniform payments (A) and are used for various financial instruments such as rent, lease, mortgage, car loans, and retirement annuities.

$$A = \frac{r(PV)}{1-(1+r)^{-t}}$$

A= periodic cash flow.
r = market interest rate per period.
PV = initial value or principal of the loan or bond.
t = total number of payment periods.

Practice: Example 1 to 5 from the CFA Institute's Curriculum.

Equity Instruments and the Time Value of Money

Equity investments, like preferred or common stock, offer ownership in a company, entitling investors to potential dividends. Contrary to fixed-income instruments, they have no set maturity date, existing until company events like sale, restructuring, or liquidation. Their valuation involves discounting future cash flows using an expected rate of return.

Three common valuation approaches based on dividend cash flows are:

- Constant Dividends
- Constant Dividend Growth Rate
- Changing Dividend Growth Rate

1. **Constant Dividends:** An investor buys stock and receives a consistent dividend (D).

For a stock paying **constant dividends** forever, the formula mirrors the perpetual bond equation, resulting in:

$$PV_t = \frac{D_t}{r}$$

2. **Constant Dividend Growth Rate:** A stock is bought at a price (PV), and the dividend received starts from one period (Dt+1) and grows consistently at rate g.

For dividends growing at a constant rate:

$$D_{t+1} = D_t(1+g)$$

and the value is:

$$PV_t = \frac{D_t(1+g)}{(r-g)}$$

assuming r – g > 0.

3. **Changing Dividend Growth Rate:** Here, dividends initially grow rapidly but slow down over time as the company matures.

For stocks with changing growth, a two-stage model is common. It begins with a high growth rate for a few years, followed by a slower, constant rate. The formula is:

$$PV_t = \sum_{i=1}^{n} \frac{D_t(1+g_s)^i}{(1+r)^i} + \frac{E(S_{t+n})}{(1+r)^n}$$

where $E(S_{t+n})$ is the stock value in n periods and can be defined as:

$$E(S_{t+n}) = \frac{D_{t+n+1}}{r-g_l}$$

Practice: Example 6 and 7 from the CFA Institute's Curriculum.

3. IMPLIED RETURN AND GROWTH

Market participants can solve for the implied return or growth rate of an asset by using the current price and future cash flow attributes. This provides insight into market expectations that are reflected in the asset's market price.

Implied Return for Fixed-Income Instruments

1. **Implied Return:** When considering fixed-income instruments, their characterized returns come from contractual interest and principal cash flows.
   - The implied return of a fixed-income instrument is the yield-to-maturity (YTM).
   - The YTM is the interest rate that equates the present value of all future cash flows from the instrument to its current market price.
2. **Discount Bond:** This type of bond offers a single principal cash flow at maturity. The implied return is calculated as:

$$r = \left(\frac{FV_t}{PV}\right)^{1/t} - 1$$

3. **Periodic Interest Payments:** These instruments offer cash flows throughout their lifespan. The YTM, representing a single implied market discount rate for all cash flows, assumes that an investor expects to receive all cash flows through maturity and will reinvest any cash received at the same YTM.
4. **YTM Calculation:** Determining YTM requires iterative methods, often using tools like spreadsheets or calculators. It's not easily solved with a standard equation but can be found by iterating to solve for r using the equation:

$$\text{PV (Coupon Bond)} = \sum_{i=1}^{N} \frac{PMT_i}{(1+r)_i}$$

Practice: Example 8 & 9 from the CFA Institute's Curriculum.

Equity Instruments, Implied Return, and Implied Growth

1. **Implied Return and Growth:** The price of a stock captures both the required return and the growth in cash flows. If dividends grow consistently, the stock's implied return is the sum of its expected dividend yield and the growth rate.

Implied Return: $r = \frac{D_t(1+g)}{PV_t} + g$

Implied Growth: $g = r - \frac{D_{t+1}}{PV_t}$

where $D_t(1+g) = D_{t+1}$

Hence, given the next expected dividend, the implied growth can be determined by subtracting the expected dividend yield from the required return.

2. **Price-to-Earnings Ratio (P/E):** Rather than just comparing share prices in monetary terms, it's common to use the P/E ratio, which relates share price to earnings per share. This ratio can be applied to individual stocks or broader indices like the S&P 500 or FTSE 100.

As we know, $PV_t = \frac{D_t(1+g)}{r-g}$ then P/E can be denoted as:

$$\frac{PV_t}{E} = \frac{D_t}{E_t} \times \frac{(1+g)}{r-g}$$

The P/E ratio is essentially the proportion of earnings paid to shareholders as dividends, known as the ***dividend payout ratio***.

3. **Implications of P/E:** Given the P/E ratio and dividend payout ratio, we can deduce either the required return or the implied dividend growth rate, considering assumptions about the other.

Analyzing the implied growth rate allows comparison with a company's expected and historical growth, aiding in valuation assessments.

4. **Forward P/E Ratio:** This ratio relates a share price to its projected earnings for the next period. It's indicative that:
   - A higher expected dividend payout ratio and higher growth will positively influence the forward P/E ratio.
   - A higher required return will negatively influence the forward P/E ratio.

$$\frac{PV_t}{E_{t+1}} = \frac{\frac{D_{t+1}}{E_{t+1}}}{r-g}$$

In essence, the implied return, growth, and P/E ratios provide valuable insights for investors, helping them to make informed decisions and evaluate stocks in relation to expected growth, dividend yields, and required returns.

Practice: Example 10 & 11 from the CFA Institute's Curriculum.

4. CASH FLOW ADDITIVITY

Cash Flow Additivity Principle - states that the present value of a series of future cash flows is the sum of the present values of individual cash flows.

- Ensuring that market prices adhere to this principle prevents the possibility of riskless profit (arbitrage) without transaction costs.

Implied Forward Rates Using Cash Flow Additivity

In the context of risk-free discount bonds, investors can use cash flow additivity to compare different investment strategies. Forward rates should be set so that investors can't earn riskless arbitrage profits.

Example:

Assume an investor is considering investing in a two-year bond with a yield of 3.5%. The investor can also invest in a one-year bond with a yield of 2.5%.

The cash flow additivity principle can be used to calculate the implied forward rate for the second year of the two-year bond.

To do this, the investor would first calculate the present value of the two-year bond using the one-year bond yield as the discount rate.

The investor would then subtract the present value of the one-year bond from the present value of the two-year bond to get the implied forward rate for the second year.

In this example, the implied forward rate would be 4.51%.

Calculation:

One-year bond rate r = 2.5%
Two-year bond rate r = 3.5%

Two-Year Bond Future Value Calculation: If an investor invests $1 in a two-year bond with a yield of 3.5%, at the end of two years, they will have:

$$FV_{2\ yrs} = 1(1+r_2)^2$$

One-Year Bond Future Value Calculation: If the investor invests that same $1 in a one-year bond with a yield of 2.5%, at the end of one year, they will have:

$$FV_{1\ yr} = 1(1+r_1)^1$$

Implied Forward Rate Calculation: Let $F_{1,1}$ be the one-year forward rate starting in one year. At the end of the second year, using the one-year bond and then reinvesting at the implied forward rate, the investor will have:

$$FV_{2\ yrs} = FV_{1\ yr}\left(1+F_{1,1}\right)$$

Using the cash flow additivity principle:
$FV_{2\ yrs} = FV_{1\ yr}\left(1+F_{1,1}\right)$

Equating our two future value equations, we have:

$$1(1+r_2)^2 = 1(1+r_1)\left(1+F_{1,1}\right)$$

Solving for $F_{1,1}$:

$$F_{1,1} = \frac{(1+r_2)^2}{(1+r_1)} - 1 = \frac{(1+0.035)^2}{1+0.025} - 1 = 4.51\%$$

Therefore, 4.51% is the implied forward rate for the second year

Practice: Example 12 & 13 from the CFA Institute's Curriculum.

Forward Exchange Rates Using No Arbitrage

The cash flow additivity principle can also be used to calculate the forward exchange rate, which is the rate at which investors expect to be able to exchange one currency for another for a given period in time.

Example:
Assume an investor is considering investing in a six-month Japanese Treasury bill with a yield of 0.05%. The investor can also invest in a six-month US Treasury bill with a yield of 2.00%. Suppose the current exchange rate is 130.61 JPY/USD.

The cash flow additivity principle can be used to calculate the forward exchange rate for the Japanese yen against the US dollar.

The no-arbitrage principle in the foreign exchange market implies that an investor should not be able to earn a risk-free profit by borrowing in one currency, converting at the spot exchange rate, investing in the other currency, and converting back at the forward exchange rate. This principle leads to the **Interest Rate Parity (IRP)** relationship.

Calculation:

Japanese 6-month Treasury bill yield = 0.05%
US 6-month Treasury bill yield = 2.00%
Spot exchange rate S_0 = 130.61JPY/USD

$$F = S_0 \times \frac{(1 + r_U)}{(1 + r_J)}$$

Where F is the 6-month forward exchange rate

Plugging in the given data:

$$F = 130.61 \times \frac{(1+0.02)}{(1+0.005)} \approx 133.155$$

So, the no-arbitrage forward exchange rate for six months is approximately 133.155 JPY/USD.

This forward rate ensures that there's no arbitrage opportunity between the spot and forward markets given the interest rate differentials between the US and Japan.

If the forward rate were any different from 133.155 JPY/USD, arbitrageurs would take advantage of the discrepancy until the rate adjusts back to this no-arbitrage level.

Practice: Example 14 from the CFA Institute's Curriculum.

Option Pricing Using Cash Flow Additivity

Cash flow additivity is used to establish no-arbitrage pricing for options. It involves creating replicating portfolios to match future cash flows with a risk-free asset, ensuring that there are no arbitrage opportunities.

The hedge ratio (proportion of the underlying asset) is used to determine option prices.

Example:

Assume an investor is considering buying a call option on a stock. The call option gives the investor the right, but not the obligation, to buy the stock at a certain price on a certain date.

The **cash flow additivity principle** can be used to price the call option. To do this, the investor would first calculate the present value of the call option's payoff in each possible scenario. The investor would then sum the present values of the call option's payoffs to get the price of the call option.

The price of the call option depends on the stock price, the strike price of the call option, the time to expiration of the call option, and the volatility of the stock.

Refer to: Exhibit 14, 15 and 16 and example mentioned under this section from the CFA Institute's Curriculum.

Practice: Example 15 from the CFA Institute's Curriculum.

Practice: End of Chapter Questions from the CFA Institute's Curriculum and from FinQuiz Question-bank.

Stanley Notes™ 2025

1. INTRODUCTION

Data is essential for investment analysis and management, however, converting it into useful information is challenging

This module provides a foundation for understanding important concepts in data analysis, which are essential for investment practitioners at all levels.

2. MEASURES OF CENTRAL TENDENCY AND LOCATION

A measure of central tendency indicates the center of the data. The most used measures of central tendency are:

- Arithmetic mean
- Median
- Mode
- Weighted mean
- Geometric mean
- Harmonic mean

The Arithmetic Mean

It is the sum of the observations in the dataset divided by the number of observations in the dataset.

The terms 'mean' and 'average' are used interchangeably.

The Sample Mean

The sample mean is the arithmetic mean value of a sample; it is computed as:

$$Sample\ mean\ \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

where,
X_i = *i^{th} observation*
n = *number of observations in the sample*

- The sample mean can be computed for individual units or overtime.
- It is not unique i.e. for a given population; different samples may have different means.

Properties of the Arithmetic Mean

Property 1: The sum of the deviations* around the mean is always equal to 0.

*The difference between each outcome and the mean is called a deviation.

Property 2:
The arithmetic mean is sensitive to extreme values i.e., it can be biased upward or downward by extremely large or small observations, respectively.

Advantages of Arithmetic Mean:

- The mean uses all the information regarding the size and magnitude of the observations.
- The mean is also easy to calculate.
- Easy to work with algebraically

Limitation: The arithmetic mean is highly affected by outliers (extreme values).

Outliers

Extreme values (outliers) in a dataset may reflect a rare value in the population or an error.

Three ways of dealing with the outliers are:

1. ***Do nothing. Use the data without any adjustment***

Choose this option when it is important to represent the whole observations and/or outliers contain meaningful information.

2. ***Delete all the outliers***

When this option is chosen, the measure of central tendency in this case is trimmed mean.

Trimmed Mean is the arithmetic mean of the distribution computed after excluding a stated small % of the lowest and highest values.

3. ***Replace the outliers with some other value***

When this option is used, the mean is called winsorized mean.

Winsorized mean: In a winsorized mean, a stated % of the lowest values is assigned a specified low value and a stated % of the highest values is assigned a specified high value and then a mean is computed from the restated data.

E.g., in a 95% winsorized mean,

- The bottom 2.5 % of values are set = 2.5th percentile value.
- The upper 2.5% of values are set = 97.5th percentile value.

The Median

Median is the middle value of a sorted (ascending or descending) list of items.

Steps to compute the Median:

1. Arrange all observations in ascending order i.e., from the smallest to the largest.
2. When the number of observations (n) is odd, the median is the center observation in the ordered list i.e.

   Median will be located at = $\frac{(n+1)}{2}$ position

- (n+1)/2 only identifies the location of the median, not the median itself.

3. When the number of observations (n) is even, then median is the mean of the two center observations in the ordered list i.e.

   Median will be located at mean of $\frac{n}{2}$ and $\frac{(n+1)}{2}$.

Advantage: Median is not affected by extreme observations (outliers).

Limitations:

- It is time consuming to calculate median.
- The median is difficult to compute.
- It does not use all the information about the size and magnitude of the observations.
- It only focuses on the relative position of the ranked observations.

Example:
Suppose current P/Es of three firms are 16.73, 22.02, and 29.30.

n = 3 → (n + 1) / 2 = 4/ 2 = 2nd position.

Thus, the median P/E is 22.02.

The Mode

The mode is the most frequently occurring value in a distribution.

Unimodal Distribution: A distribution that has only one mode is called a unimodal distribution.

Bimodal Distribution: A distribution that has two modes is called a bimodal distribution.

Trimodal Distribution: A distribution that has three modes is called a Trimodal distribution.

A distribution would have no mode when all the values in a data set are different.

Modal Interval: Data with continuous distribution (e.g., stock returns) may not have a modal outcome. In such cases, a modal interval is found i.e., an interval with the largest number of observations (highest frequency). The modal interval always has the highest bar in the histogram.

Important to note: The mode is the only measure of central tendency that can be used with nominal data.

Other Concepts of Mean

i) The Weighted Mean

Weighted mean: It is the arithmetic mean in which observations are assigned different weights. It is computed as:

$$\bar{X}_w = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \cdots + w_n X_n)$$

where,

$X_1, X_2,\ldots,X_n$ = observed values
$w_1, w_2,\ldots,w_3$ = Corresponding weights, sum to 1.

- An arithmetic mean is a special case of weighted mean where all observations are equally weighted by the factor 1/ n (or l/N).
- A positive weight represents a long position and a negative weight represents a short position.

- **Expected value:** When a weighted mean is computed for a forward-looking data, it is referred to as the expected value.

Example:

Weight of stocks in a portfolio = 0.60
Weight of bonds in a portfolio = 0.40
Return on stocks = –1.6%
Return on bonds = 9.1%

A portfolio's return is the weighted average of the returns on the assets in the portfolio i.e.

Portfolio return = ($w_{stock} \times R_{stock}$) + ($w_{bonds} \times R_{bonds}$)
= 0.60(-1.6%) + 0.40 (9.1%) = 2.7%.

ii) The Geometric Mean

Geometric mean (GM): The geometric mean can be used to compute the mean value over time to compute the growth rate of a variable.

$$G = \sqrt[n]{X_1 X_2 X_3 \dots X_n}$$

with Xi ≥ 0 for i = 1, 2, …, n.

Or

$$In\ G = \frac{1}{n}\ In(X_1 X_2 X_3 \dots X_n)$$

or as

$$In\ G = \frac{\sum_{i=1}^{n} In X_n}{n}$$

$$G = e^{lnG}$$

- It should be noted that the geometric mean can be computed only when the product under the radical sign is non-negative.

The geometric mean return over the time period can be computed as:

$$R_{Geom} = [(1 + R_1)(1 + R_2) \dots (1 + R_T)]^{1/T} - 1$$

- Geometric mean returns are also known as compound returns.

Advantages of Measures of Central Tendency:

- Widely recognized.
- Easy to compute.
- Easy to apply.

Geometric mean versus Arithmetic mean:

- The geometric mean return represents the growth rate or compound rate of return on an investment.
- The arithmetic mean return represents an average single-period return on an investment.
- The geometric mean is *always* ≤ arithmetic mean.
- When there is no variability in the observations (i.e. when all the observations in the series are the same), geometric mean = arithmetic mean
- The greater the variability of returns over time, the more the geometric mean will be lower than the arithmetic mean.
- The geometric mean return decreases with an increase in standard deviation (holding the arithmetic mean return constant).

In addition, the geometric mean ranks the two funds differently from that of an arithmetic mean.

ii) The Harmonic Mean

$$Harmonic\ Mean\ \bar{X}_H = n / \sum_{i=1}^{n} \left(\frac{1}{X_i}\right)$$

with $X_i > 0$ for i = 1,2, …, n.

- It is a special case of the weighted mean in which each observation's weight is inversely proportional to its magnitude.

Cost Averaging is an investment strategy involving periodic investments of fixed amount of money. Harmonic mean is appropriate when averaging the ratios, and the ratios are repeatedly applied to a fixed quantity to yield a variable number of units.

In cost averaging, the ratios to be averaged are prices per share at the date of the purchase, and then apply those prices to a constant amount of money to yield a variable number of shares.

Important to note:

- Harmonic mean formula cannot be used to compute average price paid when different amounts of money are invested at each date.

- When all the observations in the data set are the same, geometric mean = arithmetic mean = harmonic mean.
- When there is variability in the observations, harmonic mean < geometric mean < arithmetic mean.

Measures of Location

Quantile or Fractile is a general used for a value at or below which a stated fraction of the data lies.

The following four measures collectively are called quantiles.

1. Quartiles
2. Quintiles
3. Deciles
4. Percentiles

Quartiles, Quintiles, deciles, and Percentiles

1) **Quartiles** divide the distribution into four different parts.

- First Quartile = Q1 = 25th percentile i.e. 25% of the observations lie at or below it.
- Second Quartile = Q2 = 50th percentile i.e. 50% of the observations lie at or below it.
- Third Quartile = Q3 = 75th percentile i.e. 75% of the observations lie at or below it.

2) **Quintiles** divide the distribution into five different parts. In terms of percentiles, they can be specified as P_{20}, P_{40}, P_{60}, & P_{80}.

3) **Deciles** divide the distribution into ten different parts.

4) **Percentiles** divide the distribution into hundred different parts. The position of a percentile in an array with n entries arranged in ascending order is determined as follows:

$$L_y = (n+1)\frac{y}{100}$$

where,
y = % point at which the distribution is being divided.
Ly = location (L) of the percentile (Py).
n = number of observations.

- The larger the sample size, the more accurate the calculation of percentile location.

Calculating 10th percentile (P_{10}): Total number of observations in the table above = n = 50

$L_{10} = (50 + 1) \times (10 / 100) = 5.1$

- It implies that 10th percentile lies between 5th observation (X_5 = 0.26) and 6th observation (X_6 = 1.09).

Thus,
$P_{10} = X_5 + (5.1 – 5) (X_6 – X_5) = 0.26 + 0.1 (1.09 – 0.26)$
$= 0.34\%$

Calculating 90th percentile (P_{90}):
$L_{90} = (50 + 1) \times (90 / 100) = 45.9$

- It implies that 90th percentile lies between the 45th observation (X_{45} = 5.15) and 46th observation (X_{46} = 5.66).

Thus,
$P_{90} = X_{45} + (45.9 – 45) (X_{46} – X_45) = 5.15 + 0.90 (5.66 – 5.15)$
$= 5.61\%$

Calculating 1stQuartile (i.e.P_{25}):
$L_{25} = (50 + 1) \times (25 / 100) = 12.75$

- It implies that 25th percentile lies between the 12th observation (X_{12} = 1.51) and 13th observation (X_{13} = 1.75).

Thus,
$P_{25} = Q_1 = X_{12} + (12.75 – 12) (X_{13} – X_{12}) = 1.51 + 0.75 (1.75 –$
$1.51) = 1.69\%$

Calculating 2nd Quartile (i.e.P_{50}):
$L_{50} = (50 + 1) \times (50 / 100) = 25.5$

- It implies that P_{50} lies between the 25th observation (X_{25} = 2.65) and 26th observation (X_{26} = 2.65).
- Since, $X_{25} = X_{26} = 2.65$, no interpolation is needed.

Thus,
P_{50} = Q2 = 2.65% = Median

Calculating 3rd Quartile (i.e.P_{75}):
$L_{75} = (50 + 1) \times (75 / 100) = 38.25$

- It implies that P_{75} lies between the 38th observation (X_{38} = 3.88) and 39th observation (X_{39} = 4.06).

Thus,
$P_{75} = Q3 = X_{38} + (38.25 - 38)(X_{39} - X_{38})$
$= 3.88 + 0.25(4.06 - 3.88)$
$= 3.93\%$

Calculating 20th percentile (P_{20}) = 1st Quintile:
$L_{20} = (50 + 1) \times (20 / 100) = 10.2$

- It implies that P_{20} lies between the 10th observation (X_{10} = 1.39) and 11th observation (X_{11} = 1.41).

Thus,
1st quintile = $P_{20} = X_{10} + (10.2 - 10)(X_{11} - X_{10}) = 1.39 + 0.20(1.41 - 1.39) = 1.394\%$ or 1.39%

Quantiles in Investment Practice

Quantiles are frequently used by investment analysts to rank performance i.e., portfolio performance. For example, an analyst may rank the portfolio of companies based on their market values to compare performance of small companies with large ones i.e.

- 1st decile contains the portfolio of companies with the smallest market values.
- 10th decile contains the portfolio of companies with the largest market values.

Quantiles are also used for investment research purposes.

Interquartile range (IQR) = Third quartile - First quartile
= Q3 – Q1

- It reflects the length of the interval that contains the middle 50% of the data.
- The larger the interquartile range, the greater the dispersion, all else constant.

Refer to CFA Institute's Curriculum, Exhibit 5 and 6 for Box and Whisker Chart

Practice: Example 1, 2 and Question-Set from the CFA Institute's Curriculum.

3. MEASURES OF DISPERSION

The variability around the central mean is called Dispersion. The measures of dispersion provide information regarding the **spread** or **variability** of the data values.

Relative dispersion: It refers to the amount of dispersion/variation relative to a reference value or benchmark e.g., coefficient of variation. (It is discussed below).

Absolute Dispersion: It refers to the variation around the mean value without comparison to any reference point or benchmark. Measures of absolute dispersion include:

i. Range
ii. Mean absolute deviation
iii. Variance
iv. Standard deviation

The Range

Range = Maximum value - Minimum value

Advantage: It is easy to compute.

Disadvantages:

- It does not provide information regarding the shape of the distribution of data.
- It only reflects extremely large or small outcomes that may not be representative of the distribution.

The Mean Absolute Deviation

Mean absolute deviation (MAD) is the average of the ***absolute*** values of deviations from the mean.

$$MAD = \frac{\sum_{i=1}^{n} |X_t - \bar{X}|}{n}$$

where,
$\bar{X}$ = *Sample mean*
n = *Number of observations in the sample*

- The greater the MAD, the riskier the asset.

Example:
Suppose there are 4 observations i.e., 15, -5, 12, 22.

Mean = (15 – 5 + 12 + 22)/4 = 11%
MAD = (|15 – 11| + |–5 – 11| + |12 – 11| + |22 – 11|)/4 = 32/4 = 8%

Advantage: MAD is superior relative to range because it is based on all the observations in the sample.

Drawback: MAD is difficult to compute relative to range.

Sample Variance and Sample Standard Deviation

Variance: Variance is the average of the squared deviations around the mean.

Standard deviation (S.D.): Standard deviation is the positive square root of the variance. It is easy to interpret relative to variance because standard deviation is expressed in the same unit of measurement as the observations.

1. Sample Variance

It is computed as:

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

where,
$\bar{X}$*=Sample mean*
n = Number of observations in the sample

- The sample mean is defined as an ***unbiased estimator*** of the population mean.
- **(n – 1)** is known as the number of degrees of freedom in estimating the population variance.

2. Sample Standard Deviation

It is computed as:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Important to note:

- The MAD will always be ≤ S.D. because the S.D. gives more weight to large deviations than to small ones.
- When a constant amount is added to each observation, S.D. and variance remain unchanged.

3. Dispersion and the Relationship between Arithmetic and the Geometric Means

$$\text{Geometric mean return} \approx \text{Arithmetic mean return} - \frac{Variance\ of\ return}{2}$$

$$\bar{X}_G \approx \bar{\bar{X}} - \frac{s^2}{2}$$

The larger the variance of the sample, the wider will be the difference between the geometric mean and the arithmetic mean.

Downside Deviation and Coefficient of Variation

Downside deviation is a risk measure that focuses on returns that fall below a minimum threshold or minimum acceptable return as investors are typically only concerned about the values that fall below some minimum target return.

Standard deviation considers all deviations from the mean. Downside deviations only considers the negative deviations from the mean. Therefore, downside deviation is less than the standard deviation S.D.

Target semideviation is the measure of dispersion of observations below the stated target.

$$s_{Target} = \sqrt{\sum_{for\ all\ X_i \le B}^{n} \frac{(X_i - B)^2}{n-1}}$$

where,
B = target value,
n = number of observations.

Example: Stock returns = 16.2, 20.3, 9.3%, –11.1% and –17.0%.

Target return = B = 10%

Target semideviation = $\sqrt{\frac{[(9.3-10.0)2 + (-11.1-10.0)2 + (-17.0-10.0)2]}{5-1}}$

Target Semideviation = $\sqrt{293.675}$ = 17.14%

Coefficient of Variation

Relative dispersion is the amount of dispersion relative to a reference value or benchmark. One of such measures is coefficient of variation.

Coefficient of Variation, (CV), is the ratio of standard deviation of set of values to their mean value.

Coefficient of Variation (CV) measures the amount of risk (S.D.) per unit of mean value.

$$CV = \left(\frac{S}{\bar{X}}\right)$$

When stated in %, CV is:

$$CV = \left(\frac{S}{\bar{X}}\right) \times 100\%$$

where,

s = sample S.D.

$\bar{X}$ = *sample mean.*

- CV is a scale-free measure (i.e., has no units of measurement); therefore, it can be used to directly compare dispersion across different data sets.
- **Interpretation of CV:** The greater the value of CV, the higher the risk.
- An inverse CV $= \left(\frac{\overline{X}}{S}\right)$ ➔ It indicates unit of mean value (e.g., % of return) per unit of S.D.

Practice: Example 3, 4, 5 and Question-Set from the CFA Institute's Curriculum,

4. MEASURES OF SHAPE OF A DISTRIBUTION

Symmetrical return distribution or Normal distribution:

It is a return distribution that is symmetrical about its mean i.e. equal loss and gain intervals have same frequencies. It is referred to as normal distribution.

- A symmetrical distribution has skewness = 0

Characteristics of the normal distribution:

1) In a normal distribution, mean = median.
2) A normal distribution is completely described by two parameters i.e. its mean and variance.

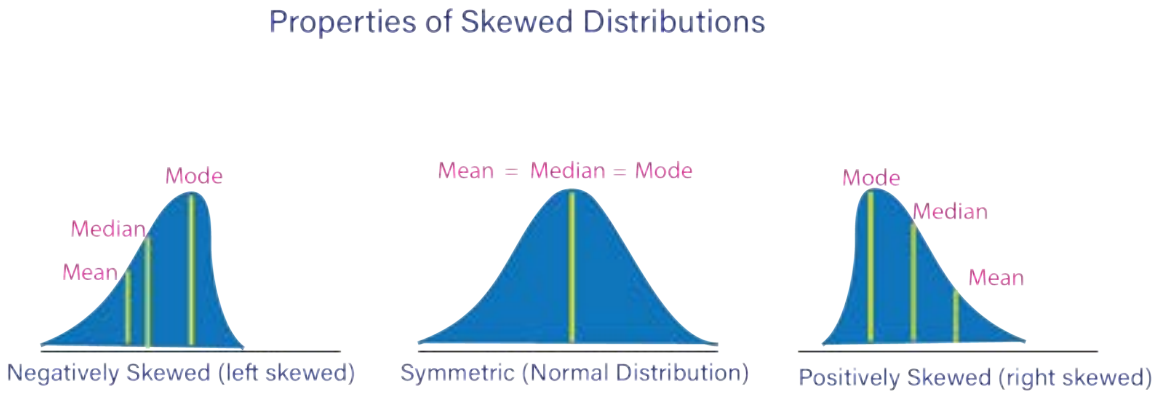
Skewed distribution: The distribution that is not symmetrical around the mean is called skewed.

a) **Positively skewed or right-skewed Distribution:** It is a return distribution that reflects *frequent* small losses and a *few* extreme gains i.e. ***limited*** but frequent downside.

- It has a long tail on its right side.
- It has skewness > 0.
- In a positively skewed unimodal distribution➔ mode < median < mean.
- Generally, investors prefer positive skewness (all else equal).

b) **Negatively skewed or left-skewed Distribution:** It is a return distribution that reflects *frequent* small gains and a *few* extreme losses i.e. ***unlimited*** but less frequent upside.

- It has a long tail on its left side.
- It has skewness < 0.
- In a negatively skewed unimodal distribution ➔ mean < median < mode.



Sample skewness (for large values of n ≥ 100) is computed as follows:

$$S_K \approx \left(\frac{1}{n}\right)\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{S^3}$$

n = number of observations in the sample
s = sample S.D.

Note: Cubing in the formula preserves the sign of the deviation from the mean.

The Shape of the Distributions: Kurtosis

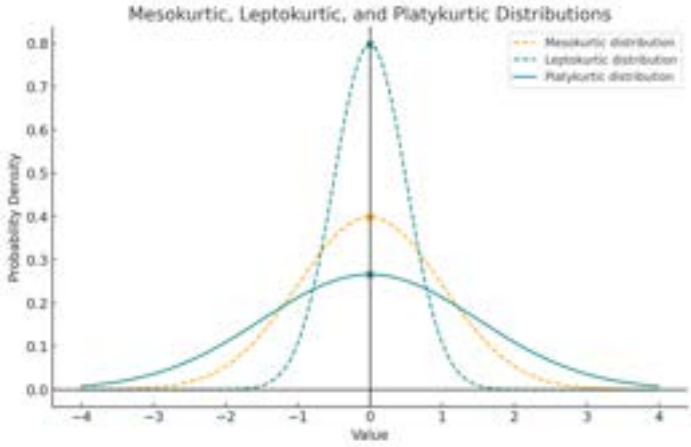
Kurtosis is used to identify how peaked or flat the distribution is relative to a normal distribution.

Leptokurtic: It is a distribution that is more peaked (i.e., greater number of observations closely clustered around the mean value) and has fatter tails (i.e., greater number of observations with large deviations from the mean value) than the normal distribution.

- It has more frequent *extremely* large deviations from the mean than a normal distribution.
- Ignoring fatter tails in analysis results in underestimation of the probability of extreme outcomes.
- The more leptokurtic the distribution is, the higher the risk.

Platykurtic: It is a distribution that is less peaked than normal.

Mesokurtic: It is a distribution that is identical to the normal distribution.



The Sample excess kurtosis (for larger sample size(n)) is computed as:

$$\boldsymbol{K_E = \left(\left(\frac{1}{n}\right)\frac{\sum_{i=1}^{N}(X_i - \bar{X})^4}{s^4}\right) - 3}$$

- For a normal distribution (mesokurtic), kurtosis = 3.0.
- For a leptokurtic distribution, kurtosis> 3.
- For a platykurtic distribution, kurtosis < 3.

NOTE: Kurtosis is free of scale (i.e., it has no units of measurement).

It is always positive number because the deviations are raised to the 4^{th} power.

Excess Kurtosis = Kurtosis – 3

- A normal or mesokurtic distribution has excess kurtosis = 0.
- A leptokurtic distribution has excess kurtosis > 0.
- A platykurtic distribution has excess kurtosis < 0.

Practice: Example 6 and Question-Set from the CFA Institute's Curriculum.

5. CORRELATION BETWEEN TWO VARIABLES

Correlation measures the linear relationship between two variables.

Firstly, determine how two variables vary together their *covariance*.

Sample covariance measures how to variables in a sample move together i.e., measures the joint variability of two random variables.

The sample covariance is calculated as:

$$s_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

where,
n = sample size
X_i *= ith observation on variable X*
$\bar{X}$ *= mean of the variable X observations*
Y_i *= ith observation on variable Y*
$\bar{Y}$ *= mean of the variable Y observations*

Positive Covariance: When both variables tend to move in the same direction, they are referred to as positively correlated and have positive covariance.

Negative Covariance: When both variables tends to move in the opposite direction, they are referred to as negatively correlated and have negative covariance.

Covariance can range from $-\alpha$ to $+\alpha$.

The covariance number doesn't tell if the relationship between two variables is strong or weak. It only tells the direction of the relationship.

Correlation coefficient measures the *direction and strength* of linear association between two variables. The correlation coefficient between two assets X and Y can be calculated using the following formula:

$$r_{XY} = \frac{covariance\ of\ X\ and\ Y}{\left(\begin{matrix}sample\ standard\\ deviation\ of\ X\end{matrix}\right)\left(\begin{matrix}sample\ standard\\ deviation\ of\ Y\end{matrix}\right)}$$

$$= \frac{cov_{XY}}{(s_X)(s_Y)}$$

or

$$r = \frac{cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}}$$

NOTE: *Unlike Covariance, Correlation has no unit of measurement; it is a simple number.*

Example:

$Cov_{xy} = 47.7$
$S_x^2 = 40$
$S_y^2 = 250$

$$r = \frac{47.78}{\sqrt{(40)(250)}} = 0.478$$

Properties of Correlation

1. The correlation coefficient can range from -1 to +1.
2. Two variables are perfectly positively correlated if correlation coefficient is +1.
3. Correlation coefficient of -1 indicates a perfect inverse (negative) linear relationship.
4. When correlation coefficient equals 0, there is no linear relationship.
5. The closer the correlation coefficient is to +1 or -1, the stronger the relationship.

Scatter plots are useful tool for a sensible interpretation of a correlation coefficient as it demonstrates the relationship graphically.

Refer to Exhibit 26, CFA Institute's Curriculum for "Scatter Plots Showing Various Degrees of Correlation".

Limitations of Correlation Analysis

1. **Linearity:** Correlation only measures linear relationships properly.

2. **Outliers:** Correlation may be an unreliable measure when outliers are present in one or both of the series.

3. **No proof of causation:** Based on correlation we cannot assume x causes y; there could be third variable causing change in both variables.

4. **Spurious Correlations:** Spurious correlation is a correlation in the data without any causal relationship. This may occur when:

   i. two variables have only chance relationships.
   ii. two variables that are uncorrelated but may be correlated if mixed by third variable.
   iii. correlation between two variables resulting from a third variable.

NOTE: *Spurious correlation may suggest investment strategies that appear profitable but actually would not be so, if implemented.*

5. **Correlation does not tell the whole story:** Knowing two variables' means, standard deviations and their correlation does not tell the whole story.

Practice: Example 7 and Question-Set from the CFA Institute's Curriculum.

Practice: End of Chapter Questions from CFA Institute's Curriculum & FinQuiz Question-bank.

2. EXPECTED VALUE AND VARIANCE

Expected value of a random variable: The expected value of a random variable is the probability-weighted average of the possible outcomes of the random variable.

Variance of a random variable: The variance of a random variable is the expected value of squared deviations from its expected value:

$$\sigma^2 (X) = E\{[X - E(X)]^2\}$$

where,

σ^2 (X) = variance of random variable X

- Variance ≥ 0.
- When variance = 0, there is no dispersion or risk → the outcome is certain and quantity X is not random at all.
- The higher the variance, the higher the dispersion or risk, all else equal.

Standard deviation: It is the positive square root of variance. It is easier to interpret than variance because it is in the same units as the random variable.

Example:

EPS ($)	Probability
2.60	0.15
2.45	0.45
2.20	0.24
2.00	0.16
	1.00

Expected value of EPS = E (EPS) = 0.15 ($2.60) + 0.45 ($2.45) + 0.24 ($2.20) + 0.16 ($2.00) = $2.3405

σ^2 (EPS) = P ($2.60) [$2.60 – E (EPS)] ^{2} + P ($2.45) [$2.45 – E (EPS)] ^{2} + P ($2.20) [$2.20 – E (EPS)] ^{2} + P ($2.0) [$2.0 – E (EPS)] ^{2}

σ^2 (EPS) = 0.15 ($2.60 – $2.34)^2 + 0.45 ($2.45 – $2.34)^2 + 0.24 ($2.20 – $2.34)^2 + 0.16 ($2.00 – $2.34)^2
= 0.01014 + 0.005445 + 0.004704 + 0.018496
= $0.038785

S.D of EPS = $\sqrt{\$0.038785}$ = $0.20

Source: CFA® Institute's Curriculum.

Conditional expected values: The conditional expected value refers to the expected value of a random variable X given an event or scenario S. It is denoted as E(X | S) i.e.

$$\mathbf{E(X \mid S)} = P(X_1|S)X_1 + P(X_2|S)X_2 \ldots + P(X_n|S)X_n$$

Conditional Variance: The conditional variance refers to the variance of a random variable X given an event or scenario.

Practice: Example 1 from the CFA Institute's Curriculum.

3. PROBABILITY TREES AND CONDITIONAL EXPECTATIONS

The Total Probability Rule for Expected Value: It is expressed as follows:

$$\mathbf{E(X)} = E(X \mid S)P(S) + E(X \mid S^C)\,P(S^C)$$

$$\mathbf{E(X)} = E(X \mid S_1)P(S_1) + E(X \mid S_2)\,P(S_2) + \ldots + E(X \mid S_n)\,P(S_n)$$

where,

$E(X \mid S_i)$ = Expected value of X given Scenario i
$P(S_i)$ = Probability of Scenario i

$S_1, S_2 \ldots, S_n$ are mutually exclusive and exhaustive scenarios or events.

Example: Suppose,

- Current Expected EPS of BankCorp = $2.34
- Probability that BankCorp will operate in a declining interest rate environment in the current fiscal year = 0.60.
- Probability that BankCorp will operate in a stable interest rate environment in the current fiscal year = 0.40.

Under declining interest rate environment:

- The probability that EPS will be $2.60 = 0.25
- The probability that EPS will be $2.45 = 0.75

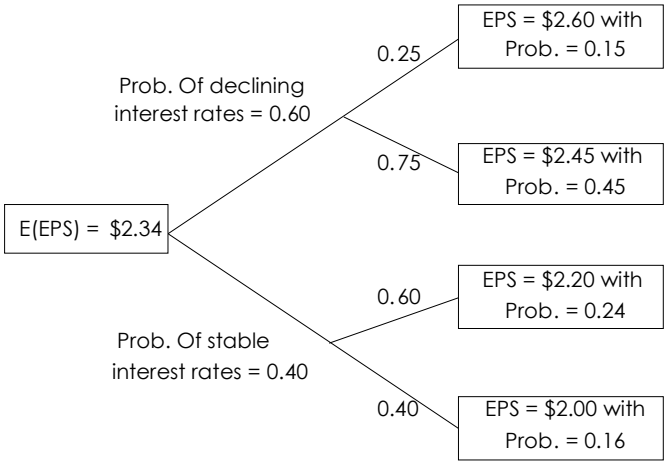
The unconditional probability that EPS will be $2.60 = Probability that BankCorp will operate in a declining interest rate environment in the current fiscal year × The probability that EPS will be $2.60 given declining interest rate environment

Stanley Notes™ 2025

The unconditional probability that EPS will be $2.60 = 0.60 × 0.25 = 0.15

The unconditional probability that EPS will be $2.45 = Probability that BankCorp will operate in a declining interest rate environment in the current fiscal year × The probability that EPS will be $2.45 given declining interest rate environment

The unconditional probability that EPS will be $2.45 = 0.60 × 0.75 = 0.45



Thus,

E (EPS | declining interest rate environment) = 0.25($2.60) + 0.75($2.45) = $2.4875

When interest rates are stable:

E (EPS | stable interest rate environment) = 0.60($2.20) + 0.40($2.00) = $2.12

E (EPS)={E (EPS | declining interest rate environment) × P(declining interest rate environment)} + {E(EPS | stable interest rate environment) × P(stable interest rate environment)}

= $2.4875 (0.60) + $2.12 (0.40) = $2.3405 ≈ $2.34.

Calculation of Conditional variances i.e. the variance of EPS given a declining interest rate environment and the variance of EPS given a stable interest rate environment.

σ^2 (EPS | declining interest rate environment) = P($2.6 | declining interest rate environment) × [$2.60 - E(EPS | declining interest rate environment)^2+ P($2.45 | declining interest rate environment) × [$2.45 - E(EPS | declining interest rate environment)^2

= 0.25($2.60 - $2.4875)^2+ 0.75($2.45 - $2.4875)^2= 0.004219

σ^2 (EPS | stable interest rate environment)=P($2.2 | stable interest rate environment) × [$2.20 - E(EPS | stable interest rate environment)^2+ P($2.00 | stable interest rate environment) × [$2.00 - E(EPS | stable interest rate environment)^2

= 0.60 ($2.20 – $2.12)^2 + 0.40 ($2.00 – $2.12)^2 = 0.0096

NOTE:

The unconditional variance of EPS = Expected value of the conditional variances + Variance of conditional expected values of EPS.
Where,

Expected value of the conditional variances = σ^2 (EPS) = P (declining interest rate environment) × σ^2 (EPS | declining interest rate environment) + P (stable interest rate environment) × σ^2 (EPS | stable interest rate environment)
=0.60 (0.004219) + 0.40 (0.0096)
=0.006371

Variance of conditional expected values of EPS = σ^2 [E (EPS | interest rate environment)] = 0.60 ($2.4875 – $2.34)^2 + 0.40 ($2.12 – $2.34)^2
= 0.032414

Thus,
Unconditional Variance of EPS = 0.006371 + 0.032414
= 0.038785

Source: Example, CFA® Curriculum.

Practice: Example 2, 3 and Question-Set from the CFA Institute's Curriculum.

4. BAYES' FORMULA

Bayes' formula is a method for updating a probability given additional information. It is also called an inverse probability. It is computed using the following formula:

Updated probability of event given the new information:

$$= \frac{Probability\ of\ the\ new\ information\ given\ event}{Unconditional\ probability\ of\ the\ new\ information} \times Prior\ probability\ of\ event$$

$$= \frac{Prob\ of\ new\ info\ given\ event}{Unconditional\ prob\ of\ new\ info} \times Prior\ prob\ of\ event$$

$$P(Event \mid Information) = \frac{P(Information \mid Event)}{P(Information)} P(Event)$$

- The updated probability is referred to as the **posterior** probability.

Diffuse priors: When the prior probabilities are equal, they are referred to as diffuse priors.

Important to Note: When the prior probabilities are equal:

Probability of information given an event
= Probability of an event given the information.

Example:

Suppose three mutually exclusive and exhaustive events i.e.

i. Last quarter's EPS of DriveMed exceeded the consensus EPS estimate.
ii. Last quarter's EPS of DriveMed exactly met the consensus EPS estimate.
iii. Last quarter's EPS of DriveMed fell short of the consensus EPS estimate.

Prior probabilities (or priors) of three events before any new information are as follows:

- P(EPS exceeded consensus) = 0.45
- P(EPS met consensus) = 0.30
- P(EPS fell short of consensus) = 0.25

Suppose the new information is ➔ DriveMed expands and the conditional probabilities (likelihoods) are:

P(DriveMed expands | EPS exceeded consensus) = 0.75
P(DriveMed expands | EPS met consensus) = 0.20
P(DriveMed expands | EPS fell short of consensus) = 0.05

Calculating the unconditional probability for DriveMed expanding i.e. P(DriveMed expands):

P(DriveMed expands) =
P(DriveMed expands | EPS exceeded consensus) ×P(EPS exceeded consensus) + P(DriveMed expands | EPS met consensus) ×P(EPS met consensus) + P(DriveMed expands | EPS fell short of consensus) × P(EPS fell short of consensus)
= 0.75(0.45) + 0.20(0.30) + 0.05(0.25)
= 0.4, or 41%

Using the Bayes' Formula, P(EPS exceeded consensus given that DriveMed expands) is estimated as:

$$P(EPS\ exceeded\ consensus \mid DriveMed\ expands) = \frac{P(DriveMed\ expands \mid EPS\ exceeded\ consensus)}{P(DriveMed\ expands)} P\left(\begin{matrix}EPS\ exceeded \\ consensus\end{matrix}\right)$$

= (0.75/0.41)(0.45) = 1.829268(0.45) = 0.823171

Source: CFA® Institute's Curriculum.

Practice: Example 4 and Question-Set from the CFA Institute's Curriculum.

Practice: End-of-Chapter from the CFA Institute's Curriculum and FinQuiz Question-Bank.

2. PORTFOLIO EXPECTED RETURN AND VARIANCE OF RETURN

Properties of Expected Value:

1. The expected value of a constant × random variable = Constant × Expected value of the random variable i.e.

$$E(w_iR_i) = w_i\,E(R_i)$$

where,

wi = weight of variable i
Ri = random variable i

2. The expected value of a weighted sum of random variables = Weighted sum of the expected values, using the same weights i.e.

$$E(w_1R_1 + w_2R_2 + \ldots + w_nR_n) = w_1E(R_1) + w_2E(R_2) + \ldots + w_nE(R_n)$$

Expected return on the portfolio: The expected return on the portfolio is a weighted average of the expected returns on the component securities i.e.

$$E(R_p) = E(w_1R_1 + w_2R_2 + \ldots + w_nR_n)$$
$$= w_1E(R_1) + w_2E(R_2) + \ldots + w_nE(R_n)$$

Covariance: The covariance is a measure of how two assets move together. Given two random variables R_i and R_j, the covariance between R_i and R_j is stated as:

$$Cov(R_i, R_f) = \Sigma_{i=1}^{n}[p(R_i - ER_i)(R_j - ER_f)]$$

When the returns on both assets tend to move together i.e. there is a positive relationship between returns
➔Covariance of returns is ***positive (i.e. >0)***.

When the returns on both assets are inversely related
➔Covariance of returns is ***negative (i.e. < 0)***.

When returns on the assets are unrelated ➔ Covariance of returns is 0.

- As the number of assets (securities) increases, importance of covariance increases, all else equal.
- Like variance, covariance is difficult to interpret.

Important to Note:

- The covariance of a random variable with itself (own covariance) is its own variance i.e.
  Cov (R, R) = E {[R - E(R)] [R - E(R)]} = E {[R - E(R)] ^{2}}
  = σ^2(R)
- Cov (R_i, R_j) = Cov (R_j, R_i)

Covariance Matrix: It a square format of presenting covariances.

Portfolio variance: It is calculated as:

$$\sigma^2(R_p) = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j Cov(R_i, R_j)$$

For example, given three assets with returns R_1, R_2 and R_3, portfolio variance is calculated as:

$$\sigma^2(R_p) = \omega_1^2\sigma^2(R_1) + \omega_2^2\sigma^2(R_2) + \omega_3^2\sigma^2(R_3) + 2\omega_1\omega_2 Cov(R_1, R_2) + 2\omega_1\omega_3 Cov(R_1, R_3) + 2\omega_2\omega_3 Cov(R_2, R_3)$$

Where,

σ^2 = Corresponding variance of each asset in the portfolio

- The smaller the covariance between assets, the greater the diversification benefits and the greater the cost of ***not*** diversifying (in terms of risk-reduction benefits forgone), all else equal.

When the returns on the three assets are independent, covariances = 0 and S.D. of portfolio return would be:

$$S.D. = [w^2_1\sigma^2(R_1) + w^2_2\sigma^2(R_2) + w^2_3\sigma^2(R_3)]^{1/2}.$$

Generally, for **n** number of securities, we need to estimate:

- n (n - 1)/2 distinct covariances.
- n distinct variances.

Properties of Variance and Covariance:

a) The variance of a constant multiplied by a random variable = Constant squared multiplied by the variance of the random variable i.e.

$$\sigma^2(w \times R) = w^2 \times \sigma^2 \times (R)$$

b) Variance of a constant = 0.
c) The variance of a constant + random variable = Variance of the random variable.
d) The covariance between a constant and a random variable is 0.

Correlation: The correlation between two random variables, R_i, and R_j, is estimated as follows:

$$\rho(R_i, R_j) = Cov(R_i, R_j) \div \sigma(R_i)\,\sigma(R_j)$$

- The value of correlation lies between -1 and + 1 i.e., for two random variables, X and Y:
  $$-1 \le \rho(X, Y) \le +1$$
- When correlation = 0, variables are unrelated and do not have any linear relationship.

Stanley Notes™ 2025

- When correlation > 0, variables have positive linear relationship.
- When correlation < 0, variables have negative (inverse) linear relationship.
- When correlation = +1, variables have ***perfect*** positive linear relationship.
- When correlation = -1, variables have ***perfect*** negative (inverse) linear relationship.

NOTE:

- When the correlation is positive (negative): $R_1 = a + bR_2 + \text{error} \rightarrow b > (<) 0$.

- When the correlation is zero: $R_1 = a + bR_2 + \text{error} \rightarrow b = 0$.

NOTE:

Correlation only deals with ***linear*** relationships.

Practice: Example 1 and Question-Set from the CFA Institute's Curriculum.

3. FORECASTING CORRELATION OF RETURNS: COVARIANCE GIVEN A JOINT PROBABILITY FUNCTION

JOINT PROBABILITY FUNCTION:

Let, R_A = Return on stock BankCorp and R_B = Return on stock NewBank.

Joint Probability Function of BankCorp and NewBank Returns (Entries Are Joint Probabilities)

	R_B = 20%	R_B = 16%	R_B = 10%
R_A = 25%	0.20	0	0
R_A = 12%	0	0.50	0
R_A = 10%	0	0	0.30

Source: Table 12, CFA® Curriculum.

Expected return on BankCorp stock = 0.20(25%) + 0.50(12%) + 0.30(10%) = 14%.

Expected return on NewBank stock = 0.20(20%) + 0.50(16%) + 0.30(10%) = 15%

$$Cov(R_A, R_B) = \sum_i \sum_j P(R_{A,i}, R_{B,j}) (R_{A,i} - ER_A)(R_{B,i} - ER_B)$$

Cov(R_A, R_B) = P(25, 20) [(25 – 14)(20–15)] + P(12, 16) [(12 – 14)(16 – 15)] + P(10, 10)[(10 – 14) (10 – 15)]
= 0.20(11)(5) + 0.50(–2)(1) + 0.30(–4)(–5)
=11 – 1 + 6 = 16

Independent Random Variables: Two random variables X and Y are independent *if and only if*:

$$P(X, Y) = P(X) P(Y)$$

- Independence is a stronger property compared to a correlation of 0 because correlation deals with only linear relationships.

Multiplication Rule for Expected Value of the Product of Uncorrelated Random Variables: When two random variables (e.g. X & Y) are uncorrelated,

Expected value of (XY)= Expected value of X × Expected value of Y

➔E (XY) = E(X) E(Y)

Practice: Example 2 from the CFA Institute's Curriculum.

4. APPLICATIONS OF THE NORMAL DISTRIBUTIONS

- The mean-variance analysis is based on the assumption that returns are normally distributed.
- **Safety-first rule:** Safety-first rule focuses on ***shortfall risk*** i.e. the risk that portfolio value will fall below some minimum acceptable level over some specified time horizon. For example, the risk that the assets in a defined benefit plan will fall below plan liabilities.

According to ***Roy's safety-first criterion***, the ***optimal*** portfolio is the one that ***minimizes*** the probability that portfolio return (R_P) falls below the threshold level (R_L).

When returns are normally distributed, the safety-first optimal portfolio is the portfolio that ***maximizes*** the safety-first ratio (SFRatio):

$$SFRatio = [E(R_P) - R_L]/\sigma_P$$

- Investors prefer the portfolio with the highest SFRatio.
- Probability that the portfolio return < threshold level = P (Rp< R_L) = N (-SFRatio).
- The optimal portfolio has the lowest P (Rp< R_L).

Example:

- Portfolio 1 expected return = 12% and S.D. = 15%
- Portfolio 2 expected return = 14% and S.D. = 16%
- Threshold level = 2%
- Assumes that returns are normally distributed.

SFRatio of portfolio 1 = (12 – 2) / 15 = 0.667
SFRatio of portfolio 2 = (14 – 2) / 16 = 0.75

- Since SFRatio of portfolio 2 > SFRatio 1, the superior Portfolio is Portfolio 2.

Probability that return < 2% = N (–0.75)
= 1 – N (0.75)
= 1 – 0.7734*
≈ 23%.

**value taken from the z-table provided on the previous page.*

Sharpe Ratio: Sharpe ratio = [E (Rp) – R_f] / σ_p

- The portfolio with the *highest* Sharpe ratio is the one that *minimizes* the probability that portfolio return will be less than the risk-free rate (assuming returns are normally distributed).

Managing Financial risk: Two important measures used to manage financial risk include:

- **Value at risk (VAR):** It provides the *minimum* value of losses (in money terms) expected over a specified time period (e.g. a day, quarter, year etc.) at a specified level of probability (e.g. 5%, 1%). VAR estimated using variance-covariance or analytical method assumes that returns are normally distributed.
  Example:
  A one week VAR of $10 million for a portfolio with 5% probability implies that portfolio is expected to loss $10 million or more in a single week.
- **Stress testing/scenario analysis:** It involves a use of set of techniques to estimate losses in extremely worst combinations of events or scenarios.

Practice: Example 3 and Question-Set from the CFA Institute's Curriculum .

Practice: Questions from FinQuiz Question bank + CFAI Curriculum End of Chapter Practice Problems

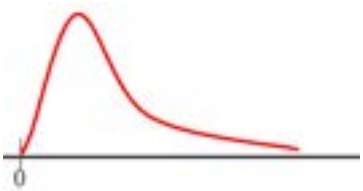
2. LOGNORMAL DISTRIBUTIONS AND CONTINUOUS COMPOUNDING

The Lognormal Distribution

A random variable (i.e. Y) whose natural logarithm (i.e. ln Y) has a normal distribution, is said to have a Lognormal distribution.

- Unlike Normal distribution, Lognormal random variables cannot be negative.

Reason:
Since, negative values do not have logarithms, Y is always > 0 and thus the distribution is positively skewed (unlike normal distribution that is bell-shaped).



- Like normal distribution, it is completely described by two parameters i.e. the mean and variance of ln Y, given that Y is lognormal.

Mean (μ_L) of a lognormal random variable = $\exp(\mu + 0.50\sigma^2)$

Variance (σ_L^2) of a lognormal random variable = $\exp(2\mu + \sigma^2) \times [\exp(\sigma^2) - 1]$.

Continuously Compounded Rates of Return

It is important to note that when a stock's continuously compounded return is normally distributed, then future stock price is necessarily lognormally distributed.

$$S_T = S_0 \exp(r_{0,T})$$

Where,
exp = e
$r_{0,t}$ = Continuously compounded return from 0 to T

- Since S_T is proportional to the log of a normal random variable → S_T is lognormal.

Price relative = Ending price / Beginning price = $S_{t+1}/S_t = 1 + R_{t,t+1}$

where,
$R_{t,t+1}$ = *holding period return on the stock from t to t + 1.*

Continuously compounded return associated with a holding period from t to t + 1:

$$r_{t,t+1} = \ln(1 + \text{holding period return})$$
Or
$$r_{t,t+1} = \ln(\text{price relative}) = \ln(S_{t+1}/S_t) = \ln(1 + R_{t,t+1})$$

NOTE:

The continuously compounded return < associated holding period return.

Continuously compounded return associated with a holding period from 0 to T:

$$R_{0,T} = \ln(S_T / S_0)$$

Or

$$r_{0,T} = r_{T-1,T} + r_{T-2,T-1} + \cdots + r_{0,1}$$

Where,
$r_{T-1,T}$ = One-period continuously compounded returns

Example:

Suppose, one-week holding period return = 0.04.

Equivalent continuously compounded return = one-week continuously compounded return = ln (1.04) = 0.039221

- The intervals within which a certain percentage of the observations of a ***normally*** distributed random variable are expected to lie are symmetric around the mean.
- The intervals within which a certain percentage of the observations of a ***lognormally*** distributed random variable are expected to lie are *not* symmetric around the mean.

In many investment applications, it is assumed that returns are **independently and identically distributed (IID)**.

- *Returns are independently distributed* implies that investors cannot forecast future returns using past returns (i.e., weak-form market efficiency).
- *Returns are identically distributed* implies that the mean and variance of return do not change from period to period (i.e. stationarity).

When one-period continuously compounded returns (i.e. $r_{0,1}$) are IID random variables with mean μ and variance σ^2, then

$$E(r_{0,T}) = E(r_{T-1,T}) + E(r_{T-2,T-1}) + \cdots + E(r_{0,1}) = \mu T$$
And
$$Variance = \sigma^2(r_{0,T}) = \sigma^2 T$$

$$\text{S.D.} = \sigma(r_{0,T}) = \sigma\sqrt{T}$$

Stanley Notes™ 2025

- It implies that when the one-period continuously compounded returns are normally distributed, then the T holding period continuously compounded return (i.e. $r_{0,T}$) is also normally distributed with mean μT and variance $\sigma^2 T$.
- According to Central limit theorem, the sum of one-period continuously compounded returns is approximately normal even if they are not normally distributed.

Volatility:

Volatility reflects the deviation of the continuously compounded returns on the underlying asset around its mean. It is estimated using a historical series of continuously compounded daily returns.

Annualized volatility = sample S.D. of one period continuously compounded returns $\times \sqrt{T}$

where,
T = Number of trading days in a year = 250.

Practice: Example 8, CFA Institute's Curriculum .

3. MONTE CARLO SIMULATION

Monte Carlo simulation involves the use of a computer software to generate a large number of random samples from a probability distribution.

Uses:

- It is widely used to estimate risk and return in investment analysis using simulation (i.e. measuring portfolio performance through simulated frequency distribution of portfolio returns)
- It can be used in valuing complex securities e.g. European-style options, mortgage-backed securities.

Steps of Monte Carlo simulation technique to examine a model's sensitivity to changes in assumptions:

1) Specify the underlying variable or variables e.g. stock price for an equity call option. Then specify the beginning values of the underlying variables e.g. stock price.

- C_{iT} = Value of the option at maturity T. The subscript I reflects a value resulting from the i^{th} simulation trial.

2) Specify a time period. Time increment = Δt = Calendar time / Number of sub-periods (K).

3) Specify the distributional assumptions for the key risk factors that drive the underlying variables. For example specify the regression model for changes in stock price.

$$\Delta(Stock\ price) = (\mu \times Prior\ stock\ price \times \Delta t) + (\sigma \times Prior\ stock\ price \times Z_k)$$

where,

Z_k= *Risk factor in the simulation. It is a standard normal random variable.*

4) K random variables are drawn for each risk factor using a computer program or spreadsheet function.

5) Now the underlying variables are estimated by substituting values of random observations in the model specified in Step 4.

6) The value of a call option at maturity i.e. C_{iT} is calculated and then this value is discounted back at time period 0 to get Ci0.

7) This process is repeated until a specified number of trials, i, is completed (e.g., tens of thousands of trials).

NOTE:

For obtaining each extra digit of accuracy in results, the appropriate increase in the number of trials depends on the problem. For example, in option value, tens of thousands of trials may be appropriate. Generally, the number of trials should be increased by a factor of 100.

Finally, mean value and S.D. for the simulation are calculated.

Mean value = Average value of the option over all trials in the simulation

- The mean value will be the Monte Carlo estimate of the value of the call option.

Random number generator: An algorithm that generates uniformly distributed random numbers between 0 and 1 is referred to as random number generator. It is important to note that random observations from any distribution can be generated using a uniform random variable.

Steps to generate random observations on variable X:

1) Generate a uniform random number (i.e. T) between 0 and 1 using the random number generator.
2) Evaluate the inverse of cumulative distribution function F(x) i.e. F^{-1} (x) to obtain a random observation on variable X.

Note: Monte Carlo simulation is a complement to analytical methods. It only provides statistical estimates, not exact results.

Practice: Example 2 & Question-Set from the CFA Institute's Curriculum.

4. BOOTSTRAPPING

Resampling: A technique that repeatedly draws samples from the observed data samples for the statistical inference of population parameter.

Bootstrapping

Bootstrap: A resampling technique that repeatedly draws samples (by putting back sample observations every time) from the observed data samples. Each observation drawn is put back into the group, therefore it can be drawn more than once.

Bootstrap is a popular resampling method that uses computer simulation for statistical inference without traditional analytical formulas (z-or t-statistics).

It mimics random sampling from a population to construct a sampling distribution, using the observed sample as if it were the actual population.

Bootstrapping and Monte Carlo simulation

- Both bootstrap and Monte Carlo simulation rely on repetitive sampling.
- Bootstrapping treats the dataset as the true population and infers statistical distribution parameter values (e.g., mean, variance).
- Monte Carlo simulation generates random data based on known statistical distribution parameters.

Bootstrap Resampling: Bootstrap involves drawing repeated samples from the original sample, each of the same size.

- Each item drawn is replaced, allowing it to be drawn multiple times.
- While some items might appear frequently in resamples, others might not appear at all.

The main difference between 'simulating' and 'bootstrapping' the valuation of a contingent claim is the **source of the random variable**. In simulation, the random variable is drawn from a probability distribution. In bootstrapping, the random variable is drawn from the sample.

The bootstrap and Monte Carlo simulation steps are the same except for:

- The bootstrap draws the random variable from the original data.
- The Monte Carlo simulation draws the random variable from a probability distribution.

Steps in Implementing Simulation using Bootstrapping

Step 1	Define the quantity of interest using underlying variables, such as the stock price, and set starting values.
Step 2	Specify the time grid that is consistent with the periodicity of the sample observations by dividing the horizon into subperiods.
Step 3	Determine the data generation method for the simulation, using observed stock price changes or returns.
Step 4	Simulate stock prices for valuing the contingent claim by drawing random values and converting stock price changes into a sequence of stock prices.
Step 5	Compute the average stock price and the contingent claim's value, then calculate its present value by discounting using an appropriate interest rate.
Step 6	Repeat the simulation steps (3 and 4) as needed, then produce summary values and statistics. The mean value from all trials gives the bootstrap estimate for the contingent claim's value.

Practice: Question-Set and End-of-Chapter Questions from the CFA Institute's Curriculum and Questions from FinQuiz Question-bank.

1. INTRODUCTION

Analysts often use sample information to assess the behavior of the underlying population.

Sample is the subset of the population. **Sampling** is the process of obtaining a sample from the population.

2. SAMPLING METHODS

Parameter is a quantity computed from or used to describe a population of data (typically represented by Greek letters).

Sample Statistic (a.k.a. statistic) is a quantity computed from or used to describe a sample of data.

Benefits of Sampling: Sampling saves:

- ***time and energy*** because it is difficult or entirely impossible to examine every member of the population.
- ***money***; thus, it is more economically efficient.

Two types of sampling methods are:

1. **Probability sampling** - gives every member of the population equal chance of being selected- therefore its sample is
   - representative of population.
2. **Non-probability sampling** – where the chance of a member of population being selected depends on factors other than probability (such as sampler's judgement, ease of access to data) therefore:
   - its sample is non-representative of population.

Note: Probability sampling is more accurate and reliable than non-probability sampling.

***Two types of* probability sampling:**

i. Simple random sampling
ii. Stratified random sampling

1 Simple Random Sampling

Sampling Plan: Sampling plan is a set of rules that specify how a sample will be taken from a population.

Simple Random Sample or random sample: A simple random sample is a sample selected from a population in such a way that every possible sample of the same size has equal chance/probability of being selected. This implies that every member is selected independently of every other member.
Simple random sampling: The procedure of drawing a random sample is known as Simple random sampling.

Random sample (for a finite/limited population) can be obtained using random numbers table. In this method, members of the population are assigned numbers in sequence e.g. if the population contains 500 members, they are numbered in sequence with three digits, starting with 001 and ending with 500.

Systematic sampling: It is the sampling process that involves selecting individuals within the defined population from a list by taking every K^{th} member until a sample of desired size is selected. The gap, or interval between k successive elements is equal and constant.

Sampling Error: Since all members of the population are not examined in sampling, it results in sampling error. The sampling error is the difference between the sample mean and the population mean.
Sampling distribution of a Statistic: The sampling distribution of a statistic is the probability distribution of a sample statistic over all possible samples of the same size drawn randomly from the same population.

2 Stratified Random Sampling

In stratified random sampling, the population is divided into homogeneous subgroups (strata) based on certain characteristics. Members within each stratum are homogeneous, but are heterogeneous across strata. Then, a simple random or a systematic sample is taken from each stratum proportional to the relative size of the stratum in the population. These samples are then pooled to form a stratified random sample.

- The strata should be mutually exclusive (i.e. every population member should be assigned to one and only one stratum) and collectively exhaustive (i.e. no population members should be omitted).
- The size of the sample drawn from each stratum is proportionate to the relative size of that stratum in the total population.
- Stratified sampling is used in pure bond **indexing** or full-replication approach in which an investor

2025 Stanley Notes™

attempts to fully replicate an index by owning all the bonds in the index in proportion to their market value weights. However, pure bond indexing is difficult and expensive to implement due to high transaction costs involved.

Advantages: Stratified random sampling generates more precise sample and generates more precise parameters (i.e. smaller variance) relative to simple random sampling.

Drawback: Stratified Random Sampling approach generates a sample that is just approximately (i.e. not completely) random.

Example:

Suppose, population of index bonds is divided into 2 issuer classifications, 10 maturity classifications and 2 coupon classifications.

Total strata or cells = (2) (10) (2) = 40

- A sample, proportional to the relative market weight of the stratum in the index to be replicated, is selected from each stratum.
- For each cell, there should be ≥ 1 issuer i.e. the portfolio must have at least 40 issuers.

Practice: Example 1 from the CFA Institute's Curriculum.

3 Cluster Sampling

Cluster Sampling – a technique where the population is divided into subpopulation groups (clusters). Each cluster is a mini representation of the population. Then certain clusters are randomly chosen (using simple random sampling) to form a sample.

i. **One-stage sampling:** Certain clusters are randomly chosen from the entire clusters. All the members from these chosen clusters are selected.
ii. **Two-stage sampling**: Certain clusters are randomly chosen from the entire clusters. A subsample is then randomly selected from each chosen cluster.

Advantages: Time-efficient and cost-efficient

Disadvantages: Cluster sampling attains lower accuracy because a sample from a cluster may not fully represent the entire population.

Difference between Cluster and Stratified Samples:

- **Cluster Sampling -** A whole cluster is viewed as a sampling unit and then sample is made from the sampled clusters.
- **Stratified Sampling -** Specific elements from each stratum makes the sampling unit.

4 Non-Probability Sampling

Non-probability sampling methods depends on researcher's sample selection capabilities instead of a selection process.

Two major types include:
i. Convenience sampling
ii. Judgmental sampling

1. Convenience sampling: The element is selected based on the ease of its accessibility to the researcher.

- **Advantages:** It is cost efficient and time-efficient i.e., data is collected quickly.
- **Disadvantages**: Limited level of sample accuracy i.e., sample may not represent the entire population

2. Judgmental Sampling: The elements are selectively handpicked from the population based on researcher's knowledge and professional judgement.

- **Advantage:** Experienced researchers may select a more accurate representative sample compared to other methods when there is a time constraint or when the specialty of researcher is critical.
- **Disadvantage:** Sample may not represent the entire population. Results may be skewed because of the researcher's biasness.

Practice: Example 2 from the CFA Institute's Curriculum.

5 Sampling from Different Distributions

Analysts should be careful when sampling from population with more than one distribution. Sample should represent a homogenous distribution.

Sampling should not be done from more than one distribution because when random samples are selected from more than one distribution (e.g. combining data collected from a period of fixed exchange rates with data from a period of floating exchange rates), the sample statistics computed from such samples may not be the representatives of one underlying population.

Practice: Example 3 and Question-Set from the CFA Institute's Curriculum.

3. CENTRAL LIMIT THEOREM AND INFERENCE

The Central Limit Theorem

According to central limit theorem: When the sample size is large,

1) Sampling distribution of mean ($\bar{X}$) will be approximately normal regardless of the probability distribution of the sampled population (with mean μ and variance σ^2) when the sample size (i.e. n) is large".

- Generally, when n ≥ 30, it is assumed that the sample mean is approximately normally distributed.

2) Sample mean = Population mean ➔ $\mu_{\bar{X}} = \mu$

3) The sampling distribution of sample means has a standard deviation equal to the population standard deviation divided by the square root of *n*.

Variance of the distribution of the sample mean = $\frac{\sigma^2}{n}$

Standard Error of the Sample Mean

Standard Error: S.D. of a sample statistic is referred to as the standard error of the statistic.

When the population S.D. (σ) is known,

Standard Error of the Sample Mean = $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

When the population S.D. (σ) is not known,

Standard Error of the Sample Mean = $s_{\bar{X}} = \frac{s}{\sqrt{n}}$

where,
s = sample S.D.

The estimate of s = $\sqrt{Sample\ Variance} = \sqrt{s^2}$

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Note: Standard deviation S.D. and standard error are two different concepts:

- S.D measures dispersion of data from the mean
- Standard error measures how much inaccuracy of a population parameter estimates comes from sampling.

Practice: Question-Set from the CFA Institute's Curriculum.

4. BOOTSTRAPPING AND EMPIRICAL SAMPLING DISTRIBUTIONS

Resampling: A technique that repeatedly draws samples from the observed data samples for the statistical inference of population parameter.

1. **Bootstrap:** A resampling technique that repeatedly draws samples (by putting back sample observations every time) from the observed data samples. Each observation drawn is put back into the group, therefore it can be drawn more than once.

   For example, to calculate the standard error of sample mean, the process takes many resamples and then compute the mean of each resample.

   Bootstrap method uses computer simulation. It mimics the process by considering the randomly drawn sample as if it were the population.

   Advantages: *Bootstrap sampling:*

   - is potentially more accurate than other methods
   - is widely used tool for statistical inference i.e., to find standard error or to construct confidence interval or any complicated estimators.
   - does not rely on analytical formula to estimate the distribution of the estimators.

2. **Jackknife:** It is another resampling technique that repeatedly draws samples by taking the observed

data sample and leaving out one observation at a time from the set.

Unlike bootstrap this method does not put back sample observation.

This method is used to reduce the bias of the estimator as well as to find standard error and confidence interval of an estimator.

For a sample size n, Jackknife typically requires n repetitions whereas with bootstrap researcher determines how many repetitions are appropriate.

Practice: Question-Set from the CFA Institute's Curriculum.

Practice: CFA Institute's end of Chapter Practice Problems and Questions from FinQuiz Question Bank.

1. INTRODUCTION

Statistical inference refers to a process of making judgments regarding a larger group (i.e., population) based on information obtained from a sample.

Hypotheses Testing is a part of statistical inference.

- A hypothesis is a statement about one or more populations.

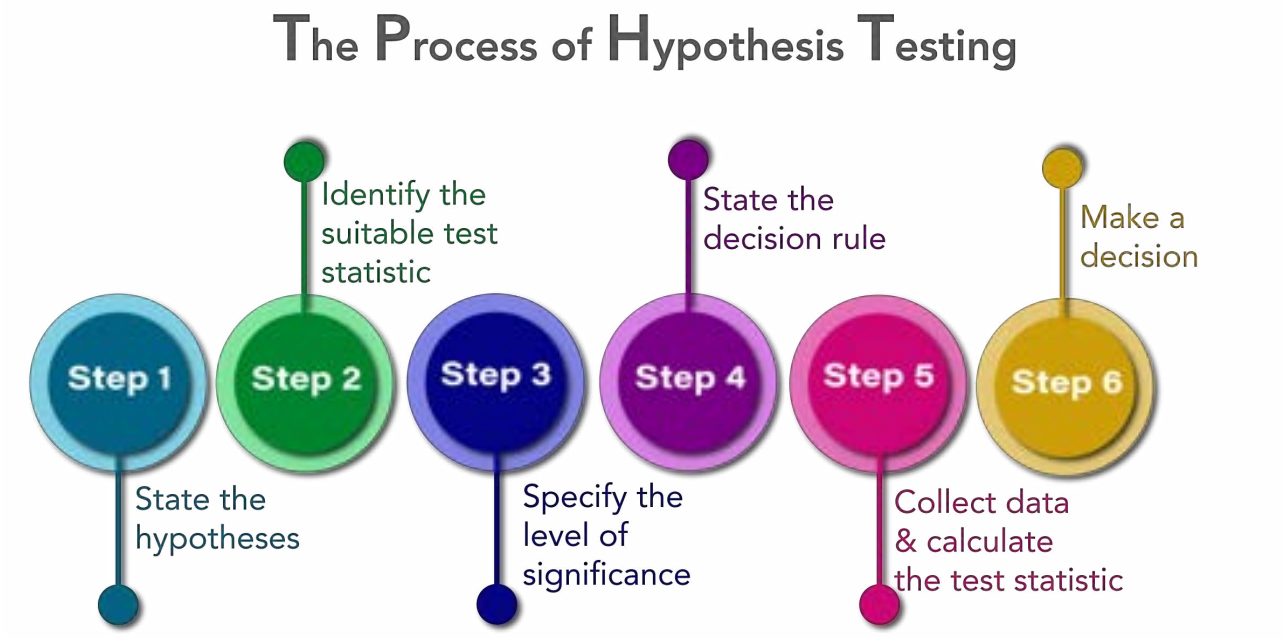
Hypothesis testing objectively gauges whether a sample statistic is likely to come from a population with the hypothesized value of the population parameter.

- In *hypothesis testing*, we have a hypothesis about a parameter's value and seek to test that hypothesis.

2. HYPOTHESIS TESTS FOR FINANCE

Statistical inference involves:

i. **Hypothesis testing** – making statements about a population using a sample.
ii. **Estimation** - estimating the value of population parameters using information obtained from a sample. Estimation includes ***point estimates*** and ***interval estimates***.



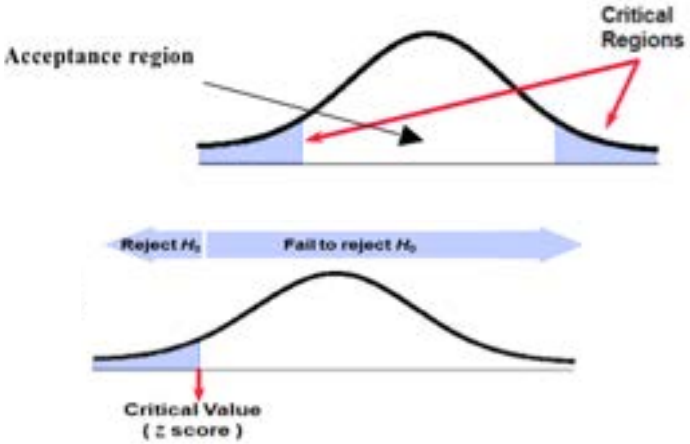
Steps in Hypothesis Testing:

1. **Stating the hypotheses:** It involves formulating the null hypothesis (H_0) and the alternative hypothesis (H_a).
2. **Determining the appropriate test statistic and its probability distribution:** It involves defining the test statistic and identifying its probability distribution.
3. **Specifying the significance level:** The significance level should be specified before calculating the test statistic.
4. **Stating the decision rule:** It involves identifying the rejection/critical region of the test statistic and the rejection points (critical values) for the test.

- **Critical Region** is the set of all values of the test statistic that may lead to a rejection of the null hypothesis.

Stanley Notes™ 2025

- **Critical value** of the test statistic is the value for which the null is rejected in favor of the alternative hypothesis.
- **Acceptance region** is the set of values of the test statistic for which the null hypothesis is not rejected.



5. **Collecting the data and calculating the test statistic:** The data collected should be free from measurement errors, selection bias and time period bias.

6. **Making the statistical decision:** It involves comparing the calculated test statistic to a specified possible value or values and testing whether the calculated value of the test statistic falls within the acceptance region.

7. **Making the economic or investment decision:** The hypothesized values should be both statistically significant and economically meaningful.

Stating the Hypotheses

Step 1 'stating the hypotheses' involves stating the following two hypotheses.

a) Null Hypothesis (H_0): It is the hypothesis that is tested. The null hypothesis (H_0) is the claim that is *initially assumed to be true* and is to be tested.

It is a statement about population parameter that is considered to be true unless sample used to conduct the hypotheses test may provide evidence that the hypothesis is false. Then we reject the null hypothesis in favor of alternative hypothesis.

For example, it is hypothesized that the population mean risk premium for Canadian equities ≤ 0.

- *The null hypothesis will always contain equality.*

b) Alternative Hypothesis: The alternative hypothesis (H_a) is the claim that is contrary to H_0. It is accepted when the null hypothesis is rejected.

For example, the alternative hypothesis is that the population mean risk premium for Canadian equities > 0.

- *The alternative hypothesis will always contain an inequality.*

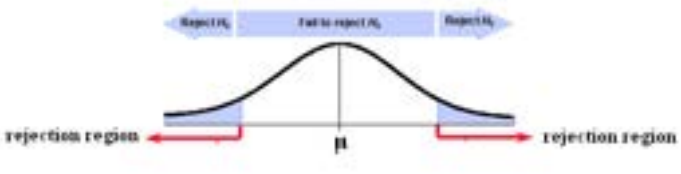
Two-Sided vs. One-Sided Hypothesis

Formulations of Hypotheses: The null and alternative hypotheses can be formulated in three different ways:

1) Two-sided alternative
2) One-sided alternative (right side)
3) One-sided alternative (left side)

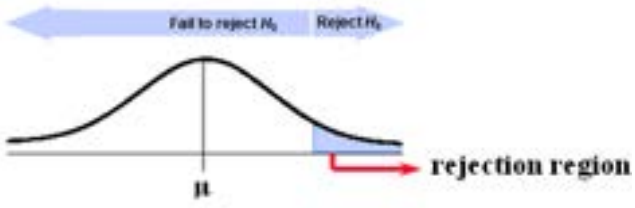
1. Two-sided alternative: H_0: $\mu = \mu_0$ versus H_a: $\mu \neq \mu_0$

- It is a two-sided or two-tailed hypothesis test.
- In this case, the H_0 is rejected in favor of H_a if the population parameter is either < or > μ_0.



2. One-sided alternative (right side): H_0: $\mu \leq \mu_0$ versus H_a: $\mu > \mu_0$ -

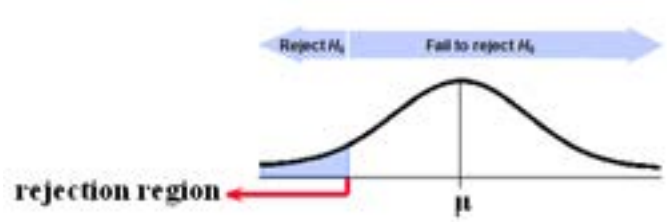
- It is a one-sided right tailed hypothesis test.
- In this case, the H_0 is rejected in favor of H_a if the population parameter is > μ_0.



Note: The side reference is based on where to reject the null in the probability distribution. In this case, the null is rejected on the right side.

3. One-sided alternative (left side): H_0: $\mu \geq \mu_0$ versus H_a: $\mu < \mu_0$

- It is a one-sided left tailed hypothesis test.
- In this case, the H_0 is rejected in favor of H_a if the population parameter is < μ_0.



where,

μ = *Value of population parameter*
μ_0 = *Hypothesized value of population parameter*

NOTE:

Null hypothesis is always tested at the point of equality:

- H_0: $\mu = \mu_0$
- H_0: $\mu \leq \mu_0$
- H_0: $\mu \geq \mu_0$

Selecting the appropriate Hypothesis

- The null hypothesis is what we 'hope to reject'.
- The alternative hypothesis is what we are 'hoping for'.
- One-sided hypothesis [H_a: $\mu > \mu_0$ or H_a: $\mu < \mu_0$] more strongly reflects the beliefs of the researcher than two-sided hypothesis [H_a: $\mu \neq \mu_0$].

Identify the Appropriate Test Statistic

Test Statistics

A test statistic is a value that is calculated using the information obtained from a sample and is used to decide whether or not to reject the null hypothesis.

- It is the focal point of statistical decision
- It depends on what we are testing.

Two key elements of hypothesis testing are identifying:

1) appropriate test statisitc for the hypothesis
2) underlying distribution of the population

When the population stanrad deviation (S.D) is known, the ***standard error of the distribution of sample mean*** is equal to the ratio of the population S.D to the square root of the sample size i.e.,

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Z-distributed is the normally distributed test statistic for the test of mean when the population variance is known.

$$z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$\bar{X}$ = sample mean
μ_0 = hypothesized value
σ = population standard deviation
n = sample size

If μ_0 = 0, the above equation can be simplified as: $z = \frac{\bar{X}}{\frac{\sigma}{\sqrt{n}}}$

Identifying the Distribution of the Test Statistic

Test Statisitcs and their Distribution

Testing for the	Test Statisitc	Prob. Dist.
Means 1. Single Mean	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$, $df = n-1$	t-distributed
2. Differences in Means	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$, $df = n_1 + n_2 - 2$	t-distributed
3. Mean of Differences	$t = \frac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$, $df = n-1$	t-distributed
Correlation	$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$, $df = n-2$	t-distributed
Independence (Categorical data)	$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$, $df = (r-1)(c-1)$	chi-square distributed
Variance 1. Single Variance	$\chi^2 = \frac{s^2(n-1)}{\sigma_0^2}$, $df = n-1$	chi-square distributed
2. Difference in Variances	$F = \frac{s_1^2}{s_2^2}$, $df = n_1 - 1, n_2 - 1$	f-distributed

Refer to: Exhibit 2 from the CFA Institute's Curriculum.

Specify the Level of Significance

When a null hypothesis is tested, it may result in four possible outcomes i.e.

1. A false null hypothesis is rejected → this is a correct decision and is referred to as the ***power of the test.***

   Power of a test = 1 – Probability of a Type-II error

   - Power of test is correctly rejecting the null hypotheses when it is true.
   - When more than one test statistic is available to conduct a hypothesis test, then the ***most powerful*** test statistic should be selected.
2. A true null hypothesis is rejected → this is an incorrect decision and is referred to as a ***Type-I error.***
3. A false null hypothesis is *not* rejected → this is an incorrect decision and is referred to as a ***Type-II error***.

4. A true null hypothesis is *not* rejected → this is a correct decision.

Type I and Type II Errors in Hypothesis Testing

	True Situation	
Decision	**H_0 True**	**H_0 False**
Do not reject H_0	Correct Decision	Type II Error (false negative)
Reject H_0 (Accept H_a)	Type I Error (false positive)	Correct Decision

- Type-I and Type-II errors are mutually exclusive errors.
- The probability of a Type-I error is referred to as a level of significance and is denoted by alpha, α. Confidence level = 1 - α
  - The lower the level of significance at which the null hypothesis is rejected, the stronger the evidence that the null hypothesis is false.

    For example, if 5% level of confidence is selected for a hypothesis test, it means the confidence level is 95%.

- The probability of a Type-II error is denoted by beta, β. The probability of type-II error is difficult to quantify.

Tradeoff between Type I and Type II Error

- All else equal, the smaller the significance level, the smaller the probability of making a type-I error and the greater the probability of making a type-II error.
- Higher the power of test, lower will be the chances of making type I error and higher will be the chances of making type II error.
- Type I and II errors probabilities can be simultaneously reduced by increasing the sample size (n).
- Type II error is difficult to quantify as there may be many different false hypotheses therefore, we only specify the probability of type I error as denoted by α.
- Type-I error is more serious than Type-II error.

Practice: Question-Set from the CFA Institute's Curriculum.

3 TESTS OF RETURN AND RISK IN FINANCE

Collect the Data and Calculate the Test Statistic

1. $H_0: \mu_1 - \mu_2 = 0$ versus $H_a: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$
2. $H_0: \mu_1 - \mu_2 \leq 0$ versus $H_a: \mu_1 - \mu_2 > 0$ or $\mu_1 > \mu_2$
3. $H_0: \mu_1 - \mu_2 \geq 0$ versus $H_a: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

where,
μ_1 = population mean of the first population
μ_2 = population mean of the second population

Test Statistic for a Test of the Difference between Two Population Means (Normally Distributed Populations, Population Variances Unknown *but Assumed Equal*) based on *Independent* samples:

A t-test based on independent random samples is given by:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}\right)}}$$

where,
S_p^2= Pooled estimator of the *Common* variance.

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

The number of degrees of freedom is $n_1 + n_2 - 2$.

Practice: Example 1 from the CFA Institute's Curriculum.

Test Concerning Differences between Means with Dependent Samples

When samples are ***dependent***, the test concerning mean differences is referred to as **paired comparisons test** and is conducted as follows.

1. $H_0: \mu_d = \mu_{d0}$ versus $H_a: \mu_d \neq \mu_{d0}$
2. $H_0: \mu_d \leq \mu_{d0}$ versus $H_a: \mu_d > \mu_{d0}$
3. $H_0: \mu_d \geq \mu_{d0}$ versus $H_a: \mu_d < \mu_{d0}$

where,

- d = difference between two paired observations = $X_{Ai} - X_{Bi}$
- X_{Ai} and X_{Bi} are the i^{th} pair i = 1, 2, …, n. on the two random variables.
- μ_d = population mean difference.
- μ_{d0} = hypothesized value for the population mean difference

Test Statistic for a Test of Mean Differences (Normally Distributed Populations, Unknown Population Variances):

$$t = \frac{\bar{d} - \mu_{d0}}{S_{\bar{d}}}$$

where,

$$\textbf{Sample mean difference} = \bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$\textbf{Sample variance} = S_d^2 = \frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1}$$

Sample S.D. = $\sqrt{S^2{}_d}$

n = number of pairs of observations

$$\textbf{Standard error of the sample mean difference} = s\bar{d} = \frac{S_d}{\sqrt{n}}$$

Example:

- H_0: The mean quarterly return on Portfolio A = Mean quarterly return on Portfolio B from 2000 to 2005.
- H_a: The mean quarterly return on Portfolio A ≠ Mean quarterly return on Portfolio B from 2000 to 2005.

The two portfolios share the same set of risk factors; thus, their returns are dependent (not independent). Hence, a paired comparisons test should be used.

The following test is conducted:

H_0: $\mu_d = 0$ *versus* **H_a: $\mu_d \neq 0$** at a 10% significance level.

where,
μ_d = population mean value of difference between the returns on the two portfolios 2000 to 2005.

Suppose,

- Sample mean difference between Portfolio A and Portfolio B = $\bar{d}$ = -0.60% per quarter.
- Sample S.D of differences = 6.50.
- Total sample size = n = 6 years × 4 = 24.
- The standard error of the sample mean difference = $s\bar{d}$ = 6.50 / $\sqrt{24}$ = 1.326807.
- t-value from the table with degrees of freedom = n - 1 = 24 - 1 = 23 and .10/ 2 = 0.05 significance level is t ± 1.714.

Decision rule: Reject H_0 if t > 1.714 or if t <–1.714.

Calculated test statistic = t = $\frac{-0.60 - 0}{1.326807}$ = –0.452213

- Since, calculated t statistic is **not <** -1.714, we fail to reject the null hypothesis at 10% significance level.

Thus, we conclude that the difference in mean quarterly returns is not statistically significant at 10% significance level.

Practice: Example 2 from the CFA Institute's Curriculum.

Tests of Variances

1. Tests of a Single Variance

We can formulate hypotheses as follows:

1. H_0: $\sigma^2 = \sigma^2_0$ versus H_a: $\sigma^2 \neq \sigma^2_0$
2. H_0: $\sigma^2 \leq \sigma^2_0$ versus H_a: $\sigma^2 > \sigma^2_0$
3. H_0: $\sigma^2 \geq \sigma^2_0$ versus H_a: $\sigma^2 < \sigma^2_0$

where,
σ^2_0 = hypothesized value of σ^2_0.

Test Statistic for Tests Concerning the Value of a Population Variance (Normal Population): If we have n independent observations from a normally distributed population, the appropriate test statistic is **chi-square test statistic**, denoted χ^2.

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

where,
n– 1 = degrees of freedom.
s^2 = sample variance, ca1culated as follows.

$$S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

Assumptions of the chi-square distribution:

- The sample is a random sample or
- The sample is taken from a normally distributed population.

Properties of the chi-square distribution:

- Unlike the normal and t-distributions, the chi-square distribution is asymmetrical.
- Unlike the t-distribution, the chi-square distribution is bounded below by 0 i.e. χ^2 values cannot be negative.
- Unlike the t-distribution, the chi-square distribution is affected by violations of its assumptions and give incorrect results when assumptions do not hold.
- Like the t-distribution, the shape of the chi-square distribution depends upon the degrees of freedom i.e. as the number of degrees of freedom increases, the chi-square distribution becomes more symmetric.

Rejection Points for Hypothesis Tests on the Population Variance:

1. **Two-tailed test:** $H_0: \sigma^2 = \sigma^2_0$ versus $H_a: \sigma^2 \neq \sigma^2_0$

   Decision Rule: Reject H_0 if

   i. The test statistic > upper $\alpha/2$ point ($\chi^2_{\alpha/2}$) of the chi-square distribution with df = n – 1 or
   ii. The test statistic < lower $\alpha/2$ point ($\chi^2_{1-\alpha/2}$) of the chi-square distribution with df = n – 1.

2. **Right-tailed test:** $H_0: \sigma^2 \leq \sigma^2_0$ versus $H_a: \sigma^2 > \sigma^2_0$.
   Decision Rule: Reject H_0 if the test statistic > upper α point of the chi-square distribution with df = n -1.

3. **Left-tailed test:** $H_0: \sigma^2 \geq \sigma^2_0$ versus $H_a: \sigma^2 < \sigma^2_0$
   Decision Rule: Reject H_0 if the test statistic < lower α point of the chi-square distribution with df = n -1.

Example: Suppose,
H_0: The variance, $\sigma^2 \leq 0.25$.
H_a: The variance, $\sigma^2 > 0.25$.

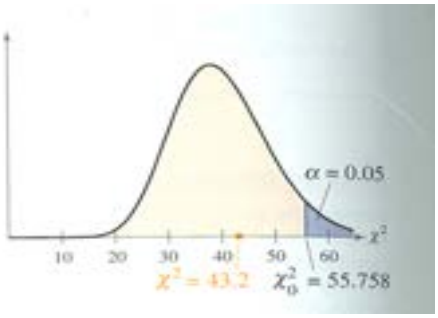
It is a right-tailed test with level of significance (α) = 0.05 and d.f. = 41 – 1 = 40 degrees. Using the chi-square table, the critical value is 55.758.

Decision rule: Reject H0 if $\chi^2 > 55.758$.

Using the X^2–test, the standardized test statistic is:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(41-1)(0.27)}{0.25} = 43.2$$

- Since, χ^2 is not > 55.758, we fail to reject the H_0.



Chi-square confidence intervals for variance: Unlike confidence intervals based on z or t-statistics, chi-square confidence intervals for variance are ***asymmetric***. A two-sided confidence interval for population variance, based on a sample of size n is as follows:

- Lower limit = L = $(n-1)\, s^2 / \chi^2_{\alpha/2}$
- Upper limit = U = $(n-1)\, s^2 / \chi^2_{1-\alpha/2}$

When the hypothesized value of the population variance lies within these two limits, we fail to reject the null hypothesis.

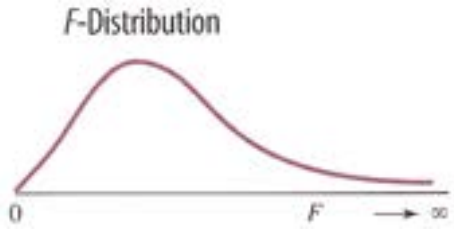
2. Test Concerning the Equality of Two Variance (F-Test)

1. $H_0: \sigma^2_1 = \sigma^2_2$ versus $H_a: \sigma^2_1 \neq \sigma^2_2$
   $\sigma^2_1 = \sigma^2_2$ implies that $\sigma^2_1 / \sigma^2_2 = 1$.
2. $H_0: \sigma^2_1 \leq \sigma^2_2$ versus $H_a: \sigma^2_1 > \sigma^2_2$
3. $H_0: \sigma^2_1 \geq \sigma^2_2$ versus $H_a: \sigma^2_1 < \sigma^2_2$

Tests concerning the difference between the variances of two populations based on independent random samples are based on an **F-test**and **F-distribution**. F-test is a ratio of sample variances.

Properties of F-distribution:

- Like the chi-square distribution, the F-distribution is non-symmetrical distribution i.e. it is skewed to the right.
- Like the chi-square distribution, the F-distribution is bounded from below by 0 i.e. $F \geq 0$.
- The F-distribution depends on two parameters n and m (numerator and denominator degrees of freedom, respectively).
- Unlike the chi-square test, the F-test is NOT sensitive to violations of its assumptions.



Relationship between the chi-square and F-distribution:

$$F = (\chi_1^2 / m) \div (\chi_2^2 / n)$$

- It follows an F-distribution with m numerator and n denominator degrees of freedom.

where,

χ_1^2 is one chi-square random variable with m degrees of freedom.
χ_2^2 is another chi-square random variable with n degrees of freedom.

Test Statistic for Tests Concerning Differences between the Variances of Two Populations (Normally Distributed Populations):

Assumption: The samples are random and independent and taken from normally distributed populations.

$$F = \frac{S^2_1}{S^2_2}$$

where,

s^2_1 = *sample variance of the first sample with* n_1 *observations.*
s^2_2 = *sample variance of the second sample with* n_2 *observations.*
df_1 = n_1 *-1 numerator degrees of freedom.*
df_2 = n_2 *-1 denominator degrees of freedom.*

NOTE:

The value of the test statistic is always ≥1.

Convention regarding test statistic: We use the larger of the two ratios s^2_1 / s^2_2 or s^2_2 / s^2_1 as the actual test statistic.

Rejection Points for Hypothesis Tests on the Relative Values of Two Population Variances:

A. When the convention of using the larger of the two ratios s^2_1 / s^2_2 or s^2_2 / s^2_1 is followed:

1. **Two-tailed test:** $H_0: \sigma^2_1 = \sigma^2_2$ versus $H_a: \sigma^2_1 \neq \sigma^2_2$

   Decision Rule: Reject H_0 at the α significance level if the test statistic > upper $\alpha / 2$ point of the F-distribution with the specified numerator and denominator degrees of freedom.

2. **Right-tailed test:** $H_0: \sigma^2_1 \leq \sigma^2_2$ versus $H_a: \sigma^2_1 > \sigma^2_2$

   Decision Rule: Reject H_0 at the α significance level if the test statistic > upper $\alpha / 2$ point of the F-distribution with the specified numerator and denominator degrees of freedom.

3. $H_0: \sigma^2_1 \geq \sigma^2_2$ versus $H_a: \sigma^2_1 < \sigma^2_2$

   Decision Rule: Reject H_0 at the α significance level if the test statistic > upper $\alpha / 2$ point of the F-distribution with the specified numerator and denominator degrees of freedom.

Practice: Example 3 and Question-Set from the CFA Institute's Curriculum.

4. PARAMETRIC VS. NONPARAMETRIC TESTS

Parametric versus Nonparametric Tests

Parametric test: A parametric test is a hypothesis test regarding a parameter or a hypothesis test that is based on specific distributional assumptions.

- Parametric tests are robust i.e. they are relatively unaffected by violations of the assumptions.
- Parametric tests have greater statistical power relative to corresponding non-parametric tests.

Nonparametric test: A nonparametric test is a test that is either not regarding a parameter or is based on minimal assumptions about the population.

- Nonparametric tests are considered distribution-free methods because they do not rely on any underlying distributional assumption.
- Nonparametric statistics are useful when the data are not normally distributed.

Uses of Nonparametric Tests

A non-parametric test mainly used in three situations

1) When data do not meet distributional assumptions.
2) When data are given in ranks.
3) When the hypothesis is not related to a parameter.

In a nonparametric test, generally, observations (or a function of observations) are converted into ranks according to their magnitude. Thus, the null hypothesis is stated as a thesis regarding ranks or signs. The non-parametric test can also be used when the original data are already ranked.

Nonparametric Inference: Summary

Nonparametric statistical procedures rely on fewer assumptions, accommodate ranked data, and address issues independent of parameters, thus allow for greater inference. They are supplied with parametric tests to assess the impact of assumptions on statistical results.

However, when the assumptions of parametric tests are satisfied, parametric are often preferred due to their increased power in rejecting false null hypotheses.

Practice: End of Chapter Practice Problems and Questions from FinQuiz Question Bank.

Stanley Notes™ 2025

1. INTRODUCTION

Correlation coefficient significance tests are used to determine if the relationship between two random variables is due to chance.

There are both parametric and non-parametric methods for determining the correlation between two variables.

If the relationship is not due to chance, this data can be used for modeling or forecasting using regression models or machine learning.

2. TEST CONCERNING CORRELATION

1. Parametric Test of a Correlation

The Pearson correlation coefficient, also known as bivariate correlation, is a statistical measure that quantifies the strength and direction of the linear relationship between two continuous variables.

It ranges from -1 to +1, with a value of +1 indicating a perfect positive correlation, -1 indicating a perfect negative correlation, and 0 indicating no linear correlation.

The closer the absolute value of the correlation coefficient is to 1, the stronger the correlation between the variables.

Consider two variables X and Y, the sample correlation r is :

$$r_{XY} = \frac{s_{XY}}{s_X s_Y}$$

s_{XY} = sample covariance between X and Y.
s_X & s_Y = standard deviation of X and Y respectively

If the two variables are normally distributed, the sample correlation, r, is used to assess whether the null hypothesis (H_0: = 0), should be rejected, using the following formula.

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Degree of freedom = df = n-2

Practice: Example 1 and 2 from the CFA Institute's Curriculum.

2. Non-Parametric Test of Correlation: The Spearman Rank Correlation Coefficient

When the population under consideration does not meet the assumptions, a test based on the Spearman rank correlation coefficient r_S can be used.

Steps of Calculating r_S:

1. Rank the observations on X from in descending order i.e. from largest to smallest.

   - The observation with the largest value is assigned number 1.
   - The observation with second-largest value is assigned number 2, and so on.
   - If two observations have equal values, each tied observation is assigned the average of the ranks that they jointly occupy e.g. if the 4th and 5th-largest values are tied, both observations are assigned the rank of 4.5 (the average of 4 and 5).

2. Calculate the difference, d_i, between the ranks of each pair of observations on X and Y.

3. The Spearman rank correlation is calculated as:

$$r_S = 1 - \frac{6\sum_{i=1}^{n} d_1^2}{n(n^2-1)}$$

a) For small samples, the rejection points for the test based on r_S are found using Table 11 below.

b) For large samples (i.e. n> 30), t-test can be used to test the hypothesis i.e.

$$t = \frac{(n-2)^{1/2} r_S}{(1-r_S^2)^{1/2}}$$

With degrees of freedom = n – 2.

Refer to: Illustration under Exhibit 4 and Practice Example 3 from the CFA Institute's Curriculum.

Practice: Question-Set, End-of-Chapter Questions from the from the CFA Institute's Curriculum as well as Questions from FinQuiz Question-bank

3. TEST OF INDEPENDENCE USING CONTINGENCY TABLE DATA

The **test of independence** determines whether there is a relationship between two ***categorical variables***. This nonparametric test assesses whether the occurrence of one variable is independent of the occurrence of another variable by comparing actual frequencies with those expected on the basis of independence.

- The data is organized in a ***contingency table***. A contingency table displays the frequency counts of each combination of categories for the two variables (in rows and columns). It is also called two-way table.
- The most commonly used test statistic for a test of independence is the **chi-square statistic**.
- It measures the difference between the observed frequencies and the expected frequencies. The formula for calculating the chi-square statistic is:
- $$\chi^2 = \sum_{i=1}^{m} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

  where
  - Σ represents the sum of all cells in the contingency table, O_{ij} is the observed frequency, and E_{ij} is the expected frequency in each cell of row I and column j.
  - E_{ij} = The expected frequencies E_{ij} for each cell in the contingency table, calculated under the assumption of independence. The expected frequency for each cell can be computed as $E_{ij} = \frac{(Total\ row\ i) \times (Total\ row\ j)}{Overall\ Total}$
  - m = the number of cells in the table, calculated by multiplying the number of groups in the rows by the number of groups in the columns.
- The degree of freedom (df) for this test is calculated as (r - 1) × (c - 1), where r is the number of rows and c is the number of columns in the contingency table.
- Determining the critical value: Chi-square distribution tables provide the crucial value for the test statistic. The critical value is based on your desired level of significance (e.g., 0.05).
- Comparing the test statistic with the critical value:
  - If the test statistic is greater than the critical value, reject the null hypothesis and conclude that there is evidence of a relationship between the variables.
  - If the test statistic is less than or equal to the critical value, fail to reject the null hypothesis and conclude that there is no evidence of a relationship.

Refer to: Illustration under Exhibits 7, 8, 9 and 10 from the CFA Institute's Curriculum.

Practice: Example 4 and Question-Set form the CFA Institute's Curriculum.

Practice: End of Chapter Practice Problems and Questions from FinQuiz Question Bank.

1. SIMPLE LINEAR REGRESSION

Regression analysis:

- is a tool used to examine whether a variable is useful to explain another variable.
- predicts the value of a dependent variable based on the value of at least one independent variable.

Simple Linear Regression (SLR): A regression that summarizes the relation between the dependent variable and one independent variable through estimation of a linear relationship.

If more than one variable is used, it is called ***Multiple Regression***:

Dependent variable (a.k.a. explained variable) Y: The variable whose variation is being explained by the independent variable.

Independent variable (a.k.a. explanatory variable) X: The variable used to explain the dependent variable.

Practice: Example 1, CFA Institute's Curriculum.

2. ESTIMATING THE PARAMETERS OF A SIMPLE LINEAR REGRESSION

The Basics of Simple Linear Regression

- Linear regression assumes straight line relationship between dependent and independent variables.
- It is also called least squares regression or ordinary least squares regression.
- The objective is to fit a line to the observations on Y and X to minimize the squared deviations from the line.

Simple Linear Regression (i.e., Y is regressed on X) is described as:

$$\boldsymbol{Y_i = b_0 + b_1 X_i + \varepsilon_i} \text{ where } i = 1,\ldots,n$$

Y = dependent variable
X = independent variable
b_0 = intercept
b_1 = slope coefficient
ε = error term
b_0 *and* b_1 are called regression coefficients

Estimating the Regression Line

In a regression model, we cannot observe b_0 *and* b_1. In its place, we use estimated values $\hat{b}_0$ *and* $\hat{b}_1$.

We select values for the intercept b_0 and slope b_1 that minimize the sum of the squared vertical distances between the observations and the regression line.

Value of dependent variable for the ith observation $\hat{Y}_i$

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$$

Error Term represents the difference between observation and its expected value i.e., how much estimatedl ith value differs from the actual ith value.

$$\varepsilon_i = Y_i - \hat{Y}_i$$

SSE (sum of squares error or residual sum of squares):

- Measures unexplained variation in the dependent variable.
- It is sum of squared deviations of the value of the dependent variable and the value of the dependent variable based on the estimated regression line

$$\text{SSE} = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

$$as\ \hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$$

$$\text{Therefore, SSE} = \sum_{i=1}^{n}(Y_i - (\hat{b}_0 + \hat{b}_1 X_i))^2$$

$$as\ Y_i - \hat{Y}_i = \varepsilon_i$$

$$\text{Therefore, SSE} = \sum_{i=1}^{n}(\varepsilon_i)^2$$

Note: Residuals are stated in the same measurement unit as the dependent variable.

Interpreting the Regression Coefficients

How to calculate and interpret regression coefficient b_1 and b_0

1) ***Slope Coefficient (b_1):*** A change in the dependent variable for a one unit change in the independent variable.

Stanley Notes™ 2025

Interpretation:
If slope is positive (negative), the change in the dependent and independent variable will be in the same(opposite) direction.

$$b_1 = \frac{cov(x,y)}{var(x)} = \frac{\frac{\sum_{i=1}^{n}(Y_i-\bar{Y})(X_i-\bar{X})}{n-1}}{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}} = \frac{\sum_{i=1}^{n}(Y_i-\bar{Y})(X_i-\bar{X})}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$$

2) ***Intercept (b_0):*** The predicted value of the dependent variable when the independent variable is set to zero.

$$b_0 = \bar{Y} - \hat{b}_1\bar{X}$$

where $\bar{Y}$ *and* $\bar{X}$ are mean values of X and Y.

Interpretation:
In some cases, zero intercept is meaningful but in some cases it does not make sense.

For example, if independent variable is:
- **money supply**, 0 intercept is meaningless becase zero money supply is not possible.
- **money supply growth**, 0 intercept is meaningful because zero money supply growth is possible.

Cross-Sectional vs. Time-Series Regressions

Two main types of data used in regression analysis are time series and cross-sectional:

1) Time-series: It uses many observations from ***different time periods*** for the ***same*** company, asset class or country etc.

2) Cross-sectional: It uses many observations for the ***same time period***of ***different*** companies, asset classes or countries etc.

Mix of two types

Panel data: It is a mix of time-series and cross-sectional data.

Practice: Example 1, 2 & Question-Set from the CFA Institute's Curriculum.

3. ASSUMPTIONS OF THE SIMPLE LINEAR REGRESSION MODEL

The four key assumptions of the simple linear regression model are:

1) Linearity
2) Homoskedasticity
3) Independence
4) Normality

Assumption 1: Linearity

'Relation between the dependent variable and independent variable is linear.'

- If the relationship is nonlinear, the model will be biased (i.e., over or underestimate the dependent variable).
- Linearity assumption also implies that independent variable must not be random (i.e., non-stochastic). Otherwise, there will be no linear relationship.

Assumption 2: Homoskedasticity

'The variance of residuals is the same for all observations. It is known as Homoskedasticity (same scatter) assumption.'

- If the variance of the residuals differs across observations, this state is called heteroskedasticity (different scatter).
- In real-world data, structural changes (regime changes) often involve heteroskedasticity.

Assumption 3: Independence

'The observations (pairs of Xs and Ys) are independent of each other, which implies the residuals are uncorrelated across observations.'

- If variables are not independent, the residuals will be correlated (display a pattern). This is an indication of autocorrelation.

Assumption 4: Normality

'The regression residuals must be normally distributed.'

- In large sample sizes, dropping the normality assumption does not noticeably influence results.

Note: Normality assumption does not mean that dependent or independent variables must be normally distributed.

Practice: Question-Set from the CFA Institute's Curriculum.

4. HYPOTHESIS TESTS IN THE SIMPLE LINEAR REGRESSION MODEL

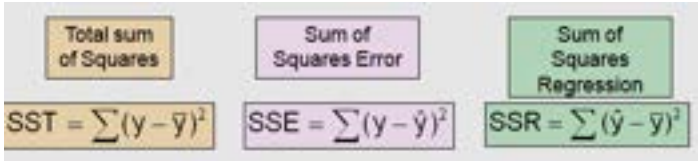
Analysis of Variance

Breaking down the Sum of Squares Total into Its Components

Total variation is made up of two parts:

Sum of squares total (SST) = sum of squares error (SSE) + sum of squares regression (SSR)

SST = SSE + SSR



where,
$\bar{y}$ = *Average value of the dependent variable*
y = Observed values of the dependent variable
$\hat{y}$ = *Estimated value of y for the given value of x*

- **SST (total sum of squares):** Measures total variation in the dependent variable i.e., the variation of the y_i values around their mean y.
- **SSE (error sum of squares):** Measures unexplained variation in the dependent variable.
- **SSR / RSS (regression sum of squares):** Measures variation in the dependent variable explained by the independent variable.

Measures of Goodness of Fit

Goodness of fit (i.e., how well the regression model fits the data) can be measured using several methods such as:

1. Coefficient of determination R^2
2. F-statistic
3. Standard error of regression S_e

1. Coefficient of Determination R^2

- The coefficient of determination is the percentage of the total variation in the dependent variable that is explained by the independent variable.
- The coefficient of determination is also called R-squared and is denoted as R^2.
- It is descriptive measure.

$$Coefficient\ of\ determination\ (R^2) = \frac{Explained\ Variation(SSR)}{Total\ Variation\ (SST)}$$

$$= \frac{Total\ Variation\ (SST) - Unexplained\ Variation\ (SSE)}{Total\ Variation\ (SST)}$$

where, $0 \leq R^2 \leq 1$

In case of a single independent variable, the coefficient of determination is: $R^2 = r^2$

where,
R^2 = *Coefficient of determination*
r = Simple correlation coefficient

Example:
Suppose coefficient of determination between returns of two assets is 0.64. This means that approximately 64 percent of the variability in the returns of one asset (or dependent variable) can be explained by the returns of the other asset (or indepepnent variable).

2. F-distributed test Statistic or F-Test

F-distributed test statistic tests whether the slopes b_1 in regression are equal to zero, against the alternative hypothesis that at least one slope is not equal to zero.

H_0: $b_1 = 0$
H_1: $b_1 \neq 0$

The F statistic is calculated as the ratio of mean square regression (MSR) to mean squared errors (MSE).

$$\mathrm{F} = \frac{\mathrm{MSR}}{\mathrm{MSE}} = \frac{(\frac{\mathrm{RSS}}{\mathrm{k}})}{(\frac{\mathrm{SSE}}{\mathrm{n-k-1}})} = \frac{\mathrm{MSR}}{\mathrm{MSE}}$$

$$\mathrm{MSR} = \frac{\mathrm{SSR}}{\mathrm{k}} = \frac{\sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2}{k} = \sum_{i=1}^{n}(\hat{Y}_i - \bar{Y})^2$$

where df numerator = k = 1

$$\mathrm{MSE} = \frac{\mathrm{SSE}}{\mathrm{n-k-1}} = \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-k-1} = \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-2}$$

where df denominator = n – k – 1 = n – 2 (in simple linear regression)

Note: F-test is always a one-tailed test (one sided).

3. The Standard Error of Estimate S_e

- Standard Error of Estimate (S_e) measures the degree of variability of the actual y-values relative

to the predicted y-values from a regression equation.
- Smaller the S_E, better the fit.
- S_E is also called standard error of regression or root mean square error.
- $Standard\ Error\ of\ Estimate: S_E = \sqrt{MSE} = \sqrt{\frac{SSE}{n-k-1}} = \sqrt{\frac{\sum(y_i-\hat{y})^2}{n-k-1}}$

where n = Sample size
k = no. of independent variables = 1.

Example:
n = 100
SSE = 2,252,363
Thus,

$$S_e = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{2,252,363}{98}} = 151.60$$

Note:
- Co-efficient of determination and F-statistic are relative measures of fit
- Standard error of the estimate is absolute measure.

ANOVA and Standard Error of Estimate in Simple Linear Regression

Analysis of Variance (ANOVA) is a statistical method used to divide the total variance in a study into meaningful pieces that correspond to different sources.

Analysis of Variance Table for Simple Linear Regression

ANOVA	df	SS	MS	F
Regression	1	$SSR = \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$	$MSR = \frac{SSR}{k}$	$F = \frac{MSR}{MSE} = \frac{SSR/1}{SSE/(n-2)}$
Error	n-2	$SSE = \sum_{i=1}^{n}(y_i - \hat{y})^2$	$MSE = \frac{SSE}{n-2}$	
Total	n–1	$SST = \sum_{i=1}^{n}(y_i - \bar{y})^2$		

Or

Source of Variability	df	Sum of Squares	Mean Sum of Squares
Regression (Explained)	1	SSR	MSR = RSS/1
Error (Unexplained)	n-2	SSE	MSE = SSE/n-2
Total	n-1	SST=SSR + SSE	

Practice: Example 5, , CFA Institute's Curriculum.

Hypothesis Testing of Individual Regression Coefficients

Hypothesis Tests of the Slope Coefficient

t-statistic is used to test the significance of the individual coefficients (e.g., slope) in a regression. It is used to test whether the population slope is different from a specific value.

Suppose we want to test a hypothesis about the slope.

Null and Alternative hypotheses
H_0: $b_1 = 0$ (no linear relationship)
H_1: $b_1 \neq 0$ (linear relationship does exist)

A t-test statistic is calculated by subtracting the hypothesized population slope B_1 from the estimated slope coefficient $\hat{b}_1$ and then dividing the difference by the standard error of the slope coefficient $s_{\hat{b}_1}$.

Test statistic $t = \frac{\hat{b}_1 - B1}{s_{\hat{b}_1}}$

where,
$\hat{b}_1$ = *Sample regression slope coefficient*
b_1 = *Hypothesized slope*
S_{b_1} = *Standard error of the slope*
df= n-k-1 = n–2

Decision Rule:
If test statistic is <– t-critical or > + t-critical with n-2 degrees of freedom, (if absolute value of t > t_c), Reject H_0; otherwise, Do not Reject H_0.

Standard Error of Slope Coefficient $(s_{\hat{b}_1})$
It is the ratio of standard error of estimate S_e to the square root of variation of independent variable. for simple linear regression:

$$s_{\hat{b}_1} = \frac{s_e}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$$

The greater the variability of the independent variable the lower will be the standard error of slope and therefore greeter will be the calculated t-statistic.

For details: Refer to Exhibit 21 to 23 from the CFA Institute's Curriculum.

For detailed example: Refer to Exhibit 26 from the CFA Institute's Curriculum.

Hypothesis Tests of the Intercept

This test is useful to determine wthether the population intercept is a specific value.
Intercept is the predicted value of the dependent variable when the independent variable is set to zero

Standard error of the intercept = $s_{\hat{b}_0} = \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$

The test is whether the intercept is different from the hypothesized value B_0 , using the following formula.

$$t_{intercept} = \frac{\hat{b}_0 - B_0}{s_{\hat{b}_0}} = \frac{\hat{b}_0 - B_0}{\sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}}$$

For details: Refer to Exhibit 24 from the CFA Institute's Curriculum.

Hypothesis Tests of Slope When Independent Variable Is an Indicator Variable

- Indicator variable (or dummy variable) is a variable that takes on a value of either 1 or 0.
- 1 if a particular condition is true and 0 if that condition is false.
- In simple linear regressions, the slope is the difference in the dependent variable for the two conditions.

Test of Hypothesis: Level of Significance and p-Values

p-value: The p-value is the smallest level of significance at which the null hypothesis can be rejected.

For example, if the p-value is 0.05 i.e., 0.05 significance level, this indicates that there is a 5% chance of rejecting the hypothesis when actually it is true actually. This is a Type 1 error.

- The smaller the p-value, the smaller the chance of making type I error (i.e., the greater the probability of rejecting the null hypothesis).
- Type I error = False positive = rejecting the null hypothesis when it is true.
- Type II error = False negative = not rejecting the null hypothesis when it is wrong.

For details: Refer to the CFA Institute's Curriculum for this section.

Practice: Question-Set from the CFA Institute's Curriculum.

6. PREDICTING USING SIMPLE LINEAR REGRESSION AND PREDICTION INTERVALS

Regression results are used to make prediction about the dependent variable.

Prediction intervals are used to realize how sure we are about the predicted results.

Prediction interval for a forecasted value of a dependent variable is created by using equation
$\hat{Y} \pm t_c s_f$

where,

$$s_f^2 = s_e^2 \left[1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{(n-1)s_X^2}\right] = s_e^2 \left[1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}\right]$$

Standard error of forecast = $s_f = s_e \sqrt{1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{\sum_{i=1}^{n}(X_i - \bar{X})^2}}$

as

$$s_f = \sqrt{s_f^2}$$

s_e = standard error of estimate
n = number of observations
X = value of independent variable
$\bar{X}$ = estimated mean of X
s^2_X= variance of independent variable
t_c = critical t-value for n −k −1 degrees of freedom.
The following can be taken from the equation:

1. The better the fit of the regressions, the smaller will be the s_e and therefore, the smaller will be the s_f.
2. The larger the n (sample size), the smaller will be the s_f.
3. The closer the X_f is to $\bar{X}$, the smaller will be the s_f.

where X_f is forecasted independent variable and $\bar{X}$ is forecasted mean of the independent variable.

For detailed example: Refer to Exhibit 34 and the example following the exhibit from the CFA Institute's Curriculum.

Practice: Question-Set from the CFA Institute's Curriculum.

6. FUNCTIONAL FORMS FOR SIMPLE LINEAR REGRESSION

Economic and financial data often exhibit nonlinear relationships between two variables. To apply simple linear regression model on such data, we need to modify either the dependent or the independent variable.

Many functional forms can be used to transform data to enable their use in linear regression. Three commonly used functional forms that involve log transformation are as follows:

1. The log-lin model
2. The lin-log model
3. The log-log model

The Log-Lin Model

In this model, the dependent variable is in logarithmic form, but the independent variable is in linear form.

$$\ln Y_i = b_0 + b_i X_i$$

Slope coefficient → relative change in the dependent variable for an absolute change in the independent variable.

Note:
Directly comparing the different model values is not possible because variables are not in the same form.

The Lin-Log Model

In this model, the independent variable is in logarithmic form, but the dependent variable is in linear

$$Y_i = b_0 + b_i \ln X_i$$

Slope coefficient → absolute change in the dependent variable for a relative change in the independent variable.

The Log-Log Model

In this model, both the dependent and independent variables are in logarithmic form. This model is also called double-log model.

$$\ln Y_i = b_0 + b_i \ln X_i$$

Slope coefficient → relative change in the dependent variable for a relative change in the independent variable.

This model is suitable in calculating elasticities.

Selecting the Correct Functional Form

Selection of the suitable functional form depends on examining the goodness of fit measures (R^2, F-statistic & S_e) as well as patterns in the residuals.

Many statistical software packages enable us to visually examine and inspect the distribution of the residuals

Practice: Question-Set from the CFA Institute's Curriculum.

Practice: End of Chapter Practice Problems & Questions from FinQuiz Question-bank.

1. INTRODUCTION

Fintech is changing investment management. Progress in this subject entails using Big Data, AI, and machine learning to evaluate investments, improve portfolios, and manage risks.

These advances affect asset managers who use these tools and technologies to make investment decisions.

2. HOW IS FINTECH USED IN QUANTITATIVE INVESTMENT ANALYSIS?

Fintech (finance + technology) is playing a major role in the advancement and improvement of:

- **investment management industry** (such as assessment of investment opportunities, portfolio optimization, risk mitigation etc.).
- **investment advisory services** (e.g., Robo-advisors with or without intervention of human advisors are providing tailored, low-priced, actionable advice to investors).
- **financial record keeping, blockchain and distributed ledger technology (DLT**) through finding improved ways of recording, tracking or storing financial assets.

What is FinTech

For the scope of this reading, term **'Fintech'** is referred to as technology-driven innovations in the field of financial services and products.

Note: In common usage, fintech may also refer to companies associated with new technologies or innovations.

Initially, the scope of fintech was limited to data processing and to the automation of routine tasks.

Today, advanced computer systems are using artificial intelligence and machine learning to perform decision-making tasks including investment advice, financial planning, business lending/payments etc.

Some salient fintech developments related to the investment industry include:

- **Analysis of large data sets:** These days, professional investment decision making process uses extensive amounts of traditional data sources (e.g., economic indicators, financial statements) as well as non-traditional data sources (such as social media, sensor networks) to generate profits.
- **Analytical tools:** There is a growing need of techniques involving artificial intelligence (AI) to identify complex, non-linear relationships among such gigantic datasets.
- **Automated trading:** Automated trading advantages include lower transaction costs, market liquidity, secrecy, efficient trading etc.
- **Automated advice:** Robo-advisors or automated personal wealth management are low-cost alternates for retail investors.
- **Financial record keeping:** DLT (distributed ledger technology) provides advanced and secure means of record keeping and tracing ownership of financial assets on peer-to-peer (P2P) basis. P2P lowers involvement of financial intermediaries.

Big Data

Big data refers to huge amount of data generated by traditional and non-traditional data sources.

Details of traditional and non-traditional sources are given in the table below.

Traditional		Non-traditional (alternate)	
Sources	Institutions, Businesses, Government, Financial Markets	Sources	Social media, Sensor networks Company-used data, Electronic devices, Smart phones, Cameras, Microphones, Radio-frequency identification (RFID)
Forms of Data	Annual reports, Regulatory	Forms of Data	Posts, Tweets, Blogs, Email, Text messages,

Stanley Notes™ 2025

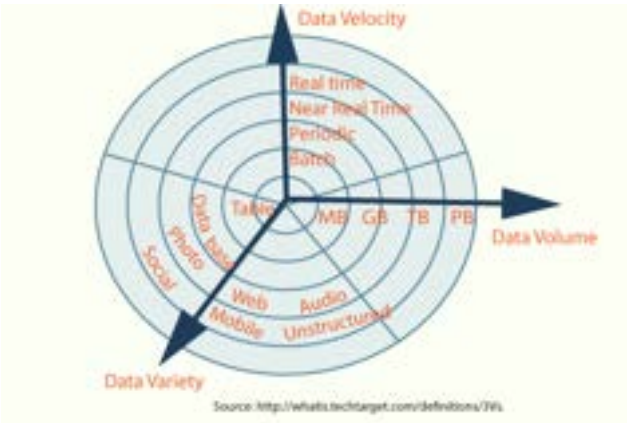
Traditional		Non-traditional (alternate)	
	filings, Sales & earnings, Conference calls, Trade prices & volumes		Web-traffic, Online news sites

Big data typically have the following features:

1. **Volume:** Quantities of data denoted in millions, or even billions, of data points. Exhibit below shows data grow from MB to GB to larger sizes such as TB and PB.
2. **Velocity**: Velocity determines how fast the data is communicated. Two criteria are Real-time or Near-time data, based on time delay.
3. **Variety**: Data is collected in a variety of forms including:

   - **structured data** – data items are often arranged in tables where each field represent a similar type of information. (e.g. SQL tables, CSV files)
   - **unstructured data** – cannot be organized in table and requires special applications or programs (e.g. social media, email, text messages, pictures, sensors, video/voice messages)
   - **semi-structured data** – contains attributes of both structured and unstructured data (e.g. HTML codes)

Exhibit: Big Data Characteristics: Volume, Velocity & Variety



Sources of Big Data

In addition to traditional data sources, alternative data sources are providing further information (regarding consumer behaviors, companies' performances and other important investment-related activities) to be used in investment decision-making processes.

Main sources of alternative data are data generated by:

1. **Individuals:** Data in the form of text, video, photo, audio or other online activities (customer reviews, e-commerce). This type of data is often unstructured and is growing considerably.
2. **Business processes:** data (often structured) generated by corporations or other public entities e.g. sales information, corporate exhaust. Corporate exhaust includes bank records, point of sale, supply chain information.
   Note:
   - Traditional corporate metrics (annual, quarterly reports) are lagging indicators of business performance.
   - Business process data are real-time or leading indicators of business performance.
3. **Sensors:** data (often unstructured) connected to devices via wireless networks. The volume of such data is growing exponentially compared to other two sources. **IoT** (internet of things) is the network of physical devices, home appliances, smart buildings that enable objects to share or interact information.

Alternative datasets are now used increasingly in the investment decision making models. Investment professionals will have to be vigilant about using information, which is not in the public domain regarding individuals without their explicit knowledge or consent.

Big Data Challenges

In investment analysis, using big data is challenging in terms of its quality (selection bias, missing data, outliers), volume (data sufficiency) and suitability.

Most of the times, data is required to be sourced, cleansed and organized before use, however, performing these processes with alternative data is extremely challenging due to the qualitative nature of the data.

Therefore, artificial intelligence and machine learning tools help addressing such issues.

3. ADVANCED ANALYTICAL TOOLS: ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING

Artificial intelligence (AI) technology in computer systems is used to perform tasks that involve cognitive and decision-making ability similar or superior to human brains.

Initially, AI programs were used in specific problem-solving framework following 'if-then' rules. Later, advanced processors enabled AI programs such as neural networks (which are based on how human brains process information) to be used in financial analysis, data mining, logistics etc.

Machine learning (ML) algorithms are computer programs that perform tasks and improve their performance overtime with experience. ML requires large amount of data (big data) to model accurate relationships.

ML algorithms use inputs (set of variables or datasets), learn from data by identifying relationships in the data to refine the process and model outputs (targets). If no targets are given, algorithms are used to describe the underlying structure of the data.

ML divides data into two sets:

- **Training data:** that helps ML to identify relationships between inputs and outputs through historical patterns.
- **Validation data**: that validates the performance of the model by testing the relationships developed (using the training data).

ML still depends on human judgment to develop suitable techniques for data analysis. ML works on sufficiently large amount of data which is clean, authentic and is free from biases.

The problem of overfitting (too complex model) occurs when algorithm models the training data too precisely. Over-trained model treats noise as true parameters. Such models fail to predict outcomes with out-of-sample data.

The problem of underfitting (too simple model) occurs when models treat true parameters as noise and fail to recognize relationships within the training data.

Sometimes results of ML algorithms are unclear and are not comprehensible i.e. when ML techniques are not explicitly programmed, they may appear to be opaque or 'black box'.

Types of Machine Learning

ML approaches are used to identify relationships between variables, detect patterns or structure data. Two main types of machine learning are:

1. **Supervised leaning:** uses labeled training data (set of inputs supplied to the program), and process that information to find the output. Supervised learning follows the logic of 'X leads to Y'. Supervised learning is used to forecast a stock's future returns or to predict stock market performance for next business day.

2. **Unsupervised learning:** does not make use of labelled training data and does not follow the logic of 'X leads to Y'. There are no outcomes to match to, however, the input data is analyzed, and the program discovers structures within the data itself e.g. splitting data into groups based on some similar attributes.

Deep Learning Nets (DLNs): Some approaches use both supervised and unsupervised ML techniques. For example, deep learning nets (DLNs) use neural networks often with many hidden layers to perform non-linear data processing such as image, pattern or speech recognition, forecasting etc.

There is a significant role of advanced ML techniques in the evolution of investment research. ML techniques make it possible to

- render greater data availability
- analyze big data
- improve software processing speeds
- reduce storage costs

As a result, ML techniques are providing insights into individual firms, national or global levels and are a great help in predicting trends or events. Image recognition algorithms are used in store parking lots, shipping/manufacturing activities, agriculture fields etc.

4. TACKLING BIG DATA WITH DATA SCIENCE

Data science is interdisciplinary area that uses scientific methods (ML, statistics, algorithms, computer-techniques) to obtain information from big data or data in general.

The unstructured nature of the big data requires some specialized treatments (performed by data scientist) before using that data for analysis purpose.

Data Processing Methods

Various data processing methods are used by scientists to prepare and manage big data for further examination. Five data processing methods are given below:

Capture: Data capture refers to how data is collected and formatted for further analysis. **Low-latency** systems are systems that communicate high data volumes with small delay times such as applications based on real-time prices and events. **High-latency** systems suffers from long delays and do not require access to real-time data and calculations.

Curation: Data curation refers to managing and cleaning data to ensure data quality. This process involves detecting data errors and adjusting for missing data.

Storage: Data storage refers to archiving and storing data. Different types of data (structured, unstructured) require different storage formats

Search: Search refers to how to locate requested data. Advanced applications are required to search from big data.

Transfer: Data transfer refers to how to move data from its storage location to the underlying analytical tool. Data retrieved from stock exchange's price feed is an example of direct data feed.

Data Visualization

Data visualization refers to how data will be formatted and displayed visually in graphical format.

Data visualization for

- **traditional structured** data can be done using tables, charts and trends.
- **non-traditional unstructured** data can be achieved using new visualization techniques such as:
  - interactive 3D graphics
  - multidimensional (more than three dimensional) data requires additional visualization techniques using colors, shapes, sizes etc.
  - tag cloud, where words are sized and displayed based on their frequency in the file.
  - Mind map, a variation of tag cloud, which shows how different concepts are related to each other.

Data visualization Tag Cloud Example

Source: https://worditout.com/word-cloud/create

Text Analytics and Natural Language Processing

Text analytics is a use of computer programs to retrieve and analyze information from large unstructured text or voice-based data sources (reports, earning calls, internet postings, email, surveys). Text analytics helps in investment decision making. Other analytical usage includes lexical analysis (first phrase of compiler) or analyzing key words or phrases based on word frequency in a document.

Natural language processing (NLP) is a field of research that focuses on development of computer programs to interpret human language. NLP field exists at the intersection of computer science, AI, and linguistics.

NLP functions include translation, speech recognition, sentiment analysis, topic analysis. Some NLP compliance related applications include reviewing electronic communications, inappropriate conduct, fraud detection, retaining confidential information etc.

With the help of ML algorithms, NLP can evaluate persons' speech – preferences, tones, likes, dislikes – to predict trends, short-term indicators, future performance of a company, stock, market or economic events in shorter timespans and with greater accuracy.

For example, NLP can help analyze subtleties in communications and transcripts from policy makers (e.g. U.S Fed, European central bank) through the choice of topics, words, voice tones.

Similarly, in investment decision making, NLP may be used to monitor financial analysts' *commentary* regarding EPS forecasts to detect shifts in sentiments (which can be easily missed in their written reports). NLP then assign sentiment ratings ranging from negative to positive, potentially ahead of a change in their recommendations.

Note:
Analysts do not change their buy, hold and sell recommendations frequently; instead, they may offer nuanced commentary reflecting their views on a company's near-term forecasts.

Practice: CFA Institute's end of Chapter Questions and Questions from FinQuiz Question Bank.

1. INTRODUCTION TO MARKET STRUCTURES

This learning module covers the following concepts:

- **Demand concepts:** Own-price elasticity of demand, cross-price elasticity of demand, and income elasticity of demand.
- **Supply concepts**: Total, average, and marginal product of labor; total, variable, and marginal cost of labor; and total and marginal revenue.
- **Market structures:** Economists classify market structures based on the number of sellers, the degree of differentiation of their products, and the barriers to entry.

These concepts are used to calculate the breakeven and shutdown points of production, and to understand demand and supply relations, optimal price and output, and the factors affecting long-run profitability in different market structures.

2. PROFIT MAXIMIZATION: PRODUCTION BREAKEVEN, SHUTDOWN AND ECONOMIES OF SCALE

Revenue under Conditions of Perfect and Imperfect Competition

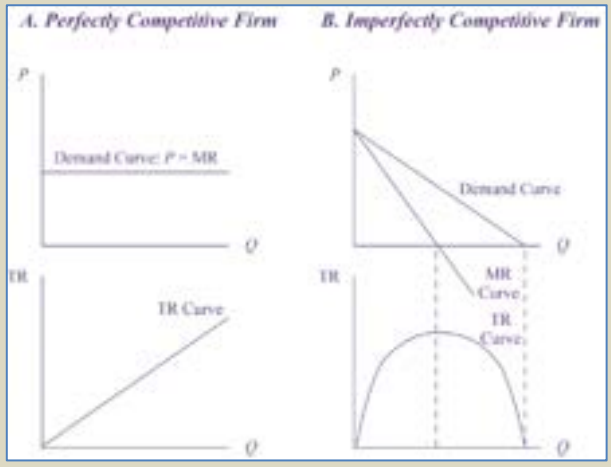
Perfectly competitive market: If a market is perfectly competitive, the firm must take the market price of its output as given, so it faces a perfectly elastic, horizontal demand curve. In this case, the firm's will be equal to price of its product.

Additionally, the firm's average revenue (AR), or revenue per unit, is also equal to price per unit. Under conditions of perfect competition, TR (as always) is equal to price times quantity.

- However, a firm that faces a negatively sloped demand curve must lower its price to sell an additional unit, so its MR is less than price (P).
- The TR curve for the firm under conditions of perfect competition is linear, with a slope equal to price per unit.

Imperfect Competition: Under conditions of imperfect competition (e.g. monopoly), price is a function of quantity: P = f(Q), and TR = f(Q) × Q.

- Initially, a decrease in price increases total expenditure by buyers and TR to the firm because the decrease in price is outweighed by the increase in units sold. But as price continues to fall, the decrease in price outweighed the increase in quantity, and total expenditure (revenue) falls.
- The TR curve for the monopolist first rises (in the range where MR is positive, and demand is elastic) and then falls (in the range where MR is negative and demand is inelastic) with output.



Profit-Maximization, Breakeven, and Shutdown Points of Production

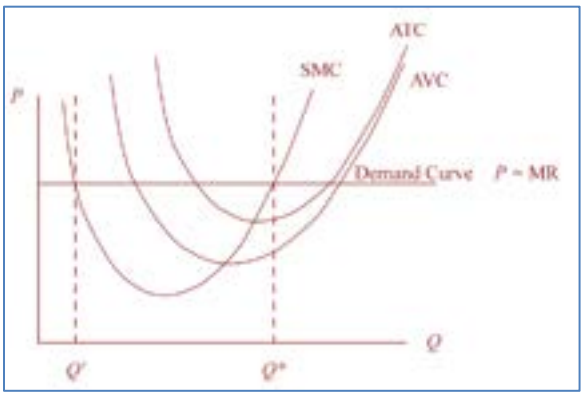
Demand and Average and Marginal Cost Curves for the Firm under Conditions of Perfect Competition

- The following graph shows that the firm is maximizing profit by producing Q*, where price is equal to SMC and SMC is rising.
- Note that at output level, Q", where P = SMC, but at that point, SMC is still falling, so this cannot be a profit-maximizing output.
- If market price rises, the firm's demand and MR curve would simply shift upward, and the firm would reach a new profit-maximizing output level to the right of Q*.

Stanley Notes™ 2025

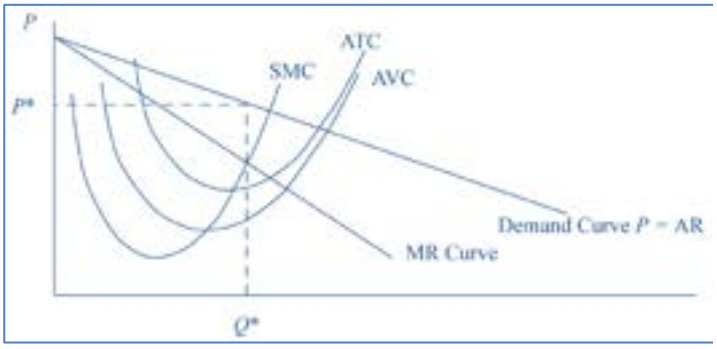
Copyright © FinQuiz.com. All rights reserved.

- If market price falls, the firm's demand and MR curve would shift downward, resulting in a new and lower level of profit-maximizing output.
- In following case, this firm is currently earning a positive economic profit because market price exceeds ATC at output level Q*. This profit is possible in the short run, but in the long run, competitors would enter the market to capture some of those profits and would drive the market price down to a level equal to each firm's ATC.



Demand and Average and Marginal Cost Curves for the Monopolistic Firm

Refer to the graph below. Q* is the level of output where SMC is equal to MR. At this output, the optimal price to charge is given by the firm's demand curve at P*. This monopolist is earning positive economic profit because its price exceeds its ATC. Due to monopolistic power of this firm, the outside competitors would not be able to compete away this firm's profits.

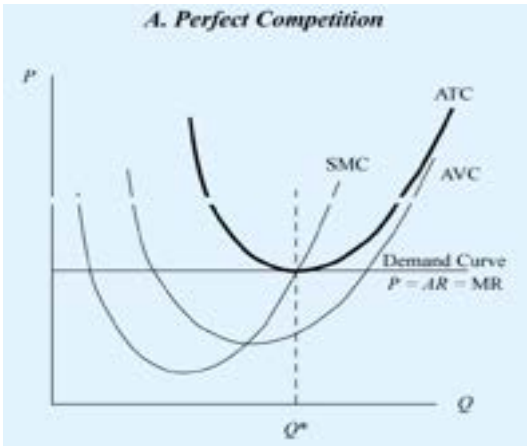


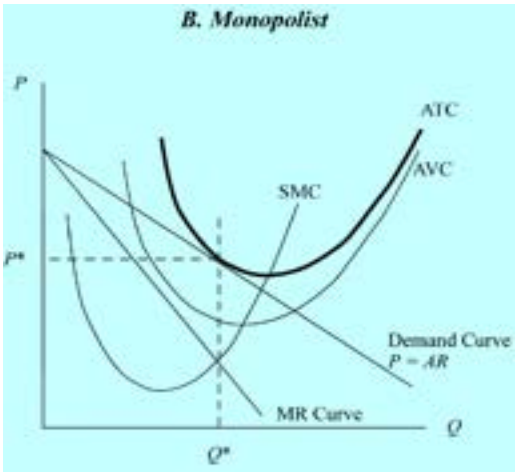
Breakeven Analysis and Shutdown Decision

Break-even price: It refers to a price where economic profit is zero i.e. P = ATC. It is the output level where P = AR = MR = ATC or where TR = TC.

Economic costs = Total accounting costs + Implicit opportunity costs

- When a firm's revenue is equal to its economic costs, it means it is covering the opportunity cost of all of its factors of production, including capital. Such a firm is earning normal profit, but not positive economic profit.
- A firm that operates in a very competitive environment with no barriers to entry from other competitors cannot earn a positive economic profit because the excess rate of return would attract entrants who would produce more output – pushing downward pressure on price. However, this situation does not imply that the firm is earning zero accounting profit.





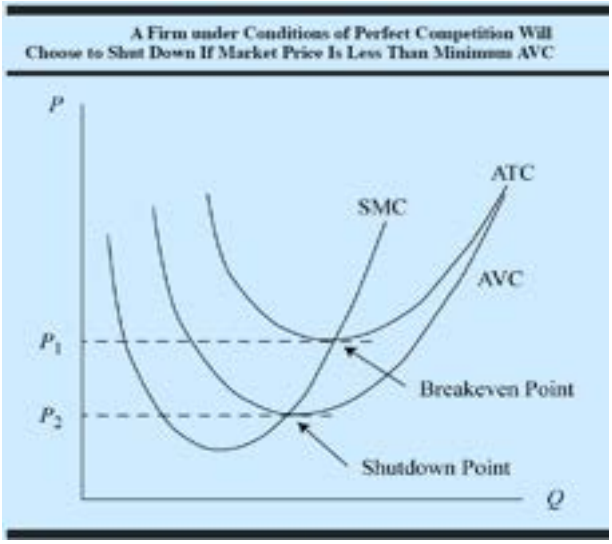
In figure 'B' above, the level of output at which SMC is equal to MR, price is just equal to ATC. This shows that a firm is breaking even and earning economic profit.

The Shutdown Decision

The Shutdown Point: The firm will shut down if total revenues generated by a firm are not enough to cover average variable costs.

- A firm should continue to produce as long as Price > Average variable cost. However, it should be noted that that at that price, firm incurs losses. Sunk costs must be ignored in the decision to continue to operate in the short run.
- When Price < AVC, it is preferable for a firm to shut down temporarily in order to save the variable costs. However, firm still has to pay fixed costs.
- If price is greater than AVC, the firm is not only covering all of its variable cost but also a portion of fixed cost.
- If all fixed costs are sunk costs, then the shutdown point is when the market price falls below minimum average variable cost. At this price, the firm incurs only fixed cost and loses less money than when operating at a price that does not cover variable cost.

Refer to the graph next: At any price above P_1, the firm can earn a positive profit and clearly should continue to operate. At a price below P_2, the minimum AVC, the firm could not even cover its variable cost and should shut down. At prices between P_2 and P_1, the firm should continue to operate in the short run because it is able to cover all of its variable cost as well as some of its fixed costs. The **shutdown point** is the minimum AVC point and the minimum ATC point is the **breakeven point**.



In the long-run (under perfect competition), profit is maximized at the level of output where the firm's long-run average total cost is at minimum level i.e. minimum point of the firm's long-run average total cost curve. In the short-run, supply curve of a competitive firm is that part of MC curve that lies above the Average Variable Cost. However, in the long run, a firm must have its MC > AC in order to remain in the industry. If MC < AC, firm will exit the industry in the long run.

Summary:

- The firm must cover its variable cost to remain in business in the short run; if TR cannot cover TVC, the firm shuts down production to minimize loss.

Loss = Amount of fixed cost

- If TVC exceeds TR in the long run, the firm will exit the market to avoid the loss associated with fixed cost at zero production.
- When TR is enough to cover TVC but not all of TFC, the firm can continue to produce in the short run but will be unable to maintain financial solvency in the long run.

Short run & Long run Decisions to Operate or Not

	Revenue-Cost relationship		
	TR = TC	TR = TVC but < TC	TR< TVC
Short run	stay in market	stay in market	shut down production
Long run	stay in market	exit market	exit market

Practice: Example 1 & 2 from the CFA Institute's Curriculum.

Economies and Diseconomies of Scale with Short-Run and Long-Run Cost Analysis

The firm selects an operating size or scale that maximizes profit over any time frame. The short run is the time period during which at least one of the factors of production, such as technology, physical capital, and plant size, is fixed.

The long run is the time period during which all factors of production are variable. In addition, in the long-run, firms

can enter or exit the market based on decisions regarding profitability.

The long run is also referred to as the "***planning horizon***" in which the firm can choose the short-run position or optimal operating size that maximizes profit over time. The firm always operates in the short run but plans in the long-run.

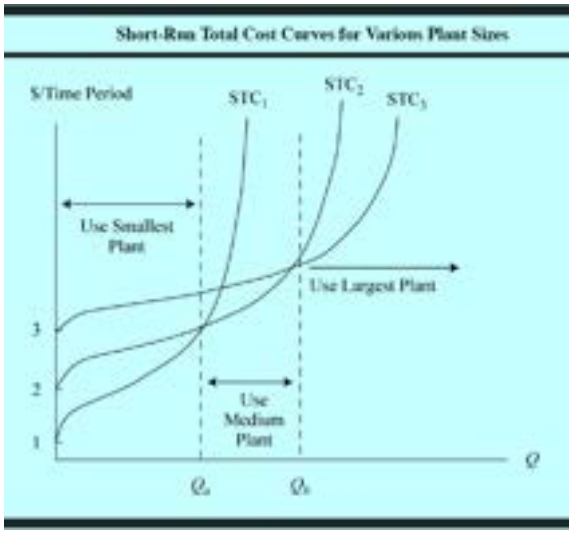
The time required for long-run adjustments varies by industry. For example, for a small business, the long-run may be less than a year, whereas, for a capital-intensive firm, the long run may be more than a decade. Costs and profits would differ between the short run and the long run.

Short- and Long-Run Cost Curves

The short-run total cost includes all the inputs (i.e., labor and capital) the firm is using to produce output.

Typically, a short-run total cost (STC) curve tends to rise with output, first at a decreasing rate because of specialization economies and then at an increasing rate, reflecting the law of diminishing marginal returns to labor.

Vertical intercept of the STC curve is determined by total fixed cost (the quantity of capital input multiplied by the rental rate on capital). At higher levels of fixed input, both TFC and production capacity of the firm are greater.



- In the above graph, Plant Size 1 is the smallest and has the lowest fixed cost; hence, its STC_1 curve has the lowest vertical intercept. It is important to note that STC_1 begins to rise more steeply with output, reflecting the lower plant capacity.
- Plant Size 3 is the largest of the three and has both a higher fixed cost and a lower slope at any level of output. If a firm decided to produce an output between zero and Qa, it would plan on building Plant Size 1 because for any output level in that range, its cost is less than it would be for Plant Size 2 or 3. Accordingly, if the firm were planning to produce output greater than Qb, it would choose Plant Size 3 because its cost for any of those levels of output would be lower than for Plant Size 1 or 2. In this case, Plant Size 2 would be chosen for output levels between Qa and Qb.
- The long-run total cost curve is derived from the lowest level of STC for each level of output because in the long run, the firm is free to choose which plant size it will operate. This curve is called an "***envelope curve***."

Note: For each STC curve, there is also a corresponding short-run average total cost (SATC) curve and a corresponding long-run average total cost (LRAC) curve, the envelope curve of all possible short-run average total cost curves.

Defining Economies of Scale and Diseconomies of Scale

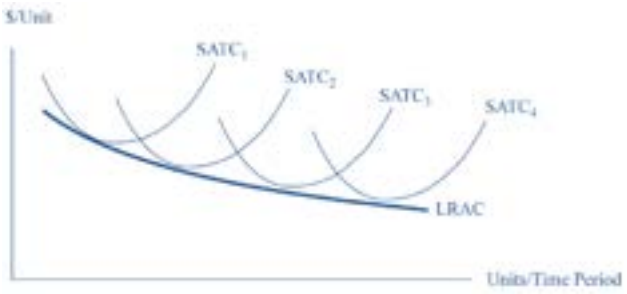
Economies of scale or Increasing returns to scale: It occurs when the % change in output > % change in inputs. E.g. a 20% rise in factor inputs leads to a 35% rise in output.

- When an industry/firm enjoys external economies i.e. has increasing returns to scale, its long-run average total cost decreases as the quantity of output increases i.e. long-run supply curve is downward sloping. Such an industry is called a decreasing-cost industry.
- However, it should be noted that individual firm's supply curve is still upward sloping when industry's long-run supply curve is negatively sloped i.e. in the long-run, when industry costs ↓, firm supply curve shifts rightward and it results in decrease in price charged by a firm for each quantity.
- ***Sources of economies of scale:*** These include specialization due to greater production, workers become more efficient due to specialization, better use of market information, discounted prices on resources when bought in large quantities etc.
- When a firm faces economies of scale, costs can be lowered, and profit can be increased by increasing production capacity.

Scale-up production: It involves increasing all of inputs in order to increase the level of output in the long-run.

Scaling down: It involves decreasing all of the inputs in order to produce less in the long run.

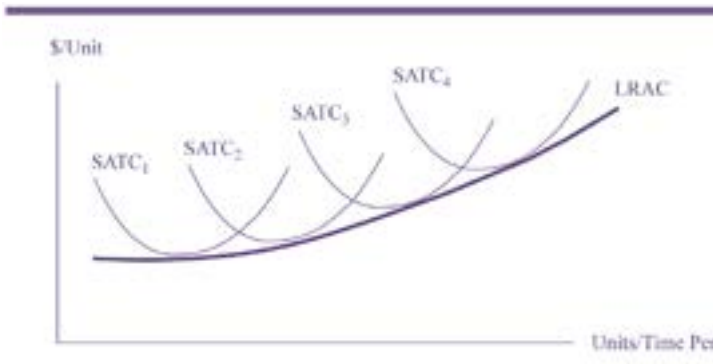
Short-run Average Total Cost Curves for Various Plant Sizes and Their Envelope Curve, LRAC: Economies of Scale



Diseconomies of scale or Decreasing returns to scale: It occurs when the % change in output < % change in inputs. E.g. a 50% rise in factor inputs raises output by only 25%.

- When an industry/firm has decreasing returns to scale, its long-run average total cost increases as the quantity of output increases i.e. long-run supply curve is upward sloping. Such an industry is called Increasing-cost industry.
- Sources: problems associated with large organizations e.g. Problems of management, maintaining effective communication, coordinating activities – often across the globe, De-motivation and alienation of staff, Divorce of ownership, and control etc.
- When a firm faces diseconomies of scale, costs can be lowered and profit can be increased by downsizing and becoming more competitive.

Short-run Average Total Cost Curves for Various Plant Sizes and Their Envelope Curve, LRAC: Diseconomies of Scale



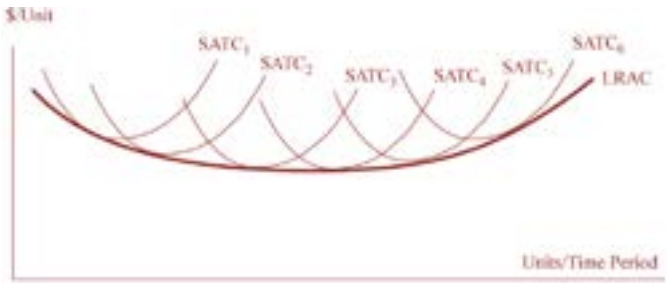
As the firm's size increases, it benefits from economies of scale and a lower ATC owing to following factors:

a) Increasing returns to scale, which occurs when a production process allows for increases in output that are proportionately larger than the increase in inputs.
b) Having a division of labor and management in a large firm with numerous workers, which allows each worker to specialize in one task rather than perform many duties.
c) Using more expensive & efficient equipment and adapting the latest in technology that increases productivity.
d) Effectively reducing waste and lowering costs through marketable byproducts, less energy consumption, and enhanced quality control.
e) Using market information and knowledge in a better way for more effective managerial decision making.
f) Obtaining discounted prices on resources when buying in larger quantities.

The factors that can lead to diseconomies of scale, inefficiencies, and rising costs when a firm increases in size include the following:

a) Decreasing returns to scale, which occurs when a production process leads to increases in output that are proportionately smaller than the increase in inputs.
b) Difficulty in managing the firm properly owing to its large size.
c) Overlapping and duplication of business functions and product lines.
d) Higher resource prices due to supply constraints when buying inputs in large quantities.

Economies and diseconomies of scale can occur at the same time. If economies of scale dominate diseconomies of scale, LRAC decreases with increases in output. Opposite is true if diseconomies of scale dominate. We can see in the graph below that there is certain range of output over which LRAC falls (economies of scale) and then a range over which LRAC is constant, followed by a range over which diseconomies of scale prevail



Minimum efficient scale: The minimum point on the LRAC curve is referred to as the minimum efficient scale. The minimum efficient scale is the optimal firm size under perfect competition over the long run. Theoretically, under perfect competition, the firm should operate at

this level over the long-run in order to maintain its viability.

Practice: Example 3 and Question-set from the CFA Institute's Curriculum.

3. INTRODUCTION TO MARKET STRUCTURES

Characteristics: In the long run, the forces associated with the market structure within which the firm operates determine the profitability of the firm.

- In a highly competitive market, long-run profits are decreased by the forces of competition.
- In less competitive markets, large profits can persist in the long-run.
- In the short-run, any outcome is possible.

Analysis of Market Structure

Economists' Four Types of Structure

Market: A market is a group of buyers and sellers that are aware of each other and are able to agree on a price for the exchange of goods and services.

Some markets are highly concentrated i.e. sales generated from a small number of firms represent the majority of total sales; whereas some markets are very fragmented.

There are four types of market structure:

1) Perfect competition
2) Monopolistic competition
3) Oligopoly
4) Monopoly

Factors That Determine Market Structure

Following five factors determine market structure.

1) The **number and relative size of firms supplying the product** i.e., greater the number of firms supplying the product, greater will be the competition.
2) The **degree of product differentiation**: Higher product differentiation provides pricing leverage i.e., control over pricing decisions.
3) The **power of the seller over pricing decisions**.
4) The **relative strength of the barriers to market entry and exit**: Barriers can result from large capital investment requirements, patents, high exit costs etc. Smaller the barriers to entry and exit, greater will be the competition.
5) The **degree of non-price competition**: Non-price competition refers to product differentiation through marketing. It prevails in market structures where product differentiation is essential e.g. monopolistic competition.

Market Structure	Number of Sellers	Degree of Product Differentiation	Barriers to Entry	Pricing Power of Firm	Non-Price Competition
Perfect Competition	Many	Homogeneous/ Standardized	Very Low	None	None
Monopolistic Competition	Many	Differentiated	Low	Some	Advertising and Product Differentiation
Oligopoly	Few	Homogeneous/ Standardized	High	Some or Considerable	Advertising and Product Differentiation
Monopoly	One	Unique Product	Very High	Considerable	Advertising

- From the consumers' perspective, the most desirable market structure is the one with the greatest degree of competition, due to low prices.
- From the perspective of producers the most desirable market structure is the one in which the seller has the most control over prices.

Porter's Five Forces and Market Structure

A financial analyst should evaluate the following factors when analyzing market conditions in which firm operates and its profitability:

1. Threat of substitutes i.e. evaluate whether the product is differentiated or not.
2. Threat of entry
3. Intensity of competition among incumbents i.e. evaluate how many sellers are there in the industry.
4. Bargaining power of customers
5. Bargaining power of suppliers

Practice: Question-set from the CFA Institute's Curriculum.

4. MONOPOLISTIC COMPETITION

Monopolistic competition is a hybrid market because each firm may have a tiny 'monopoly' due to differentiation of their product and each firm generates zero economic profit in the long-run.

Characteristics:

1. Many buyers and sellers
2. Products differentiated i.e. each firm produces a product that is at least slightly different from those of other firms. The products represent close substitutes for products offered by other firms.
3. Entry and Exit possible with fairly low costs.
4. Firm has some control over price.
5. Suppliers differentiate their products through advertising and other non-price strategies.

Demand Analysis in Monopolistically Competitive Markets

Monopolistic competition firm has a downward sloping demand curve due to its product differentiation.

Along the demand curve, at higher prices, demand is elastic and at lower prices, demand is inelastic.

Like monopoly, price exceeds marginal cost.

Like competitive market, price equals average total cost in the long-run.

Due to downward-sloping demand curve, marginal revenue is less than price.

The Monopolistically Competitive Firm in the Short Run:

In the short-run, Profit is maximized where MR = MC.

There is no well-defined supply schedule in monopolistic competition i.e. supply curve is neither represented by MC nor AC curve.

Output level is determined at a point where MR = MC and price is charged on the basis of market demand.

Short-run economic profits encourage new firms to enter the market. This leads to:

- Increase in the number of products offered.
- Reduction in demand faced by firms already in the market.
- Incumbent firms' demand curves shift to the left.
- Demand for the incumbent firms' products fall, and their profits decline.

Short-run economic losses encourage firms to exit the market. This leads to:

- Decrease in the number of products offered.
- Increase in the demand faced by the remaining firms.
- Shifts the remaining firms' demand curves to the right.
- Increase in the remaining firms' profits.

Long-Run Equilibrium in Monopolistic Competition

Like in case of perfect competition, free entry and exit drive economic profit to zero.

Differences between Monopolistic Competition and Perfect Competition:

In perfect competition, there is no excess capacity in the long run. Due to free entry, competitive firms produce at

Copyright © FinQuiz.com. All rights reserved.

the point where average total cost is minimized, which is the efficient scale of the firm.

But in monopolistic competition, equilibrium occurs at a higher level of average cost instead of output level where AC is the minimized.

In monopolistic competition, output is less than the efficient scale of perfect competition.

For a competitive firm, P = MC; whereas for a monopolistically competitive firm, P > MC. Consequently, an extra unit sold at the posted price indicates greater profit for the monopolistically competitive firm. This results in deadweight loss.

Unlike perfect competition, in monopolistic competition, economic costs include advertising or marketing costs.

Practice: Question-set from the CFA Institute's Curriculum.

5. OLIGOPOLY

Oligopoly and Pricing Strategies in

Characteristics:

1. Few sellers offering similar or identical products.
2. Industry dominated by small number of large firms.
3. Products offered by each seller are close substitutes of products offered by other firms.
4. Interdependent firms i.e. their price decisions are interdependent on each other.
5. Barriers to entry and exist are high i.e. fairly high costs.
6. Firms have substantial control over price.
7. Products are differentiated through advertising and other non-price strategies.

Duopoly: A duopoly is an oligopoly with only ***two*** firms. It is the simplest type of oligopoly.

Price Collusion: Price collusion refers to agreement among firms on the quantity to produce and price to charge. However, even in absence of price collusion, a ***dominant firm*** can easily become a price maker in the market. When firms collude:

- Profit increases.
- Uncertainty of cash flows reduces.
- Provide opportunities to create barriers to entry.

Cartel: Cartel refers to collusive agreements that are made openly and formally e.g. OPEC cartel.

Oligopoly firms can generate higher profits by ***cooperating*** / **joining** together and acting like a monopolist i.e. by producing a small quantity of output and charging a price above marginal cost.

Factors necessary for a collusion to be successful:

1) There is small number of firms in the industry.
2) Product produced by the firms is identical / similar.
3) Firms have similar cost structure.
4) Orders received by firms are small in size and are frequent i.e. receive on a regular basis.
5) Firms face severe threat of retaliation by other firms in the market; therefore, there are few opportunities to keep actions secret.
6) The degree of external competition.

Demand Analysis and Pricing Strategies in Oligopoly

In oligopoly, demand depends on the degree of price interdependence.

In case of price collusion, aggregate market demand curve is composed of individual sellers.

In case of non-collusion, each firm faces an individual demand curve. In non-colluding oligopoly, market demand depends on the pricing strategies of the firms. There are three basic pricing strategies.

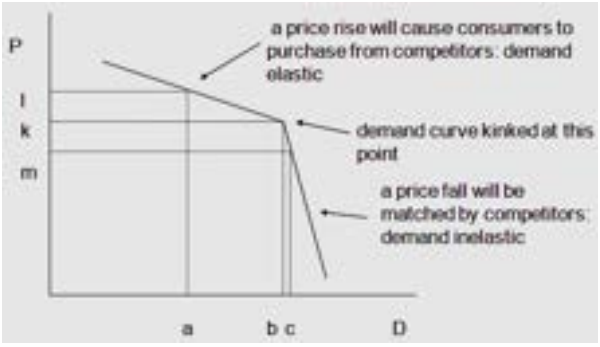
1) Pricing Interdependence: In this strategy, firm's pricing decisions depend on each other. This strategy is used in the market where there are price wars e.g. Commercial Airline industries.

In this strategy, when a firm reduces its price, the competitor follows the same and ignores price increase. This implies that firms face two demand structures i.e. one related to price increase and other related to price decrease. It is referred to as **kinked demand curve** i.e.

- When one firm increases price, competitor firm's market share increases, when other does not match price increase.

- When one firm reduces price, there is no change in competitor firm's market share, when other matches price decrease.

Reason: Price elasticity of demand is greater at higher prices because firm's rivals have lower prices; whereas price elasticity of demand is lower at lower prices because firm's rivals will match decrease in price.



This implies that in case of a kinked demand curve, individual firms are disadvantaged by reducing prices. Therefore, these markets tend to be characterized by price rigidities.

NOTE:

In case of Kinked demand curve, each demand curve will have its own marginal revenue structure.

- Demand function (DP ↑) and Marginal revenue structure (MRP ↑) associated with higher prices.
- Demand function (DP ↓) and Marginal revenue structure (MRP ↓) associated with lower prices.

The two demand structures intersect at the prevailing price i.e. where price increase = price decrease = 0.

See: Exhibit 16 & 17 form the CFA Institute's Curriculum.

In oligopoly, firm's demand curve is represented by relevant portion of demand schedule when price increases and relevant portion of demand schedule when price decreases.

Overall Demand = DP ↓+ DP ↑
= Demand segment associated with price decrease + Demand segment associated with price increase

- Due to kinked demand curve, oligopoly markets have stable and rigid pricing structure.

The Cournot Assumption

Cournot Assumption: In a Cournot model, it is assumed that firms make pricing and output decisions *simultaneously*. In Cournot assumption, profit maximizing output by each firm is determined by assuming no change in other firm's output i.e. there is no retaliation by other firms.

In this strategy, this pattern continues until each firm reaches its long-run equilibrium position. In the long-run, both price and output are stable and change in either price or output will not increase profits for any firm.

- Compared to perfect price competition, Cournot equilibrium price will be higher and equilibrium output level will be lower.
- Compared to monopoly, Cournot equilibrium price will be lower and equilibrium output level will be higher. This implies that total profits are less than the monopoly profit. **See exhibit 14.**
- As the number of sellers in an oligopoly grows larger, an oligopolistic market looks more and more like a competitive market. The price approaches marginal cost, and the quantity produced approaches the socially efficient level.

The Nash Equilibrium

Nash Equilibrium: Game theory refers to the study of how people behave in strategic situations. It is used by decision makers to analyze responses by rival decision makers.

In game theory, a Nash equilibrium is a situation in which different firms in oligopoly interacting with one another, choose their best strategy, given the strategies that all the others have chosen (i.e. Dominant Strategy). In oligopoly, this implies that in Nash equilibrium, no firm can increase its profits by independently changing its pricing strategy. Thus,

- Firms have interdependent actions because their profit depends not only on how much they produce but also on how much the other firms produces.
- These actions are non-cooperative i.e. each firm's decisions are made to maximize its own profits.
- Firms do not collude to maximize joint profits.
- Equilibrium occurs when all firms choose their best strategy given the actions of their rivals.

- Joint profit is maximized when both firms charge higher prices for their products i.e. joint profit = $500 + $300 = $800.

Stackelberg Model: In a Stackelberg model, it is assumed that firms make pricing and output decisions sequentially i.e.

- The leader firm chooses its output first because it has the first mover advantage.
- After observing the leader's output, the follower firm chooses its output.
- To force the follower firm to reduce production or to exit the market, the leader firm may aggressively overproduce. It is referred to as "Top Dog" strategy.
- Compared to Cournot model, the leader firm earns more while the follower firm earns less in a Stackelberg model.

Practice: Question-set from the CFA Institute's Curriculum.

6. DETERMINING MARKET STRUCTURE

Econometric Approaches

Measures to estimate market power:

1) Market power can be measured by estimating the elasticity of demand and supply in a market.

- When demand is highly elastic → it indicates that market is close to perfect competition.
- When demand is inelastic → it indicates that firms may have market power.

Limitations:

i. This analysis has endogeneity problem i.e. the equilibrium price and output are jointly determined by the interaction of demand and supply.
Thus, a model with two separate equations (i.e. one equation for Quantity demanded and one for Quantity supplied) is needed to correctly estimate the demand and supply.
ii. Elasticity can be computed using regression analysis but it requires large number of observations.
iii. The regression analysis is based on historical data, which is not necessarily a good predictor of future.

2) Using cross-sectional regression analysis instead of time-series analysis. In this analysis, sales from different firms in the market are analyzed during the same year or for single transactions from many buyers & companies.

Limitations:

i. This method is complex.
ii. Different specifications of explanatory variables e.g. using total GDP instead of per-capita GDP to use as a proxy for income will provide different estimates.

Simple Measures

1) Concentration Ratio: It is the sum of the market shares of the N largest firms. It is computed as follows:

CR = Sum of sales values of the largest 10 firms / Total market sales

- CR is always between 0% and 100%.
- CR = 100% for monopoly.
- CR ≈ 0% for a perfectly competitive industry.

Advantage: It is simple and easy to compute.
Disadvantages:

i. It is not a direct measure of market power i.e. a high CR does not necessarily indicate a monopoly power; because when barriers to entry are low, even a single firm in the industry behaves like a firm in perfect competition.

ii. It ignores the affect of mergers among the top market players i.e. CR does not change much when the largest and second-largest incumbents merge; they are likely to have greater pricing power after merger.

Practice: Example 'Calculating The Concentration Ratio ', CFA Institute's Curriculum.

2) Herfindahl-Hirschman index (HHI): The Herfindahl-Hirschman Index equals the sum of the square market share of the top N companies in an industry.

$$\textbf{HHI} = \sum X_i^2$$

where,

X_i^2 is squared market share of the i^{th} firm.
HHI = 1 for monopoly.
HHI ≈ 0 for a perfectly competitive industry.

When there are M firms in the market with *equal market share*, then

$$\textbf{HHI} = (1 / M)$$

Interpretation: For example, HHI of 0.20 means that market is shared equally by five firms in the market.

Disadvantages:

i. Like CR, it is not a direct measure of market power i.e. a high HHI does not necessarily indicate a monopoly power; because when barriers to entry are low, even a single firm in the industry behaves like a firm in perfect competition.
ii. It is less useful for financial analyst estimating potential profitability of firm or group of firms because it ignores elasticity of demand.

Practice: Example 4 and Question-set from the CFA Institute's Curriculum.

Practice: CFA Institute's End of Chapter Practice Problems and Questions from FinQuiz Question Bank.

1. INTRODUCTION

Business cycles are cyclical fluctuations in economic activity. They are influenced by factors such as population, technology, capital, expectations, political developments, natural disasters, and fiscal and monetary policy decisions.

Credit cycles are cyclical fluctuations in the availability and cost of credit. They often move in tandem with business cycles.

Business cycle theories attempt to explain the causes and dynamics of business cycles.

2. OVERVIEW OF THE BUSINESS CYCLE

Business Cycles: Recurrent expansion and contractions in economic activity that affect the major sectors of the economy.

- Business cycles are associated with economies that are based on business enterprises rather than agrarian societies or centrally planned economies.
- A business cycle has expected sequence of phases such as upturns are followed by downturns.
- These phases occur at about the same time throughout the economy.
- It is important to note that cycles are recurrent rather than periodic; this implies that all phases of cycles do not have the exact same intensity and/or duration.
- Duration of cycles is between 1 and 12 years.

Phases of the Business Cycle

A business cycle is primarily divided into two segments with two turning points. The two segments are:

1. Expansion or upswing
2. Contraction or downturn

The two turning points are:

i. Peak (highest point of the cycle)
ii. Trough (lowest point of the cycle)

A business cycle can be divided into several phases. However, the above mentioned two segments are easy to identify in retrospect.

Types of Cycles

- Classical cycle
- Growth cycle
- Growth rate cycle

1. <u>Classical Cycle:</u>

- Fluctuations in the level of economic activity measured by GDP in volume.
- Contraction phase:
  - is short in duration
  - starts from a peak and ends at a trough
- Expansion phase:
  - is long in duration
  - starts from a trough and ends at a peak

2. <u>Growth Cycle:</u>

- Fluctuations in business activity around the long-term potential or trend growth level.
- The focus is on whether the actual economic activity is below or above the trend growth in economic activity.
- It divides economic activities into long-run and short-run trends.
- Compared to classical cycle, in this cycle peaks are reached earlier and troughs later in time.

3. <u>Growth Cycle:</u>

- Fluctuations in growth rate of economic activity (GDP growth rate).
- Compared to the other two cycles, this cycle recognizes peaks and troughs earlier.

<u>Practice:</u> Refer to Exhibit 2, 3 & 4 from the CFA Institute's Curriculum.

Practical Issues

- Typically, definitions of business cycle are somewhat ambiguous and are used interchangeably.
- Classical cycle concept is rarely used in practice.
- Growth cycle concept is often used to identify fluctuations around potential output.

Stanley Notes™ 2025

Four Phases of the Cycle

The four phases of business cycle are:

1. Recovery
2. Expansion
3. Slowdown
4. Contraction

1. Recovery

- Economy is going through trough phase
- Actual output is at its lowest level relative to potential output.
- Economic output is below potential but is starting to increase

2. Expansion

During expansion, aggregate economic activity is increasing, and actual output is above potential output.

- Spending ↑
- Production ↑
- Employment ↑
- Prices & interest rates ↑
- GDP ↑

3. Slowdown

- Actual output is highest relative to potential output i.e., largest positive output gap and that gap begins to narrow.
- Inflation and price levels may decrease
- Businesses may rely on overtime than new hiring
- Consumers are still optimistic

4. Contraction:

- Actual output is below potential output
- Consumer and business confidence ↓
- Employment and overtime ↓
- Economic activity ↓
- Usually, this phase is shorter than expansion phase
- If decline is prolonged it may lead to recession & then depression

Refer to Exhibit 5: Business Cycle Phase Characteristics from the CFA Institute's Curriculum.

Leads and Lags in Business and Consumer Decision Making

Relative to the expected turning points, sometimes the turning points may exhibit leads and lags due to the behavior of businesses and households.

Market Conditions and Investor Behavior

Many macro-economic variables (e.g., GDP growth) and economic sectors (e.g., consumer discretionary) have exhibited changes cyclically in the past. Investors can benefit from portfolio management if they can forecast future changes in these variables. This benefit can be for diversified portfolios (e.g., when to increase allocation to equities) as well as for equity positions concentrated in sectors (e.g., when to increase exposure to the auto sector).

1. Recovery phase

Risky assets (e.g., stocks) become more attractive for investors (i.e. prices shoot up) when a recession is expected to end soon. Expectations cause stock prices to start rising a few months before the actual pickup in the economic activity.

2. Expansion phase

The economy experiences a 'boom' as the economic expansion continues for some time. Firms may increase production capacity and shortages of qualified human resources can become an issue. Concerns for overheating of the economy may bring actions from the government/central bank.

3. Slowdown phase

Stocks perform better than bonds during economic booms (i.e., higher returns). Higher production and higher corporate profits lead to higher stock prices. Bonds are unattractive for investors and are available at lower prices (offering higher yields compared to the past).

4. Contraction phase

Investors prefer to invest in bonds. Stocks of companies with stable cash flows (and dividends) also become attractive.

Practice: Question-set from the CFA Institute's Curriculum.

3. CREDIT CYCLES

Credit cycles - fluctuations in prices and availability of credit, particularly private sector credit which is critical for business investments and real estate properties.

Cyclical developments of financial variables are examined using ***credit*** and ***property prices***.

When economy is:

- strong/improving – lenders extend credit
- weak/weakening – lenders tighten credit

Applications of Credit Cycles

Loose credit conditions cause asset price and real estate bubbles. Duration and magnitude of recessions or recoveries are intensified and affected by:

- **financial frictions** (variation in access to external financing)
- **relation between busines and credit cycles**. Credit cycles tend to be longer and deeper than business cycles.

Consequences for Policy

Recognizing credit cycle stage helps investors in assessing:

1. business cycle expansions and contractions
2. growth in housing and construction markets
3. policy makers' actions

Practice: Question-Set from the CFA Institute's Curriculum.

4. ECONOMIC INDICATORS OVER THE BUSINESS CYCLE

The Workforce and Company Costs

Contraction Phase:

- Businesses eliminate overtime but retain workers.
- Companies secure a bond of loyalty with workers due to the high cost of training newcomers.
- As the contraction phase prolongs, businesses liquidate inventories and cut costs such as advertising, consultants, new equipment, etc.
- Banks reduce lending as bankruptcy risk increases.
- Lower aggregate demand lowers wages, wage growth, input prices, and interest rates.

Turning Point of Busines Cycle

- Lower prices and interest rates stimulate consumer and business spending, and as a result, aggregate demand and economic activity start to increase.

Refer to Exhibit 7 'Business Cycle Phases- Level of Employment', CFA Institute's Curriculum.

Fluctuations in Capital Spending

- Capital spending fluctuates with business cycle and changes in economic activity
- Companies' spending decisions are driven by:
  - expectations
  - business conditions
  - level of capacity utilization
- Investment is one of the most procyclical and volatile element of GDP.

Refer to Exhibit 8 'Capital Spending during the Economic Cycle', CFA Institute's Curriculum.

Practice: Example 1 from the CFA Institute's Curriculum.

Fluctuations in Inventory Levels

Although, inventories represent a small part of the overall economy, they can have greater effect on economic growth because fluctuations in inventory levels occur rapidly.

The key indicator used to analyze fluctuations in inventory levels is the ***inventory-sales ratio.*** It reflects the outstanding stock of available inventories to the level of sales.

Refer to Exhibit 9 'Inventories throughout the Cycle', CFA Institute's Curriculum.

Practice: Example 2 from the CFA Institute's Curriculum.

Economic Indicators Over the Business Cycle

An economic indicator is a variable that provides information on the state of the overall economy.

Types of Indicators

There are three types of economic indicators.

1. **Leading economic indicators (LEI)**: Leading indicators are variables that change before the changes in the overall economy. They are useful for predicting the economy's future state, usually short-term.
2. **Coincident economic indicators:** Coincident economic indicators are variables that change close to the changes in the overall economy. They provide information regarding present/current state of the economy.
3. **Lagging economic indicators:** Lagging economic indicators are variables that change after the changes in the overall economy. They provide information regarding the economy's past condition.

Refer to Exhibit 10 'Types of Economic Indicators', CFA Institute's Curriculum.

Composite Indicators

Composite economic indicator: It is an aggregate measure of leading, lagging and coincident indicators to measure the cyclical state of the economy.

Leading Indicators

In the U.S., a composite leading indicator is known as the ***Conference Board Leading Economic Index (LEI)*** - (published by conference board). It is composed of 10 components.

Refer to Exhibit 11 'Index of Leading Indicators', CFA Institute's Curriculum.

Using Economic Indicators

Refer to Exhibit 12 'Use of Statistics to Identify Business Cycle Phase', CFA Institute's Curriculum.

Other Composite Leading Indicators

- Organization of Economic Co-operation Development (OECD) calculates:
  - OECD CLI (Composite Leading Indicators) to measure business cycle of the economy using growth cycle concept.
- These indices are consistent across countries, and therefore can be easily used for comparison purposes.

Surveys

Composite indicators at regional or country level are used to form economic tendency surveys. These monthly or quarterly surveys:

- are usually released by central banks, statistical offices, research institutes and trade associates.
- mostly include qualitative questions about finances, level of activity and confidence in the future.

The Use of Big Data in Economic Indicators

Because of the advent of big data and massive increase in information, the number of variables in various composite indicators has been increased.

Nowcasting

Nowadays, policymakers and experts are monitoring financial and economic variables in real-time using a large variety of data.

Nowcast – a term contracted from 'now' and 'forecasting' to monitor the economic condition in real-time such as GDP growth, inflation, unemployment etc.

GDPNow

GDPNow – a recent estimate of real GDP growth using data from the recent measured quarter to forecast GDP for the current quarter (before-quarter end)

Many entities such as investment banks, asset management, institutions are producing Nowcasts for their clients or internal use.

Practice: Question-Set from the CFA Institute's Curriculum.

Practice: CFA Institute's End of Chapter Practice Problems and Questions from FinQuiz Question Bank.

1. INTRODUCTION

Fiscal Policy: Government decisions on taxation and spending.

- Aims to affect aggregate demand through government spending and tax revenue adjustments.
- Tools include transfer payments, current government spending, capital expenditures, and taxes.

- Government deficits arise from the disparity between revenues and expenditures over a time period.
- Structural budget deficit is often analyzed to gauge a government's fiscal stance.

Monetary Policy: Central bank activities to influence money and credit quantity

2. INTRODUCTION TO MONETARY AND FISCAL POLICY

The decisions made by governments can have a much larger impact on the economy compared to decisions made by a single household or a business because:

1) In most of the developed economies, a significant proportion of the population is employed by public sector.
2) Governments are usually responsible for a significant proportion of spending in an economy.
3) Governments are the largest borrowers in world's debt markets.

Types of Government Policy: There are two types of government policy.

1. **Monetary policy** refers to policy used by central bank to influence the macro economy by changing the quantity of money and credit in the economy.
2. **Fiscal policy** refers to the policy used by the government to influence the macro economy by changing government spending and taxation.

Practice: Question-set from the CFA Institute's Curriculum.

3. ROLES AND OBJECTIVES OF FISCAL POLICY

Fiscal policy involves using government spending and taxes to affect the economic activity.

Roles and Objectives of Fiscal Policy

The ***primary*** objective of fiscal policy is to affect the overall level of aggregate demand in an economy and the level of economic activity i.e. to meet government's economic objectives of low inflation and high employment. Other objectives include:

- Distribution of income and wealth among different segments of the population.
- Allocation of resources between different sectors and economic agents.

Fiscal Policy and Aggregate Demand

In a recession with rising unemployment, ***Expansionary fiscal policy*** can be conducted through following ways:

- Decreasing personal income tax rate to raise disposable income, which eventually leads to rise in aggregate demand.
  Limitations: during recession, increase in disposable income due to lower tax rate may lead to increase in precautionary savings instead of increase in consumption spending. Hence, it may further leads to decrease in AD.
- Reducing sales (indirect) taxes to lower prices, which raises real incomes and eventually leads to increase in consumer demand.
- Reducing corporation (company) taxes to improve business profits so that capital spending increases.
- Reducing tax rates on personal savings to raise disposable income, which eventually leads to an increase in consumer demand.
- Increasing public spending on social goods and infrastructure (i.e. hospitals and schools) to improve personal incomes and the aggregate demand.

NOTE: Opposite occurs in Contractionary fiscal policy.

Stanley Notes™ 2025

Important to understand: When the economy operates at its potential GDP (full employment) level, then an expansionary fiscal policy will not have any significant effect on output or AD because at full employment level, there are few spare or unused resources (e.g. labor or idle factories); in such a case, fiscal expansion will only lead to increase in aggregate price level i.e. inflation.

- According to **Keynesians** School, fiscal policy can have significant impact on AD, output, and employment only when economy operates below its potential GDP level or when the rate of capacity utilization is low.
- According to **Monetarists**, monetary policy is a more effective tool than fiscal policy because fiscal policy can only have a ***temporary*** effect on AD.

Government Receipts and Expenditure in Major Economies

Government revenue = Tax revenues - transfer payments.

Government spending includes public spending on social goods and infrastructure (i.e. hospitals and schools) and interest payments on the government debt.

Balanced budget: When government spending = government revenues, then the budget is balanced.

Budget deficit: When government spending > government revenue, a budget is in deficit.
Budget surplus: When government spending < government revenue, a budget is in surplus.

- An increase in a budget surplus is associated with contractionary fiscal policy.
- An increase in a budget deficit is associated with expansionary fiscal policy.

Total government revenues as a percentage of GDP: It refers to the % of a country's total output that is gathered by the government through taxes, fees, charges, fines, and capital transfers. Generally, the higher the total government revenues as a % of GDP, the greater a government is involved both directly and indirectly in the economic activity of a country.

Automatic Stabilizers: Automatic stabilizers are features of fiscal policy that stabilize real GDP without any explicit action by the government e.g. income tax, VAT, and social benefits. In this case, budget surplus and deficit fluctuate with business cycle (real GDP) i.e.

- When an economy slows and unemployment rises (e.g. during recession), → tax revenues ↓, government spending on social insurance and unemployment benefits will ↑, and budget surplus decreases (or budget deficit rises).
  - This acts as a fiscal stimulus.
- Similarly, when an economy expands and employment and incomes are high, → progressive income and profit taxes ↑ and the budget surplus increases (or budget deficit falls).

Advantage of automatic stabilizers: They help to reduce output fluctuations caused by 'shocks' to autonomous parts of aggregate expenditure.

Automatic stabilizers v/s Discretionary fiscal policy: Unlike Automatic stabilizers, discretionary fiscal policies (i.e. tax changes and/ or spending cuts or increases) are actively used by the government to stabilize the AD and economy.

Structural or cyclically adjusted budget deficit: It refers to the budget deficit that exists when the economy is at full employment (or full potential output). It is the most appropriate indicator of fiscal policy.

Important to understand: When the government increases its spending, it is not always expansionary because government may simultaneously increase taxes by a larger amount than that of increase in government spending.

Practice: Question-set from the CFA Institute's Curriculum.

Deficits and the National Debt

Budget Deficit: When government spending or expenditure > government revenue, a budget is in deficit.

Government (or national) debt: National debt is the accumulated budget deficit over time.

How budget deficits are financed: Government deficits are financed by borrowing from the private sector.

- When the ratio of debt to GDP and the ratio of interest rate payments to GDP rise beyond a certain unknown point, the solvency of the country deteriorates.
- When the real growth in the economy < the real interest rate on the debt → debt burden (real interest rate × debt) grows faster than the economy; hence, the debt ratio will increase in spite of growing economy.

Effect of inflation on the real value of outstanding debt: When inflation and nominal GDP increase, → the real value of the outstanding debt ↓. Thus, the ratio of debt to GDP falls.

Effect of deflation on the real value of outstanding debt:
When inflation and nominal GDP decrease, → the real value of the outstanding debt ↑. Thus, the ratio of debt to GDP rises.

The arguments against being concerned about national debt relative to GDP are as follows:

- The increasing ratio of debt to GDP does not indicate a severe problem because the debt is owed internally to fellow citizens.
- The increasing ratio of debt to GDP is not harmful because a proportion of the borrowed money may be used for capital investment projects or training, education of human capital, which would eventually lead to higher future output and tax revenues.
- Large fiscal deficits may help to reduce distortions caused by existing tax structures by introducing tax changes.
- Fiscal deficits may have no net impact because the private sector may increase savings in expectations of higher future tax rates. It is known as "***Ricardian equivalence***".
- In case of unemployment in an economy, debt rather helps to increase employment.

The arguments in favor of being concerned about national debt relative to GDP are as follows:

- High debt to GDP ratio may lead to higher future tax rates to earn higher tax revenues to finance the deficit. Higher marginal tax rates reduce labor effort and entrepreneurial activity and results in lower growth in the long-run.
- When government deficit is financed through printing money, inflation increases.
- **Crowding out effect:** When deficits are financed through borrowing, it leads to crowding out effect i.e. due to higher government demands, cost of borrowing (interest rates) ↑ and as a result, private sector investing decreases.

NOTE: Both tax distortions and crowding out effect are more serious in the long-run rather than in the short-run.

Practice: Question-set from the CFA Institute's Curriculum.

4. FISCAL POLICY TOOLS

Fiscal Policy Tools and the Macro Economy

Government spending includes:
a) *Transfer payments* include welfare payments, payments for state pensions, housing benefits, tax credits and income support for poorer families, child benefits, unemployment benefits, and job search allowances. Transfer payments are made to:

- Guarantee a minimum level of income for poorer people.
- Redistribute income & wealth.

b) *Spending on goods and services* e.g. health, education, and defense to benefit citizens, to improve country's skill level and overall labor productivity.

c) *Spending on infrastructure projects* e.g. roads, hospitals, prisons, and schools to improve country's economic growth.

Government revenues include:
a) Direct taxes include taxes that are levied on income, wealth, and corporate profits. For example, capital gains taxes, national insurance (or labor) taxes, corporate taxes, local income or property tax for both individuals and businesses, and inheritance tax on a deceased's estate.

b) Indirect taxes include taxes on spending on a variety of goods and services in an economy, i.e. excise duties on fuel, alcohol, and tobacco as well as sales (or value-added tax).

Advantages of taxes:

- Taxes can be used to raise revenues for the government to finance expenditures.
- Taxes facilitate distribution of income and wealth among different segments of the population.

NOTE:
According to supply-side economics, income taxes may reduce the incentive to work, save and invest.

Desirable attributes of a tax policy:
1) Simplicity: The tax system ought to be easy and relatively inexpensive to adminster i.e. the tax system should have low costs of administration and compliance. Also, the final tax liability should be certain and not easily manipulated.

2) Efficiency: The tax system should not interfere with the efficient allocation of resources i.e. taxes should raise revenue without negative distortions such as reducing work-incentives for individuals and investment incentives for companies.

3) **Fairness:** The tax system ought to be fair in its relative treatment of different individuals.

   a) ***Horizontal Equity***: Individuals who are identical should pay the same taxes.
   b) ***Vertical Equity***: Individuals who have greater ability to pay and are rich, should pay more taxes.

- **Progressive tax system:** Tax rate increases proportionately with the increase in income.
- **Flat tax system:** Tax rate is constant regardless of taxable income.

4) **Revenue sufficiency:** Taxes are primarily used to raise revenue to fund government operations. Thus, a good tax system must generate sufficient revenue to fund projected government expenditures and at the same time should help in equitable distribution of income and economic stability. This implies that only one tax that yields more revenue should be imposed instead of imposing many taxes that yield low income.

Practice: Question-set from the CFA Institute's Curriculum.

The Advantages and Disadvantages of Using the Different Tools of Fiscal Policy

Advantages:

- Indirect taxes are easy to adjust, can instantly affect spending behavior and have low costs of administration and compliance.
- In addition, indirect taxes can be used to discourage alcohol or tobacco use.

Disadvantages:

- Direct taxes, welfare and other social transfers are relatively difficult to adjust in response to changes.
- Capital spending plans take longer period of time to formulate and implement; this makes such plans less effective.

NOTE:

- The announcement of future rise in income tax can lead to immediate decline in consumption.
- Generally, direct government spending has relatively greater impact on aggregate spending and output than income tax cuts or transfer increases.

Modeling the Impact of Taxes and Government Spending: The Fiscal Multiplier

The net impact of the government sector on AD is:

G – T + B = Budget surplus OR Budget deficit

where,
G = government spending
T = taxes
B = transfer benefits

Disposable income = Income – Net taxes = (1 – t) Income

where,
Net taxes = taxes – transfer payments
t = net tax rate

For example, if t = 20%, then for $1 increase in national income,

- Net tax revenue will rise by 20 cents.
- Household disposable income will rise by 80 cents.

The fiscal multiplier: Government purchases have a multiplier effect on AD i.e. each dollar spent by the government can increase AD for goods and services by more than a dollar. It is used to determine the total change in output that occurs as a result of exogenous changes in government spending or taxation.

Fiscal Multiplier (in absence of taxes) = 1/(1 - MPC)

where,
MPC = Marginal propensity to consume = Fraction of extra income that a household consumes rather than saves.

MPS = Marginal propensity to save
= Fraction of extra income that is saved; it is estimated as
MPS = 1 – MPC.

Total increase in income and spending = Fiscal multiplier × G

Example: Suppose,

- MPC = 3/4 or 0.75
- Increase in government spending = $20 billion
  - AD curve will initially shift to the right by $20 billion.

Fiscal Multiplier = 1/(1 - 3/4) = 4 Thus,
The total change in spending produced by our initial $20 billion spending increase = 4 × $20 billion = $80 billion.

- The multiplier effect tells us that an additional $60 billion of consumption will occur as the initial spending increase moves through the economy.

Fiscal multiplier in the presence of taxes: When taxes are present,

MPC (with taxes) = MPC × (1 - t)

Fiscal multiplier = $\frac{1}{1-MPC\,(1-t)}$

Total increase in income and spending = Fiscal multiplier × G

- The cumulative extra spending and income will continue to spread through the economy at a ***decreasing rate*** because 0 < MPC × (I - t) < 1.

Example: Suppose,

- Tax rate is 20%.
- MPC = 90%
- Government increases spending (G) by $1 billion.

Fiscal multiplier = $\frac{1}{1-0.90\,(1-0.20)}$ = 3.57

Total increase in income and spending = 3.57 × $1 billion = $3.57 billion.

Effects of reducing taxes:
Initial increase in consumption due to reduction in taxes = MPC × tax cut amount

The total or cumulative effect of the tax cut would be = multiplier × initial change in consumption

Example: Suppose taxes are reduced by $20 billion and MPC = 0.75, then,

Initial increase in consumption = .75 × $20 billion
= $15 billion.
The total or cumulative effect of the tax cut would be = 4 × $15 billion = $60 billion

- The total effect of the tax cut < the total effect of Increase in government spending of the same amount because households consumption do not rise immediately with reduction in taxes i.e. a portion of the tax cut is saved.
- The higher the MPC, the lower the savings (MPS).

Effects of increasing transfer payments: The effect of increasing transfer payments is the same as the effect of tax cuts i.e. a portion of the transfer payment increase is saved.

This implies that when MPC < 1, both increased transfer payments and tax cuts are less effective than increased government spending in stimulating the economy.

The Balanced Budget Multiplier

The **balanced budget multiplier** refers to the magnified effect of a ***simultaneous*** change in government spending and taxes on the Aggregate Demand that leaves the budget balance unchanged i.e. does not result in budget deficit or surplus. This implies that:

- An equal increase in autonomous government purchases and autonomous taxes will shift the AD curve to the right and lead to an increase in the equilibrium level of GDP.
- An equal decrease in autonomous government purchases and autonomous taxes will shift the AD curve to the left and lead to a decrease in the equilibrium level of GDP.

Explanation:

- When government spending increases by ΔG, → AD curve shifts to the right and leads to an increase in equilibrium GDP.
- When taxes increase by ΔT, → AD curve shifts to the left and leads to decrease in equilibrium GDP.

when **ΔG = ΔT**,

a) The government's budget deficit (or surplus) will be initially unaffected by the policy and;
b) The expansionary effect (i.e. increase in G) will be stronger than the contractionary effect (i.e. increase in T). Hence, the net result will be an ***increase in equilibrium GDP***.

Reason:

- When taxes rises e.g. by $5 billion and MPC < 1 e.g. MPC = 0.8, → private sector's level of desired spending will decline by less than the full $5 billion i.e. by $4 billion only.
- In contrast, when government spending increases e.g. by $5 billion, then the AD will be increased by full $5 billion.
- Thus, the net result would be an increase in the equilibrium level of GDP.

Important to Note: The balanced budget multiplier is ***not zero***; rather, it is exactly equal to **one** i.e. when both the government spending and taxes increase by $X, then the level of equilibrium GDP will increase by precisely $X.

NOTE:
Increase in equilibrium GDP leads to increase in induced tax revenues, thus increasing the government's budget surplus (or reducing its deficit).

Government Debt, Deficits and Ricardo:
When the government has insufficient tax revenues to meet expenditures, it needs to borrow from the public by issuing stock of IOUs.

Total size of the outstanding debt of government = Cumulative quantity of net borrowing + Fiscal (or budget) deficit in the current period

- When outstanding stock of debt falls (rises), budget surplus rises (falls).

Ricardian Equivalence: Consumers are forward-looking; therefore, they view current debt-financed tax cut as equivalent to delayed taxation (i.e. increase in future taxes). Thus, the reduction in current taxation does not make consumers better off, so they do not raise consumption. Rather, they save the full tax cut in order to repay the future tax liability.

Consequently, private saving rises by the amount public saving falls, leaving national saving unchanged. This is known as Ricardian Equivalence.

Practice: Question-set from the CFA Institute's Curriculum.

5. FISCAL POLICY IMPLEMENTATION

Deficits and Fiscal Stance

Is increase in fiscal deficit always caused by fiscal policy? Answer is no. Increase in fiscal deficit (or budget deficit) can be because of policy decision (aka discretionary fiscal adjustments) (e.g. lowering tax rate) and/or because of automatic stabilizers.

Automatic stabilizers may increase or decrease fiscal deficit without any policy decision from the government.

For example, corporate tax collection by Government will decline during recessions because of lower corporate profits. This can increase fiscal deficit (assuming other factors remain same) without any policy decision by Government.

Budget deficit can be thought of as comprising two parts. The first one is cyclical and will go away once the economy returns to full employment. The second one is structural and will not go away even when economy recovers to full employment.

Budget deficit (and changes in it) represents impacts of policy decisions and automatic stabilizers. *Structural fiscal deficit* indicates impacts of policy decisions only.

Structural budget deficit is also known as *cyclically adjusted budget deficit*.

Structural budget deficit is calculated assuming economy was operating at full employment this year.

Reported budget deficit should be adjusted for nominal vs real interest rates and impacts of inflation.

Difficulties in Executing Fiscal Policy

Fiscal policy cannot completely stabilize aggregate demand as predicted by the multiplier due to following time lags associated with executing and implementing fiscal policy.

1) **Recognition lag:** Policymakers do not have complete information on how the economy functions; hence, it takes time to collect economic statistics and data. In addition, data are subject to substantial revision. It is associated with the *problem of driving with the rear view mirror*.

2) **Action lag or Execution lag:** When necessary policy changes are decided on, it may take many months to implement a policy response. This is known as action lag.

3) **Impact lag:** Even if policy changes are implemented, it may take additional time to impact the economy.

These types of policy lags are also associated with executing discretionary Monetary Policy.

Fiscal policy v/s Monetary policy: Fiscal policy is less effective than monetary policy to deal with short-run stabilisation because:

- Fiscal policy is often very time-consuming to implement.
- It is politically easier to loosen fiscal policy than to tighten it.

Macroeconomic issues associated with Fiscal Policy:

- If the government is concerned with both unemployment and inflation in an economy, then using expansionary fiscal policy to increase AD toward the full employment level may also lead to a tightening labor market (i.e. rising wages) and further increase in inflation.
- When the budget deficit is already large relative to GDP but further fiscal stimulus is required, then the increase in the deficit by adopting expansionary fiscal policy will lead to increase in interest rates on government debt. Consequently, government may face political pressure to deal with the deficit.
- When resources are underutilized due to ***shortage of supply*** of labor or other factors of production rather than a shortage of demand, then discretionary fiscal policy will be ineffective i.e. it will only lead to increase in inflation and will not increase AD.

Practice: Question-Set from the, CFA Institute's Curriculum.

Practice: CFA Institute's end of Chapter Practice Problems and Questions from FinQuiz Question Bank.

Stanley Notes™ 2025

1. INTRODUCTION

Central banks' multifaceted roles in modern economies are:

i. Monopoly currency supplier
ii. Government's banker
iii. Bankers' bank
iv. Lender of last resort
v. Payments system regulator
vi. Conductor of monetary policy
vii. Banking system supervisor

Three key tools central banks use:

1. Open market operations
2. Refinancing rate
3. Reserve requirements

Central bank success depends on three factors:

1. Independence
2. Credibility
3. Transparency

Fiscal and monetary policies impact aggregate demand differently.

2. ROLES OF CENTRAL BANKS

The Roles of Central Banks

Roles of a Central Bank:

1. Currency Stewardship:
- Monopoly supplier of currency.
- Guardian of fiat currency value to maintain public trust.

2. Government's Bank:
- Serves as the government's banker.

3. Bankers' Bank:
- Functions as the primary bank for other banks.

4. Lender of Last Resort:
- Provides funds to banks in crises by printing money.

5. Payment System Oversight:
- Regulates and supervises payment systems.
- Manages foreign currency and gold reserves.
- Coordinates internationally with other central banks.

6. Banking System Supervisor:
- Supervises the banking system. However, in some countries, this role is assumed by a separate authority).

7. Monetary Policy Operator:
- Responsible for a country's monetary policy.

Money in all major economies today is a ***legal tender*** i.e. it must be accepted when offered in exchange for goods and services.

***Fiat Money:** Money that is not convertible into any other commodity is known as fiat money. Fiat money does not have any intrinsic value; it is used as money because of government decree and because it is accepted by people as a medium of exchange. As long as people accept fiat money as a medium of exchange and as long as the fiat money holds its value over time, it will also be able to serve as a unit of account.

The Objectives of Monetary Policy

- To maintain full employment and output.
- To maintain confidence in the financial system.
- To promote understanding of the financial sector.
- To maintain price stability (i.e. to control inflation). In other words, to achieve stable & positive economic growth with stable & low inflation.

Practice: Example 1 from the CFA Institute's Curriculum.

Practice: Question-Set from the CFA Institute's Curriculum.

Copyright © FinQuiz.com. All rights reserved.

3. MONETARY POLICY TOOLS AND MONETARY TRANSMISSION

Central banks have three primary monetary policy tools available.

1. Open Market Operations

Open Market Operations: It involves purchases and sales of government bonds from and to commercial banks and/ or designated market makers. It is one of the most direct ways of changing the money supply.

- When central bank purchases government bonds from commercial banks, the reserves of commercial banks increase, → lending to corporations and households increases and → consequently, the money supply increases.
- When open market operations tool is used, the central bank may target a desired level of commercial bank reserves or a desired interest rate for these reserves.

2. Central Bank's Policy Rate

Policy rate/Refinancing rate or Discount rate: It is the rate of interest the central bank charges on loans to banks.

- When central bank lowers the policy rate, banks' borrowing from the central bank and lending to the public increases; consequently, money supply increases.

This policy rate can be achieved by using short-term collateralized lending rates, known as repo rates. E.g., central bank can increase the money supply by buying bonds (usually government bonds) from the banks, with an agreement to sell them back at some pre-specified time in the future. This transaction is known as a ***repurchase agreement***.

- A ***repurchase agreement*** represents a secured loan to the banks (borrowers).
- Central bank (lender) earns the repo rate.
- Normally, the maturity of repo agreements ranges from overnight to 2 weeks.

Generally, when a central bank increases the policy rate, a commercial bank increases its base rate because commercial banks do not want to lend at a rate of interest that would be lower than the rate they are charged by the central bank. ***Base rate*** is the reference rate on which a bank bases lending rates to all other customers e.g. a bank may lend to a client at base rate + 2%.

- ***Federal funds rate:*** It is the interbank lending rate on overnight borrowings of reserves. It is the most important interest rate used in U.S. monetary policy.

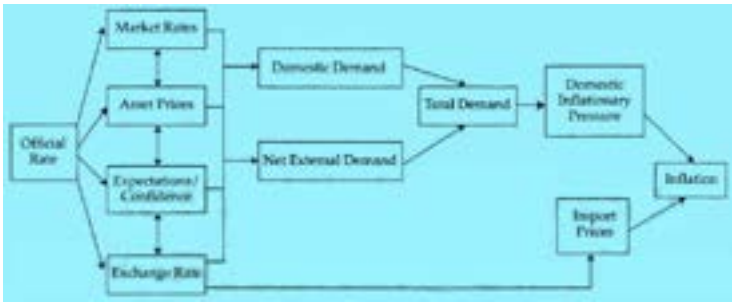
3. Reserve Requirements

Reserve requirements or Legal reserve ratio: Legal reserve is the ratio of cash reserves to deposits that banks are required to maintain.

- By lowering the ratio (or decreasing the reserve requirements), banks will have more reserves to lend and invest → consequently, money supply increases.

4. The Transmission Mechanism

The ***monetary transmission mechanism*** is the process through which a central bank's interest rate gets *transmitted* through the economy and eventually affects the increase in the aggregate price level i.e. inflation.

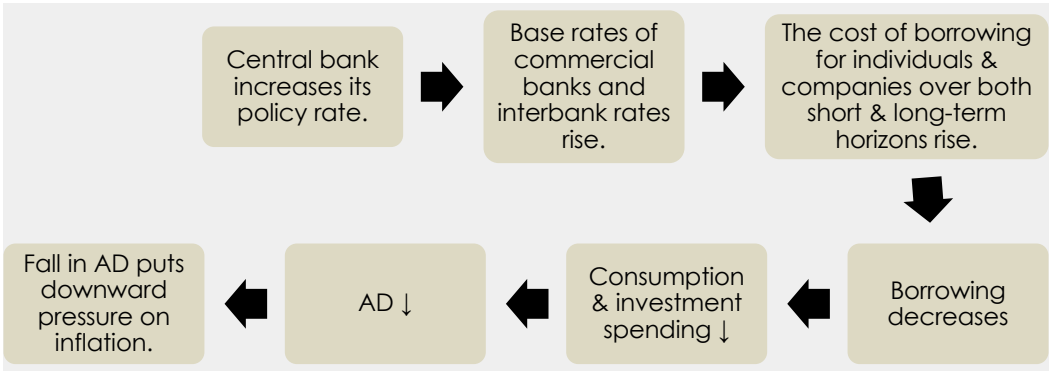


Source: Bank of England

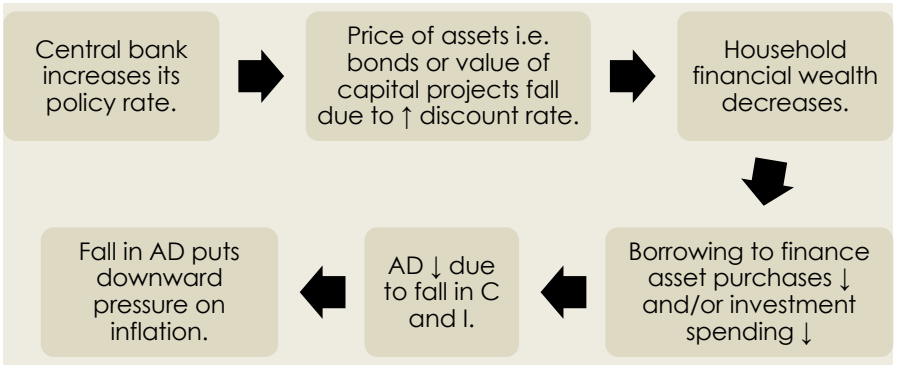
Suppose central bank increases its policy rate, it will get transmitted through the economy via ***four*** interrelated channels.

1. Bank lending rates
2. Asset prices
3. Agents' expectations
4. Exchange rates

Bank Lending Rate Channel

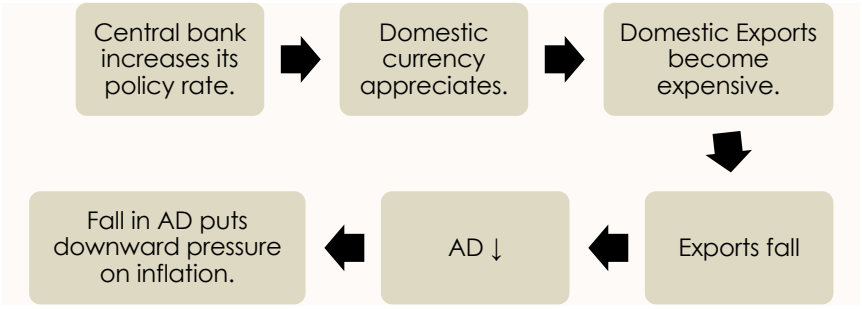


Asset Prices Channel



Agents' expectations Channel

Exchange rates Channel



Practice: Question-Set from the CFA Institute's Curriculum.

4. MONETARY POLICY OBJECTIVES

Inflation Targeting

Inflation-targeting is used to control inflation and to maintain price stability. It is recommended that the central banks should have:

a) A clear, symmetric and forward-looking medium-term inflation target.
b) Target inflation rate should be sufficiently > 0% to avoid the risk of deflation (negative inflation) but should also be low enough to maintain price stability.
- Most central banks in developing economies target an inflation rate of 2% based on a CPI.
- Central banks are permitted to use a range around the central target of + 1% or -1% e.g. with a 2% target, target is to keep inflation between 1% and 3%.

The success of inflation-targeting depends on three key concepts:

1. Central bank independence

Central bank independence: In order to be able to implement monetary policy objectively, the central bank should have a degree of independence from government. The degree of independence varies among economies.

- ***Operationally independent:*** A central bank is operationally independent when it determines the level of interest rate, the definition of inflation that it target, the target inflation rate, and the inflation-targeting horizon.

- ***Target independent:*** A central bank is target independent when it determines only the level of interest rate; the definition & level of target inflation is determined by the government.

2. Credibility

Credibility: In order to be able to implement monetary policy objectively, the central bank should be credible among economic agents because when economic agents believe that the central bank will hit the target, the belief itself could become self-fulfilling.

3. Transparency

Transparency: In order to be able to implement monetary policy objectively, the central bank should be transparent in its goals, objectives and decision making.

Central banks ***do not target current inflation***; rather, they focus on inflation two years ahead due to two reasons:

1) The target inflation rate reflects a past inflation rate because it is based on headline inflation rate which indicates rise in the price of basket of goods and services over the ***previous*** 12 months.
2) Changes in interest rates (monetary policy) affect the real economy with a time lag.

The Main Exceptions to the Inflation-Targeting Rule:

Two prominent central banks that do not use a formal inflation target include the Bank of Japan and the U.S. Federal Reserve System.

Impediments to the successful operation of any Monetary Policy in Developing Countries:

1) Due to the absence of a sufficiently liquid government bond market and developed interbank market, it is difficult to conduct monetary policy.

2) Due to rapid changes in an economy, it is difficult to determine the neutral rate and the equilibrium relationship between monetary aggregates and the real economy.

3) Due to rapid financial innovation taking place in developing countries, it is difficult to determine the standard definition of money supply.

4) Central banks in developing countries lack credibility due to poor track record in controlling inflation in the past; it makes monetary policy less effective.

5) Central banks in developing countries lack independence.

Practice: Example 2 from the CFA Institute's Curriculum.

Copyright © FinQuiz.com. All rights reserved.

Practice: Question-set from the CFA Institute's Curriculum.

Exchange Rate Targeting

Many developing economies use exchange rate targeting to operate monetary policy instead of using inflation-targeting.

Exchange rate targeting involves setting either a fixed level of band of values for the exchange rate against a major currency. The central bank supports this target by buying and selling the national/domestic currency in foreign exchange markets i.e.

- When domestic inflation rises relative to the economy of major currency, with a floating exchange rate system, domestic currency would depreciate against the major currency; to guard the domestic currency from depreciating (i.e. to meet the exchange rate target), central bank starts selling foreign currency reserves and buying its own domestic currency. As a result, domestic money supply decreases and short-term interest rates increase. Hence, inflation falls and exchange rate rises against the major currency (e.g. dollar).
- Opposite occurs when domestic inflation falls relative to the economy of major currency.

Rationale to use Exchange rate Targeting: The rationale behind using an Exchange rate targeting is to import the inflation experience of the low inflation economy by tying domestic economy's currency to the currency of an economy with a good track record on inflation.

Drawback of using exchange rate targeting: Due to exchange-rate targeting, domestic interest rates and money supply can become more volatile.

NOTE:

For exchange rate targeting to be successful, the monetary authority must be independent, credible, and transparent in decision making; otherwise, speculators may trade against the monetary authority.

Managed exchange rate policy: In managed exchange rate policy, a central bank seeks to limit the movement of domestic currency by actively intervening in the market.

Practice: Question-set from the CFA Institute's Curriculum.

Contractionary and Expansionary Monetary Policies and the Neutral Rate

Central banks adjust official policy rates to manage liquidity.

1) **Contractionary Monetary Policy**: Raising interest rates is done when anticipating inflation due to strong economic activity.

- This reduces liquidity.
- It's termed "contractionary" as it aims to reduce money supply and real economy growth.

2) **Expansionary Monetary Policy:** During economic slowdowns with weakening inflation, central banks might:

- Lower interest rates.
- Increase liquidity.
- This is called "expansionary policy."

Note: "High" and "low" policy rates are in relative terms. These terms are relative to the economic conditions and the central bank's objectives.

High rate = more restrictive or contractionary stance.
Low rate = more accommodating or expansionary stance.

Neutral Rate: The neutral rate is the rate of interest that neither overheats the economy nor slows down the economy. It should correspond to the average policy rate over a business cycle.

- When policy rates are > neutral rate, monetary policy is contractionary.
- When policy rates are < neutral rate, monetary policy is expansionary.

Two components of the Neutral rate are as follows:

1) Real trend rate of growth of the underlying economy i.e., the rate of economic growth that an economy can achieve in the long-run with a stable inflation rate.
2) Long-term expected inflation.

Neutral rate = Trend growth + Inflation target

For example,

Neutral rate = 2% + 2.5% = 4.5%

- The central bank's monetary policy will be contractionary (expansionary) when its policy rate > (<) 4.5%.

What's the Source of the Shock to the inflation Rate?

The Sources of the Shock to the Inflation Rate:

The monetary authority must first try to identify the source of the shock to the inflation rate before adopting a contractionary or expansionary monetary policy. There are two sources of the shock to the Inflation Rate:

a) Demand shock: When inflation increases due to increase in consumption and investment growth rates resulting from increase in the confidence of consumers and business leaders, it is referred to as demand shock.

- When inflation rises due to demand shock, it is appropriate to use tight monetary policy.

b) Supply shock: When inflation increases due to increase in the costs of production (e.g. costs of inputs), it is referred to as supply shock.

- When inflation rises due to supply shock, it is ***not*** appropriate to use tight monetary policy because increasing interest rates further reduce profits and consumption and eventually further increase unemployment.

Limitations of Monetary Policy

Problems in the Monetary Transmission Mechanism

Central banks cannot always control the money supply because:

- Central banks cannot control the amount of money that households and corporations put in banks on deposit.
- Central banks cannot easily control the willingness of banks to make loans.

For an effective monetary policy, the monetary authority must have credibility.

For example, suppose

- The central bank increases the interest rate but bond market participants think that short-term rates are already too high and raising interest rate will result in a recession, and the central bank will not be able to meet its inflation target.
- As a result, due to fall in expected inflation rates demand for long-term bonds ↑, → long-term interest rates decrease. Decrease in long-term rates makes long-term borrowing cheaper for companies and households, → consumption and investment spending increases.
- Hence, the more credible the monetary authority, the more stable the long end of the yield curve.

NOTE:

Bond market vigilantes are bond market participants who may reduce (increase) their demand for long-term bonds, resulting in yields to rise (fall) and bond prices to fall (rise), when it is believed that monetary authority lacks credibility in inflation rate targeting.

Interest Rate Adjustment in a Deflationary Environment and Quantitative Easing as a Response

It is more difficult for conventional monetary policy to deal with deflation than inflation because, once nominal interest rates are reduced to 0% to stimulate economy, the central bank cannot reduce them any further. During deflation,

- The real value of debt rises.
- Consumers are encouraged to postpone consumption today, leading to fall in demand that leads to further deflationary pressure.

Deflationary trap: It has following characteristics:

- Weak consumption growth
- Falling prices
- Increases in real debt levels

Liquidity trap:

Liquidity trap occurs when the demand for money becomes infinitely elastic (money demand curve is horizontal) i.e. demand for money balances increases without any change in the interest rate.

This implies that during liquidity trap, monetary policy becomes ineffective because increasing money supply will not further lower interest rates or affect real activity.

Liquidity trap is associated with the phenomenon of deflation.

Once interest rates are at 0%, central bank can use following two approaches to stimulate economy:

1) Convincing market participants that interest rates will remain low for a long time period even if the inflation picks up. This action will tend to lower interest rates along the yield curve.

2) Increasing the money supply [Quantitative easing].

Quantitative easing (QE) refers to purchasing government or private securities by the central bank from the private sector and financing those purchases by printing money.

Operationally, it is similar to open market purchase operations but conducted on a much larger scale.

NOTE:

Financing government deficit by printing money is called ***monetization of the government deficit.***

Practice: Question-set from the CFA Institute's Curriculum.

Practice: Example 3 from the CFA Institute's Curriculum.

5. INTERACTION OF MONETARY AND FISCAL POLICY

Both fiscal and monetary policies are used to stabilize aggregate demand. However, these policies are not interchangeable because they work through different channels.

A. Easy fiscal policy/tight monetary policy: When the expansionary fiscal policy is accompanied by a contractionary monetary policy, it will lead to:

- Higher aggregate output.
- Higher interest rates.
- Expansion of the size of the public sector relative to the private sector.

B. Tight fiscal policy/easy monetary policy: When contractionary fiscal policy is accompanied by expansionary monetary policy, it will result in:

- Low interest rates.
- Expansion of the size of the private sector relative to the public sector.

C. Easy monetary policy/easy fiscal policy: When both fiscal and monetary policy are easy, then the combined impact will be:

- Higher aggregate demand.
- Lower interest rates (at least if the monetary impact is larger).
- Growing private and public sectors.

D. Tight monetary policy/tight fiscal policy: When both fiscal and monetary policies are tight, then the combined impact will be:

- Higher interest rates (at least if the monetary impact on interest rates is larger).
- Lower aggregate demand from both public and private sectors.

NOTE: *In all the aforementioned cases, it is assumed that wages and prices are rigid.*

Factors Influencing the Mix of Fiscal and Monetary Policy

The dual objective of stabilizing the level of AD at close to the full employment level and increasing the growth of potential output can be best achieved by expansionary monetary policy and a tight fiscal policy.

When growth of an economy is slow due to shortage of good quality, trained workforce or modern capital infrastructure, the government should use expansionary fiscal policy.

When this expansionary fiscal policy is accompanied by an expansionary monetary policy, it may lead to increase in inflationary pressures.

Policy conclusions regarding the role of policy interactions:

A. No monetary accommodation:
Increase in government spending generally has greater impact (i.e. 6 times bigger) on GDP than similar size social transfers because the social transfers are viewed as temporary by the economic agents. With no monetary accommodation (i.e. expansionary monetary policy), increase in AD leads to higher interest rates immediately.

- ***Targeted social transfers*** to the poorest citizens have a greater effect than ***non-targeted transfers*** (i.e. 2 times bigger).
- Labor tax reductions have a slightly bigger impact on GDP than the non-targeted transfers.

B. Monetary accommodation (i.e. keeping interest rates unchanged in spite of rising AD**):**

- Fiscal multipliers are greater when loose fiscal policy is accompanied by loose monetary policy.

Quantitative Easing

The purpose of quantitative easing – purchasing securities by central bank to reduce interest rates – is to increase the supply of money as well as lending to the consumers and businesses.

Central Banks have purchased large quantities of government bonds under quantitative easing programs.

This encourages bond holders (financial institutions and others) to sell bonds to central bank and use the money for spending – thus promoting consumption and money supply.

However, this can encourage governments to issue more and more debt without any fear of lack of demand for bonds (know that central bank is a buyer in big quantity). This is similar as 'printing of money' and funding the budget.

Cumulative multiplier

$$= \frac{\text{cumulative effect on real GDP over the two years}}{\%\ of\ GDP}$$

The Importance of Credibility and Commitment

According to the IMF model, high budget deficits lead to:

- Higher real interest rates.
- Crowding out effects.
- Decreased productive potential of an economy.
- Increasing inflation expectations and higher longer-term interest rates if budget deficits are expected to persist.

Practice: Question-set and CFA Institute's end of Chapter Practice Problems + Questions from FinQuiz Question Bank.

1. INTRODUCTION

The international environment is constantly changing, and this can impact companies, industries, nations, and regional groups.

Geopolitics

Geopolitics is the study of how geography affects politics and international relations.

- Analysts study the actors (individuals, organizations, governments etc.) and their interactions in order to understand key drivers of investment performance, such as economic growth, business performance, market volatility, and transaction costs.

Geopolitical Risk

Geopolitical risk is the risk that arises from tensions or actions between actors that disrupt the normal, peaceful flow of international relations. This can affect the circumstances of the financial markets and asset allocation choices.

Impact of Geopolitical Risk on:

- **macroeconomic level:** Geopolitical risks affect macroeconomic growth, interest rates, and market volatility. Changes in capital markets can affect asset allocation decisions, including an investor's regional exposure.
- **portfolio level:** Geopolitical risk can affect a portfolio's suitability for an investor's goals, risk tolerance, and time horizon.

2. NATIONAL GOVERNMENTS AND POLITICAL COOPERATION

Actors: Actors are individuals, organizations, businesses, and national governments that engage in political, economic, and financial activities.

Types of actors:

- **State actors:** parties that directly control country's national security or resources. Governments, political groups, and leaders are state actors.
- **Non-state actors:** who participate in global political, economic, or financial issues but don't control national security or resources. Non-state actors include NGOs, multinational enterprises, charities, and influential figures (business leaders, cultural celebs etc.).

Note: These actors are not only impacted by their interaction with one another, but also by circumstances affecting their allies and opponents.

Features of Political Cooperation

Relations between nations or national governments (state actors) can be ***cooperative*** or ***competitive***.

Countries cooperate to achieve a common aim. These aims might be strategic, military, economic, or cultural. Given the scope of country ambitions and interests, their interconnections may be complicated, posing geopolitical risk.

Political Cooperation: is the degree to which countries agree on laws and standards for their operations and interactions.

1. **Cooperative country:** A co-operative country is one that engages in mutual rules standardization, tariff harmonization, free information flow, and technology transfer.
2. **Non-cooperative country:** A non-cooperative country has inconsistent, arbitrary rules, closed borders for goods and services, retaliation, and limited technology exchange.

Motivations for Cooperation

A country may collaborate with its neighbors or other nations for a variety of reasons, including military, economic, or cultural interests.

1. **National Security or Military Interest** - protects a country's residents, economy, and institutions from foreign threats. These threats might be military, terrorist, criminal, cyber, or natural calamities.

   Geographic variables influence a country's security policy and degree of cooperation.

Copyright © FinQuiz.com. All rights reserved

Stanley Notes™ 2025

- ***Landlocked countries*** depend on neighbors for resources, making collaboration crucial for international access, growth, or survival.
- ***Countries connected to trade routes***, may exploit their geographic location to exert influence on international dynamics.

2. **Economic Interest** - National security has expanded beyond military to include economic factors.

- Domestic security is achieved by growing national wealth and limiting income inequality.
- International security is achieved by domestic companies operating globally.
- Countries cooperate to secure essential resources through trade or level the global playing field through standardization.

Resource Endowment, Standardization, and Soft Power

1. **Geophysical resource endowment** includes habitable geography, climate, food, and water.

   These resources are unevenly distributed. This divergence give rise to power dynamics, which can influence the interaction between countries. When dealing with a resource-hungry country, a resource-rich country may have more political influence.

   U.S., Russia, Australia, and China are resource independent whereas western Europe, Japan, and Turkey import a lot of fossil fuels. Saudi Arabia has plentiful fossil resources yet imports many essentials.

2. **Standardization** is the process of making rules/protocols for how a product or service is produced, sold, transported, or used. When everyone agrees to follow these rules, this is called standardization.

   It supports cross-border trade and capital flows, which boost economic growth and living standards.

3. **Cultural Considerations and Soft Power**

Cultural Considerations - countries may cooperate for cultural reasons. Examples:

- cultural similarities
- historical linkages
- immigrant trends
- common experiences

Soft Power - involves influencing another country's decisions without force or coercion. Countries may build soft power over time through:

- advertising
- travel subsidies
- cultural activities
- university exchange programs

The Role of Institutions

An institution is an established organization or practice in a society. NGOs, charities, religious practices, families, the media, political parties, and educational practices are a few examples of institutions.

Strong institutions

- promote internal and external political stability.
- strengthen a country in building cooperative connections.
- such as those fostering government accountability, rule of law, and property rights, allow countries to operate with more authority and independence internationally.
- can lessen the risk of a country deviating from its cooperative duties by integrating cooperation at many societal levels.

Hierarchy of Interests and Costs of Cooperation

The national interests of a country may be thought of as a ***hierarchy***, with survival-critical concerns at the top and nice-but-not-essential issues at the bottom. Governments are guided by the hierarchy of interests. When two concerns conflict, the one that is higher on the list is given more attention.

For example, tariff harmonization may help a country on its own, but cooperation costs more when two countries are at military conflict with each other. Depending on the countries' priorities, cooperation might not be in their best interest, even if it could help.

Power of the Decision Maker

The motivations of decision makers can affect a country's cooperative and non-cooperative choices. Political parties and individual decision makers have their own influences and needs.

Priority setting is significantly affected by the ***duration of a country's political cycle***. Many countries have short political cycles, making it difficult to prioritize long-term risks like climate change or wealth disparity.

One government may prioritize military buildup or healthcare differently than its predecessor. How nations assess these problems will influence cooperation.

Copyright © FinQuiz.com. All rights reserved

Securing food and water may be prioritized above supporting a cultural activity. As social demands are addressed, national interests might become more subjective.

Political Non-Cooperation

Countries' interests in political collaboration differ, with some being more cooperative than others. Most countries cooperate to some extent, although there are some exceptions, such as those that weight self-determination above collaboration. Extreme cases of non-cooperation are rare, as other state actors may attempt to persuade non-cooperative players to cooperate.

Practice: Example 1, 2, 3 and Question-set from the CFA Institute's Curriculum.

3. FORCES OF GLOBALIZATION

Globalization

Globalization is the interaction and integration of individuals, organizations, and governments on a global scale. It is characterized by the cross-border spread of goods, information, jobs, and culture.

Cross-border trade has grown tremendously on both the microeconomic and macroeconomic levels during the last several decades.

Globalization has had a cultural and communication impact, with nations exchanging ideas and trading goods. It has also made it easier for people around the world to collaborate. The internet has increased the speed and efficiency of this process.

Features of Globalization

Globalization helps companies access new markets, talent, and learning. Globalization is characterized by economic and financial cooperation including:

- capital flows
- currency exchange
- trade of products and services
- cultural and information interaction.

Anti-globalization or nationalism promotes a country's economic interests over others'. Prime focus is on national sales and production. Anti-globalization is characterized by:

- limited currency exchange
- limited cross-border investment
- limited economic and financial cooperation

Important Note:

Globalization and cooperation are related but not always dependent, as the private sector can drive trade even with little government support.

Motivations of Globalization

For non-state actors, three potential gains are:

1. **Increasing profits -** by increasing sales or reducing costs.

   a) **Increasing sales:** Companies globalize to access new clients/markets to improve sales. This process needs considerable investment in new markets, hiring, and training.
   b) **Decreasing costs:** Globalization helps firms to access cheaper tax-operating regimes, cut labor expenses, and enhance supply chain efficiency.

2. **Access to resources and markets:** A non-state actor may globalize to improve access to resources (such as skilled workers or cheap raw materials), markets, or investment opportunities. The investment can be done through portfolio investment or foreign direct investment.

3. **Intrinsic gain -** Intrinsic gain is beyond profit. It's hard to measure yet promotes globalization. It increases empathy among actors and reduces geopolitical risks.

   Intrinsic benefits include extending one's horizons, experiencing new places, acquiring new concepts. All can lead to personal growth and education. Learning new techniques increases productivity.

Costs of Globalization and Threats of Rollback

The following are some of the possible drawbacks of globalization.

1. **Unequal Accrual of Economic and Financial Gains:** Economic theory says that aggregate economic activity improves when all actors seek profit maximization and efficiency. However, this does not mean that everyone benefits. If a factory goes overseas, it creates employment away but reduces

Copyright © FinQuiz.com. All rights reserved

them at home. Local firms in the foreign country may have to compete with the foreign firm for labor.

2. **Lower Environmental, Social, and Governance Standards:** Companies in low-cost nations follow local norms. While corporate profit may increase, the aggregate effect may be negative due to lower environmental protection, social benefits, or corporate governance standards.

3. **Political Consequences:** A third cost of globalization is the political consequences of global expansion.. Globalization may exacerbate income and wealth disparities as well as opportunity inequalities across nations.

These tendencies might manifest themselves in the domestic politics of many countries, reducing the likelihood of effective political cooperation.

4. **Interdependence:** Companies can become dependent on other countries' resources for their supply chains, which can result in the nation itself becoming dependent on other nations for certain resources. Supply chain disruptions can force companies to switch suppliers or cease operations.

Practice: Examples 4, 5 and Question-set from the CFA Institute's Curriculum.

4. INTERNATIONAL TRADE ORGANIZATIONS

International Monetary Fund (IMF), World Bank (WB), and World Trade Organization (WTO) are the three organizations that set the rules for the international trade.

1. International Monetary Fund

Objectives of IMF:

1. To promote exchange rate stability.
2. To help establish multilateral system of payments and eliminate foreign exchange restrictions.
3. To ensure the stability of the international monetary system.
4. To promote international monetary cooperation to deal with international monetary problems.
5. To ensure exchange stability and orderly exchange arrangements to

- Facilitate growth of international trade.
- Promote high employment.
- Promote sustainable economic growth and poverty reduction.

6. To provide temporary financial assistance (i.e. lends foreign exchange to member countries) to help ease balance of payments adjustment. These funds are only lent under strict conditions and IMF continually monitors borrowing countries.

Sources of funds available to IMF: Member countries provide IMF a pool of gold and currencies that it can use for lending purposes.

The IMF has redefined and deepened its operations by following ways:

1. The IMF has improved its lending facilities to better serve its members.
2. The IMF has taken several steps to improve economic and financial monitoring of global, regional, and country economies.
3. The IMF has taken several steps to help global economies to resolve global economic imbalances.
4. The IMF is devoting more resources to analyze global capital market developments and their links with macroeconomic policy. In addition, it is devoting resources for the training of country officials to assist them better manage their financial systems, monetary and exchange regimes, and capital markets.
5. IMF assesses financial sector vulnerabilities to alert countries to vulnerabilities and risks in their financial sectors.

Role of IMF from an investment perspective: The IMF helps to keep country-specific market risk and global-systemic risk under control.

2. World Bank Group

Objectives of World Bank:

- To provide assistance to developing countries to reduce poverty and enhance sustainable economic growth.
- To provide funds for development projects (i.e. highways, schools).
- To provide technical assistance in development projects.
- To provide analysis, advice, and information to its member countries for helping them achieve sustainable economic growth and improve standard of living.
- To increase the capabilities of its partner countries, people in developing countries, and its own staff.

Copyright © FinQuiz.com. All rights reserved

Factors necessary for developing countries to grow and attract business are as follows:

1. Strong governments
2. Educated government officials
3. Effective legal and judicial systems that encourage business
4. Enforcement of individual and property rights and contracts
5. Developed financial systems to support both micro-credit financing and larger corporate ventures financing.
6. Absence of corruption

Affiliated entities of the World Bank: The World Bank has two closely affiliated entities:

i. The International Bank for Reconstruction and Development (IBRD)
ii. The International Development Association (IDA)

- Both IBRD and IDA are ***non-profit*** organizations.

Role of IBRD and IDA:

To provide **low or interest-free loans** and grants to countries that either have no access to international credit markets or have unfavorable access to international credit markets.

IBRD:

- The IBRD is market-based entity and one of the most important supranational borrowers in the international capital markets.
- IBRD has strong capital position and has very conservative financial, liquidity, and lending policies. Hence, it has high credit rating.
- Because of high credit rating, IBRD can charge low interest rates to its borrowers (i.e. developing countries).
- IBRD does not obtain funds from outside sources to fund its own operating costs (i.e. overhead costs).
- IBRD also finances World Bank operating expenses, assists IDA and debt relief programs.

Sources of funds available to IBRD for lending purposes:

- Funds obtain through selling AAA-rated bonds in the world's financial markets. However, it earns a small margin on this lending.
- Income earned from lending out its own capital i.e. reserves built up over the years and money paid in from its member country shareholders. It represents the greater proportion of its total income.

Role of the World Bank from an investment perspective: The World Bank helps countries to construct the basic economic infrastructure necessary for domestic financial markets and a well-established financial industry in developing countries.

3. World Trade Organizations

The International Trade Organization (ITO): It was founded to

- Promote international economic cooperation from perspective of trade.
- Reduce and regulate customs tariffs.

World Trade Organization (WTO): WTO was founded in 1995 and is a successor to General Agreement on Tariffs and Trade (GATT). It is an international organization that

- Regulates cross-border trade relationships among countries.
- Serves as the forum for trade negotiations among countries.
- Monitors a global policy setting to promote coherent and transparent trade policies.
- Serves as a major source of economic research and analysis.

Objectives of WTO: WTO's goal is to expand trade and improve world living standards by establishing trade policies i.e.

- Promoting free trade.
- Eliminating barriers to trade (e.g. quotas, duties and tariffs).
- Settling trade disputes among countries.
- Eliminating trade discrimination through **most favored nation** (i.e. treating every country equally).
- Providing technical cooperation and training to developing, least-developed, and poor countries.
- Cooperating with the other two Briton Woods institutions, the IMF and the World Bank.

Examples of Rounds of Negotiations that took place under the GATT:

- **Tokyo round (1970s):** To deal with a wide range of non-tariff trade barriers.
- **Uruguay round (1986):** To deal with agriculture and textiles trade issues, intellectual property rights and trade in services.

Example of Ongoing round of negotiations under WTO:

Copyright © FinQuiz.com. All rights reserved

- **Doha round**: Its objective is to enhance globalization by reducing barriers and subsidies in agriculture and to deal with a wide range of cross-border services.

Role of the WTO from an investment perspective: The WTO provides the major institutional and regulatory base to facilitate establishment of multinational corporations. Stocks and bonds of these multinational corporations represent the key elements in investment portfolios.

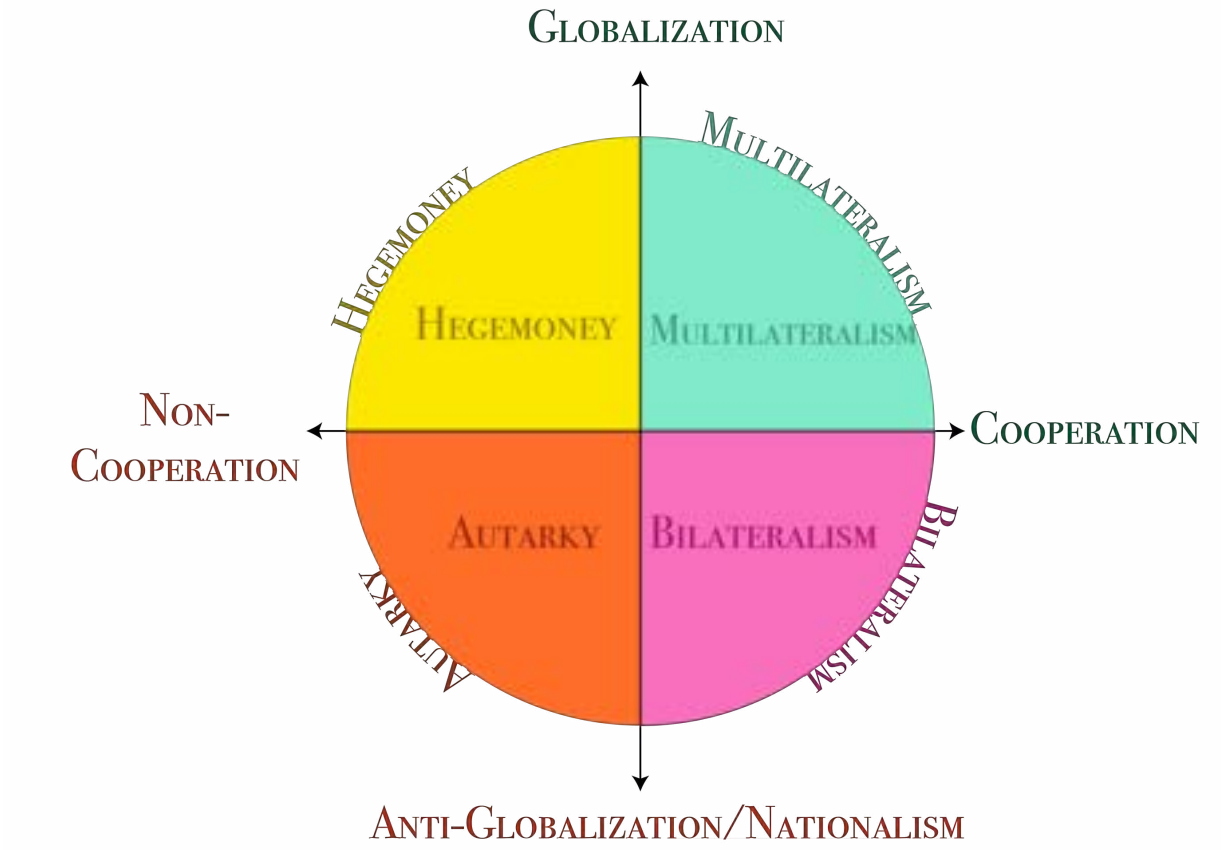
NOTE:

The GATT still exists in the form of an updated version of 1994 and it is the WTO's principal treaty for trade in goods.

Practice: Question-set from the CFA Institute's Curriculum.

5. ASSESSING GEOPOLITICAL ACTORS AND RISK

Archetypes of Globalization and Cooperation



The above framework shows four country behavior archetypes:

1. Autarky,
2. Hegemony,
3. Multilateralism, and
4. Bilateralism

The two axes refer to ***political cooperation versus non-cooperation*** and ***globalization versus nationalism***. Both axis in this framework represent a spectrum.

Each archtype has its own set of costs, benefits, and tradeoffs. This framework can assist investment analysts in

Copyright © FinQuiz.com. All rights reserved

evaluating geopolitical players and the possibility of investment outcomes in terms of risks and opportunities.

Important:

Geopolitical risk analysis - Globalization and cooperation framework - is dynamic in nature. Therefore, countries can be moving targets. Successful geopolitical risk analysis considers the country's position in this framework as well as its stability within it.

1. Autarky

Autarky is characterized by little or no foreign trade or financing. State-owned companies govern domestic industry. Autonomous countries are stronger politically because they can control technology, products, services, media, and political message.

China was mostly autocratic throughout the 20th century, with little political collaboration or globalization.

Autarky can sometimes speed up a country's economic and political growth. Autarky has disadvantages as well. North Korea and Venezuela, for example, have experienced slower economic and political development.

2. Hegemony

A hegemonic country is a regional or global leader and exerts control over others' resources. State-owned enterprises control key export markets. A hegemonic system provides benefits to the country itself and to the international system.

Countries that agree with the hegemon's norms and standards may enjoy the leader's rewards, including monitoring and enforcing the standards.

There may be costs to a hegemonic system, including increased geopolitical risk.

3. Multilateralism

Countries that cooperate in mutually beneficial economic agreements and broad rule harmonization are characterized by multilateralism. Multiple trade partners are completely integrated into global supply networks by private companies.

Germany and Singapore are two examples of multilateral countries.

4. Bilateralism

Bilateralism is the cooperation between two countries, usually involving political, economic, financial, or cultural cooperation. Countries involved in bilateralism may have several partners, but agreements are one-at-a-time.

Countries usually exist on a spectrum between ***bilateralism*** and ***multilateralism***, with ***regionalism*** in between. In regionalism, group of countries collaborate with each other.

Countries are unlikely to perfectly fit the bilateral frame, as stronger political cooperation tends to lead to globalization.

Practice: Example 6, 7 and Question-set from the CFA Institute's Curriculum.

6. THE TOOLS OF GEOPOLITICS

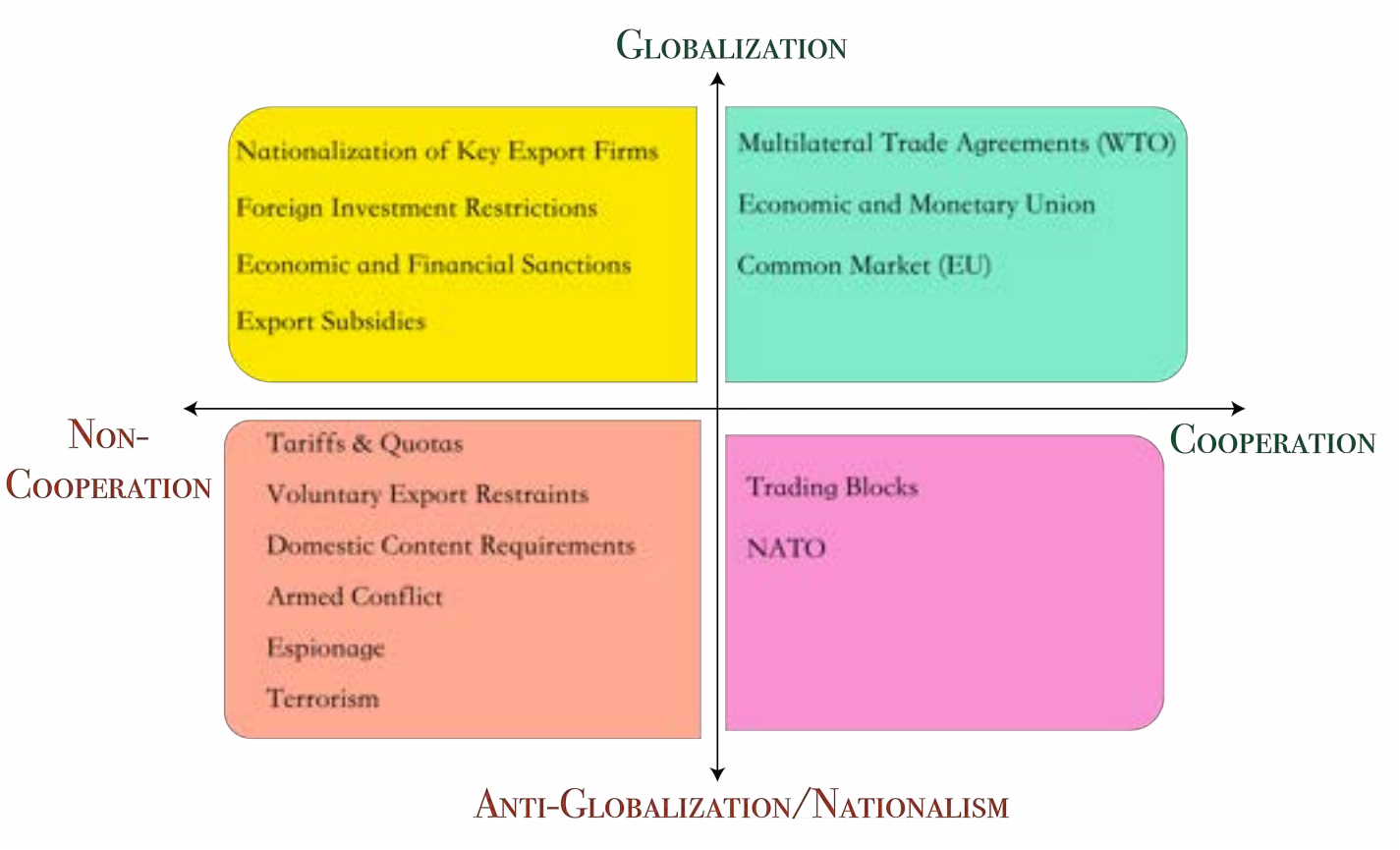
Three types of tools of geopolitics are:

1. National tools
2. Economic tools
3. Financial tools

Each tool has options that can promote cooperation or increase conflict (non-cooperation).

Copyright © FinQuiz.com. All rights reserved

Exhibit: Tools of Geopolitics



1. National Security Tools

National security tools are those that influence state actors through direct or indirect impact on resources, people, or borders.

National security tools may be active, indicating they're being utilized, or threatening, suggesting their use is probable enough to justify alarm.

National security tools can be direct, such as armed conflict, or indirect, such as espionage or military alliances. Some tools are used in a non-cooperative way, while others are used cooperatively.

For example, NATO is an alliance between the US, Canada, UK and EU, which serves as a collective effort to reduce nuclear proliferation and other common national security goals.

The most extreme example of a national security tool is ***armed conflict***, which can disrupt or destroy physical infrastructure, reshape international flows of goods, services, capital, and labor, and impact neighboring countries and refugee-accepting countries.

2. Economic Tools

Economic tools are activities that support cooperative or non-cooperative stances through economic means.

- Cooperative economic tools include ***multilateral trade agreements, common markets***, and ***common currencies.***
- Economic tools can also be non-cooperative, such as ***nationalization***, the process of transferring an activity or industry from private to state control.
- Countries may impose:
  - ***voluntary export limits*** - declining to trade as much of their goods and services as is required to fulfill demand.
  - ***domestic content criteria*** - requiring a specific amount or kind of domestic input to be included in an exported good.

3. Financial Tools

Financial tools are activities that support cooperative or non-cooperative stances through financial means.

Copyright © FinQuiz.com. All rights reserved

Cooperative financial tools include
- encouraging foreign investments
- allowing cross border currency exchanges

Non-cooperative financial tools include
- restricting foreign investments
- limiting access to local currency markets

Cooperative financial tools foster security, economic and financial support and as a result reduce geopolitical risk, but the same tools can cause international system weaknesses. Dollar domination is an example. US monetary policy can hurt countries that do not have US dollar reserves.

Multi-Tool Approach

Geopolitics and its tools are complex and include political, economic, and financial systems. These systems can be intertwined and are often multifaceted.

For example, many countries, including those with multilateral trade agreements, restrict ***cabotage*** because it is a highly complex process. Countries must cooperate on issues like physical security and economic cooperation to permit cabotage.

The term "***cabotage***" refers to a foreign company's right to transport people and products within a country.

Note: Actors that incorporate more tools of collaboration are less likely to initiate conflict.

Geopolitical Risk and Comparative Advantage

A country or an actor's core priorities may be shaped both by geopolitics and by the tools of geopolitics, which can either shift the relative advantage in one direction or the other.

Countries with certain resources and capabilities may benefit from trade if they cooperate.

Countries or regions with low geopolitical risk exposure may attract more labor and capital, whereas those with higher geopolitical risk exposure may see a decline in employment and capital. A sustained threat of conflict may drive higher asset price volatility, and consequently investors may demand a higher level of risk compensation.

Practice: Example 8, 9 and Question-set from the CFA Institute's Curriculum.

7. INCORPORATING GEOPOLITICAL RISK INTO THE INVESTMENT PROCESS

Geopolitical risk can have a significant impact on financial markets, depending on an investor's goals and risk tolerance. Some investors may consider geopolitical risk only if it affects their asset classes or strategies in the long term. For other investors, geopolitical risk is key for generating alpha.

Types of Geopolitical Risk

Three types of geopolitical risks are:

1. **Event risk** - focuses on specified dates, such as elections, new legislation, holidays, and political anniversaries. Investors' expectations of a country's cooperation are influenced by political changes. Therefore, risk analysts often use political calendars as a starting point for calculating event risk.

   Note: Predictability does not affect the likelihood, speed, or magnitude of impact on investors, but it does offer them more time to prepare a reaction.

2. **Exogenous risk -** is a sudden or unexpected risk that affects either a country's cooperative position, non-state entities' potential to globalize, or both.

3. **Thematic risk –** risks that evolve over time such as climate change, pattern migration, the rise of populist forces, cyber threats, and terrorism.

Assessing Geopolitical Threats

Geopolitical risk can affect investments at macroeconomic, industry, and company levels. Investors examine geopolitical risk in three areas:

1. **Likelihood of occurrence** - is the probability that the risk will occur. This can be challenging to measure as many risks are unpredictable.

Copyright © FinQuiz.com. All rights reserved

Collaboration and globalization reduce geopolitical risk since partners' political, economic, and financial costs are larger.

Other variables may include internal political stability, economic need, and governmental motives.

2. **Velocity of its impact** - is the pace at which the risk impacts an investor portfolio.

   i. **Short-term or 'high velocity'** - impacts include market volatility and investor flight to quality. Such impacts may damage whole industry or the whole market. *Exogenous* or *'Black swan'* events include rare but significant events.
   ii. **Medium-term risks** impact companies' processes, costs, and investment opportunities, resulting in lower valuations. These risks are sector specific.
   iii. **Long-term or 'low velocity'** risks can impact asset allocation, but the immediate impact on portfolios is likely to be limited.

3. **Size and nature of that impact -** When evaluating a risk's relevance, investors should evaluate its impact. High-impact risks need more investigation than a low-impact risks. External variables might intensify a risk's impact. For example, risk affects markets more during recessions.

   The nature of the impact may be ***discrete or broad***. Discrete affects only one firm or sector, whereas broad ones affect a sector, country, or global economy.

Practice: Example 11 and 12 from the CFA Institute's Curriculum.

Impact of Geopolitical Risk

Scenario analysis

Scenario analysis, in which investment teams examine their portfolios across several world scenarios, can assist teams in prioritizing and making sound investment decisions.

Scenarios can take the form of ***qualitative*** analysis, ***quantitative*** measurement, or ***both***.

A simple framework for qualitative scenario building begins with developing a 'base case' for the risk. Investors can then consider the upside and downside scenarios, as well as how markets are likely to recover after the event.

Quantitative scenarios can ***vary in sophistication***, from simple stylized scenarios (measuring one factor) to more complicated ones using extreme events to test portfolio resilience.

Scenario analysis is a useful tool for tracking risks, deciding on valuable portfolio actions, and avoiding groupthink. It requires a consistent commitment of investors' time and resources.

Effective scenario analysis necessitates teams establishing creative processes, identifying scenarios, tracking them, and assessing the need for action on a regular basis.

Tracking risks according to signposts

Asset managers build strategies to prioritize portfolio risks to minimize the impact of unanticipated change on investments.

Signposts:

- are indicators, market levels, data pieces, or events that signal a risk becoming more or less likely.
- can be compared to traffic lights, with green indicating no action is required, amber indicating caution, and red indicating an action plan is necessary.
- identification may take trial and error. A fundamental principle is to distinguish signal from noise. E.g., policy changes are more critical than political changes.

Economic and financial market circumstances can signal upcoming trouble. A pegged currency or rapidly declining export value can prompt a change in exchange rate policy. Data screens should be used to identify these red flags early for country-level portfolios.

Manifestations of Geopolitical Risk

Geopolitical risks are unpredictable and can have a wide impact on investor portfolios. They can cause market volatility that can cause sudden changes in asset prices.

During the COVID-19 epidemic, for example, stock prices fell while investors' "flight to safety" drove bond prices higher.

Copyright © FinQuiz.com. All rights reserved

Acting on Geopolitical Risk

Risk analysis is the process of analyzing the likelihood, velocity, and impact of a risk to determine if and how to act in the face of such threats. The asset allocator may consider geopolitical risk in their asset allocation strategy. They may consider countries with a long history of using a multilateral approach to be more reliable invest.

Portfolio managers can consider geopolitical risk in their models. Risk depends on investment objectives, risk tolerance, and time horizon.

Political, economic, and financial cooperation may boost the stakes of geopolitical risk assessments for global investors. Changes in international cooperation can affect capital markets.

Practice: Example 13, 14 and End of Chapter Questions from the CFA Institute's Curriculum and Questions from FinQuiz Question-bank.

Copyright © FinQuiz.com. All rights reserved

Stanley Notes™ 2025

1. INTRODUCTION

Understanding trade policies is crucial for global investors as it impacts trade volume, value, and return on investment.

Being aware of potential government trade policy changes is essential as they impact firm profitability and growth by modifying product demand and pricing.

2. BENEFITS AND COSTS OF INTERNATIONAL TRADE

Benefits of International trade:

- Exchange gains: Countries can sell exports at higher prices for more profit and/or buy imports cheaper than domestic production costs
- Boosts economic growth (GDP) through:
  - Efficient resource allocation.
  - Learning, productivity, and knowledge spillovers.
- Offers variety of goods and services to domestic households.
- Access to larger capital and product markets.
- Increased competition leading to greater efficiency.
- Promotes specialization via comparative advantage.
- Facilitates financial capital flow, enhancing liquidity, output, and reducing capital costs.
- Raises wages for unskilled workers in labor-intensive manufacturing sectors.
- Promotes macroeconomic stability.
- Fosters innovation, idea exchange, technical expertise flow, and factor accumulation.
- Enables economies of scale and increasing returns in industries like automobiles, steel by opening new markets.
- Increases awareness of global consumer tastes and preferences.
- Encourages development of quality institutions and policies for domestic innovation.
- Amplifies positive impacts of foreign R&D through trade partnerships.

NOTE:

Although trade increases overall welfare, it does not mean that every individual consumer and producer is better off; rather, trade creates winners and losers. However, the gains from trade are greater than losses.

Traditional trade models, i.e. ***Ricardian model*** and the ***Heckscher-Ohlin model***, focus on specialization and trade based on the concept of comparative advantage that results due to differences in technology and factor endowments, respectively.

Newer models of trade focus on the gains from trade that result from economies of scale, greater product variety and increased competition.

Intra-industry trade: It refers to a trade that occurs when a country exports and imports goods in the same product category or classification. Intra-trade benefits countries because each country can focus on the production and export of one or more varieties of the good and can import other varieties of the same good.

NOTE:

In industries where there is "***learning by doing***," the cost of production per unit falls as output increases because of expertise and experience acquired in the process of production.

Disadvantages of Free Trade:

- Trade may result in greater income inequality.
- Trade may destroy jobs in the industries that compete against imports.
- Unemployment increases when less-efficient firms are forced to exit the industry.
- The displaced workers are required to be retrained for jobs in expanding industries.

It is argued that these costs occur in the short and medium term only; in the long run, the resources (i.e. displaced workers) are likely to be more effectively (re-) employed in expanding export industries after some re-training.

But the workers who remain in import-competing industry may be permanently worse off. Thus, even in the long-run, trade does not necessarily benefit every stakeholder.

NOTE: Besides trade, other factors that affect economic growth include:

- Quality of institutions, infrastructure, and education
- Economic systems
- Degree of development
- Global market conditions

Practice: Question-set from the CFA Institute's Curriculum.

3. TRADE RESTRICTIONS AND AGREEMENTS – TARIFFS, QUOTAS AND EXPORT SUBSIDIES

Trade restrictions (or trade protection) are government policies that are used to impose limits on the ability of domestic households and firms to trade freely with other countries. Examples of trade restrictions include:

- **Tariffs:** Tariffs are taxes levied by a government on imports.
- **Import quotas:** Quota is a direct quantitative restriction on the amount of a good that can be imported into a country (generally for a specified period of time).
- **Voluntary export restraints (VER):** It is a trade restriction under which the exporting country agrees to limit its exports of the good to its trading partners to a specific number of units. VER is similar to import quotas; but unlike quotas, it is imposed by the exporting country.
- **Export Subsidies:** They are direct payments (or the granting of tax relief and subsidized loans) paid by the government to the domestic exporters or potential exporters.
- **Embargoes:** Embargo is a prohibition on trade in a particular good.
- **Domestic content requirements:** Domestic content provisions are a regulation that requires that some specified % of a final good be produced domestically. This % can be specified in physical units or in value terms.

Rationale behind imposing Trade restrictions:

1) Protecting established domestic industries from foreign competition.
2) Protecting new industries from foreign competition (a.k.a infant industry argument).
3) Protecting and increasing domestic employment.
4) Protecting strategic industries for national security reasons.
5) Generating revenues (i.e. through tariffs).
6) Retaliation against trade restrictions imposed by other countries.

- **Capital restrictions:** Capital restrictions are limits imposed on foreigners' ability to own domestic assets and/ or domestic residents' ability to own foreign assets.

Trade restrictions v/s Capital restrictions:

- Trade restrictions limit the openness of ***goods*** markets.
- Capital restrictions limit the openness of ***financial*** markets.

1. Tariffs

Tariffs are taxes levied by a government on imports.

Objectives of tariffs:

- To protect domestic industries that produces the same or similar goods.
- To reduce a trade deficit.

Disadvantages of tariffs:

- Tariffs increase the price of imports above the free trade price; as a result, demand for imported goods falls.
- Tariffs reduce the overall global welfare.

In this context:

Small Country: A small country refers to a country that is a price taker in the world i.e. it cannot influence the world market price.

- Trade barriers generate a net welfare loss in a small country.

Large Country: A large country refers to a country that is a ***large*** importer of the product and can influence the world market price. Trade barriers can generate a net welfare gain in a large country provided that:

- It imposes an even larger welfare loss on its trading partner (s).
- It does not face retaliation from its trading partner.
- The benefits of improving its terms of trade are greater than the deadweight loss as a result of the tariff.

Under Free trade:

- World price (free trade price) = P*.
- Domestic supply = Q^1.
- Domestic consumption = Q^4.

- Imports = Q^1Q^4.

Under Tariffs:

- Per-unit tariff imposed = t.
- Due to tariffs, domestic price = world price + per-unit tariff = $P^* + t = P_t$.

  Note: A tariff imposed by a small country does not change the price in the exporting country.

- Domestic production increases to Q^2.
- Domestic consumption declines to Q^3.
- Imports reduce to Q^2Q^3.

As a result of these price changes:

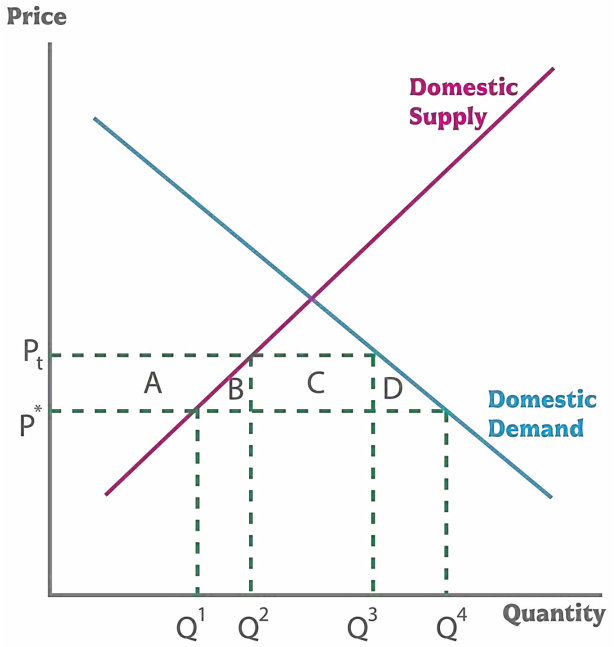
- Consumers lose in the importing country and gain in the exporting country.
- Producers gain in the importing country and lose in the exporting country.
- Government imposing the tariff gains revenue.

When a small country imposes a tariff:

- Consumer's surplus decreases due to increase in price.

  Consumer loss = A + B + C + D
- Producer's surplus increases due to increase in price.

  Producers gain = A
- Government revenue increases i.e. the government gains tariff revenue on imports = Q^2Q^3. It is represented by area C.
- Triangle B and D reflects efficiency loss because tariffs distort incentives to consume and produce.
  - Triangle B reflects production inefficiencies i.e. inefficient producers with cost of production > P* exist in the market and leads to inefficient allocation of resources.
  - Triangle D reflects consumption inefficiencies i.e. consumers are now unable to consume the good due to higher prices.



The net welfare effect = consumer's surplus loss + producer's surplus gain + government revenue

Deadweight loss to the country's welfare =B + D

- Deadweight loss occurs when loss in consumer surplus > sum of the gain in producer surplus and government revenue.

Practice: Example 1 from the CFA Institute's Curriculum.

2. Quotas

A quota is used to restrict the quantity of a good that can be imported into a country, generally for a specified period of time. This restriction is usually enforced by issuing licenses to some group of individuals or firms.

- Due to import quota, foreign producers increase the price of their goods.
- The profits received by the import license holders (foreign producers) are known as ***quota rents***.

In the fig below,

- Import quota = Q^2Q^3.
- Domestic price after the quota = P_t (same as the domestic price after the tariff t was imposed).
- Quota rent is represented by Area C.

Welfare loss to the importing country = B + D + C

- Under a quota, the welfare loss > than under an equivalent tariff.
- The welfare loss will be same under quota and tariffs only when the government of the country that imposes the quota can capture the quota rents by auctioning the import licenses for a fee i.e. it will be equal to areas B + D.

Tariffs v/s Import Quotas:

- The revenue generated from a tariff is collected by the government.
- The revenue generated from quotas is collected by the holders of import licenses.
- Under import quotas, an increase in demand will result in a higher domestic price and greater domestic production compared to an equivalent import tariff.
- Under import tariffs, an increase in demand does not change the domestic price and domestic production; rather, it results in higher consumption and imports compared to an equivalent import quota.
- Unlike quotas, effect of tariff is uncertain.

Voluntary export restraint (VER) versus Import Quota:

- Import Quota is imposed by the importer, whereas VER is imposed by the exporter.
- Under VER, all of the quota rent is captured by the exporting country, whereas under import quota at least some part of the quota rent may be earned by local importers.
- VER produces a greater welfare loss to the ***importing*** country than import quota.

3. Export Subsidies

Objective of export subsidies: To stimulate exports.

Disadvantage of export subsidies: They reduce welfare by promoting production and trade that is inconsistent with comparative advantage.

Countervailing duties: Duties that are imposed by the importing country against subsidized exports are referred to as countervailing duties.

Under export subsidies:

Exporter receives = International price + per-unit subsidy for each unit of the good exported

- It incentivizes exporters to shift sales from the domestic to the export market.

As a result,

- In case of small country, the price of a good in the domestic market/exporting country increases (i.e. Price = Price before subsidy + subsidy). The net welfare effect is negative.
- In case of large country, the world price falls as the large country increases exports and part of the subsidy is transferred to the foreign country (i.e. country's terms of trade deteriorates). The net welfare effect is negative and is larger than that of small country case.

Under export subsidy,

- Consumer surplus decreases.
- Producer surplus increases.
- Government revenue also decreases.

Panel A: Effects of Alternative Trade Policies

	Tariff	Import Quota	Export Subsidy	VER
Impact on	Importing country	Importing country	Exporting country	Importing country
Producer surplus	Increases	Increases	Increases	Increases
Consumer surplus	Decreases	Decreases	Decreases	Decreases
Government Revenue	Increases	Mixed (depends on whether the quota rents are captured by the importing country through sale of licenses or by the exporters)	Falls (government spending rises)	No change (rent to foreigners)

National welfare	• Decreases in small country • Could increase in large country	• Decreases in small country • Could increase in large country	Decreases	Decreases

Panel B: Effects of Alternative Trade Policies on Price, Consumption, Production, and Trade

	Tariff	Import Quota	Export Subsidy	VER
Impact on	Importing country	Importing country	Exporting country	Importing country
Price	Increases	Increases	Increases	Increases
Domestic consumption	Decreases	Decreases	Decreases	Decreases
Domestic Production	Increases	Increases	Increases	Increases
Trade	Imports decrease	Imports decrease	Exports increase	Imports decrease

Source: CFA Institute's Curriculum.

Practice: Example 2 from the CFA Institute's Curriculum.

Changes in the Government's trade policy may affect:

- Pattern and value of trade
- Industry structure

- Profitability and growth of firms because changes in trade policies affect:
  - Goods a firm can import/export
  - Demand for its products
  - Pricing policies
  - Production costs due to increased paperwork, procurement of licenses, approvals etc.

4. TRADING BLOCS AND REGIONAL INTEGRATION

Important examples of **regional integration** include:

1) North American Free Trade Agreement (NAFTA)
2) European Union (EU)

Regional Trading Bloc: A group of countries that have signed an agreement to get the benefits of free trade by removing tariffs and quotas between themselves while retaining the right to set tariffs and quotas towards non-members.

Types of Regional Trading Blocs:

1) **Free trade areas (FTA)**: A free trade area allows free-trade among members. However, each member can have its own trade policy towards non-member countries.

   Example: The North American Free Trade Agreement (NAFTA) creates a free trade area. Its members include United States, Canada, and Mexico.

2) **Customs union**: A customs union allows free trade among members and also requires a common external trade policy against non-member countries.

   Example: In 1947, Belgium, the Netherlands, and Luxemburg ("**Benelux**") formed a customs union.

3) **Common market:** A common market incorporates all aspects of customs union and also allows free movement of factors of production (especially labor) among members.

   Example: The Southern Cone Common Market (MERCOSUR) of Argentina, Brazil, Paraguay, and Uruguay.

4) **Economic Union:** An economic union incorporates all aspects of a common market and also requires common economic institutions and coordination of economic policies among members.

   Example: The European Union.

Monetary Union: When the members of the economic union decide to adopt a common currency, it is referred to as a monetary union.

World Trade Organization (WTO): The WTO is a negotiating forum with objectives to eliminate trade barriers (both tariff and non-tariffs) between countries and to settle trade disputes among countries. Under WTO, all countries treated on most favored nation (MFN) basis.

Practice: Example 3 from the CFA Institute's Curriculum.

Regional integration

- Relative to multilateral trade negotiations under WTO, regional integration is popular and easy to implement because it is easier, politically less conflictual, and quicker to eliminate trade and investment barriers among a **small group** of countries.
- Unlike WTO, regional integration results in preferential treatment for members compared with non-members.
- Regional integration can change the patterns of trade.

Effects of forming a Regional Trading Bloc (e.g. Custom Union):

1) **Trade creation:** Trade creation occurs when regional integration leads to replacement of high-cost domestic production by low-cost imports from other members.

- Trade creation increases welfare.

Example: Suppose, Country A and Country B are members of a regional trading bloc, whereas Country C is not.

- Before the RTA, Country A has in place a specific (per unit) tariff on imports from both B and C.
- Country B is the lower-cost producer in comparison to Country C.
  - Therefore Country A imports good from Country B.
- Once Country A joins regional integration with Country B, tariff is removed on imports from Country B. Good continues to be imported from Country B and imports increase because price has fallen due to removal of tariff.
- Consumer surplus in Country A increases while producer surplus and government tariff revenue falls. Hence, trade creation results in net increase in welfare.

2) **Trade diversion:** Trade diversion occurs when regional integration leads to the replacement of low-cost imports from non-members with higher-cost imports from member countries.

- Trade diversion reduces welfare.

Example: Suppose, Country A and Country B are members of a regional trading bloc, whereas Country C is not.

- Before the RTA, Country A has in place a specific (per unit) tariff on imports from both Country B and Country C.
- Assume Country C is now the lower-cost producer in comparison to B.
  - Country A imports the good from Country C.
- Once Country A joins a regional trading bloc with Country B, however, Country A switches to Country B as an import supplier. → Imports expand as the domestic price falls.
- Consumer surplus in Country A increases while producer surplus and government revenue falls
- Whether net welfare effect is positive or negative, is ambiguous i.e.
  - If trade-diverting effects > trade-creating effects, then regional trading bloc will reduce welfare in Country A.

Advantages of Regional Trading Blocs:

- By eliminating or reducing trade barriers against each other, member countries can allocate resources more efficiently.
- Other advantages are the same as that of free trade e.g.
  - Greater specialization
  - Reduction in monopoly power and prices due to foreign competition
  - Greater choices available to consumers
  - Economies of scale from larger market size
  - Improvements in quality due to the exposure to more competition
  - Learning by doing
  - Technology transfer, knowledge spillovers
  - Encourage FDI
- Facilitates members to negotiate better trade terms with the rest of the world than as individual nations.
- By promoting greater interdependence among members, it helps to reduce the potential for conflict.
- Increases labor mobility and hence help reduce unemployment in member countries.
- Strong growth in any RTA country can accrue other RTA member countries.

- Enhances the benefits of good policy and lead to convergence in living standards.
- Provides greater currency stability.

Disadvantages of Regional Trading Blocs:

- Regional integration may result in less-efficient allocation of resources and decrease in welfare due to trade diversion.
- They may encourage mergers and takeovers resulting in greater oligopolistic collusion.
- They create adjustment costs arising from job losses e.g. when inefficient firms exit the market due to import competition, unemployment increases. However, it is viewed as a temporary phenomenon. But when workers remain unemployed for a long period, they may face long-term losses.
- Differences in tastes, culture, and competitive conditions among members of a trading bloc reduce the potential benefits from investments within the bloc.
- Problems faced by individual member countries may quickly spread to other countries in the bloc.

Challenges in the formation of Regional Trading Agreements (RTA):

There are at least two challenges in the formation of an RTA i.e.

1) It is difficult to form an RTA due to cultural differences and historical considerations (e.g. wars and conflicts).
2) Countries are hesitant to form an RTA due to their preference to pursue independent economic and social policies.

Practice: Question-set from the CFA Institute's Curriculum.

1. INTRODUCTION & THE FOREIGN EXCHANGE MARKET

The foreign exchange market, the world's largest trading market, supports international trade and finance. It is used by many participants for a variety of purposes, including financial, business, trading, and hedging.

Exchange rates are mostly determined by international capital flows, and they affect country trade balances.

2. THE FOREIGN EXCHANGE MARKET AND EXCHANGE RATES

The balance of payments (BOP) measures the payments that flow between any individua
Foreign exchange (FX) market is a market in which currencies are traded against each other. It facilitates international trade and cross-border capital flows.

- It is the world's largest market in terms of daily turnover.
- It operates 24 hours a day, each business day.

The Foreign Exchange Market

Individual currencies are often referred to by standardized three-letter codes. Individual currency is different from exchange rate because

- An individual currency can be held e.g. $100 million deposit.
- An individual currency can be singular.

Exchange rate: An exchange rate refers to the price of one currency (price currency) in terms of another currency (base currency) i.e. number of units of one currency (called the **price currency**) that can be bought by one unit of another currency (called the **base currency**).

- In an exchange rate, always two currencies are involved.
- This exchange rate is referred to as nominal exchange rates.

Example:

A/ B refers to the number of units of currency A that one unit of currency B will buy.

USD/ EUR exchange rate of 1.356 means that 1 euro will buy 1.356 U.S. dollars

- Euro is the base currency
- U.S. dollar is the price currency.

If this exchange rate decreases, then it would mean that fewer U.S. dollars will be needed to buy one euro. It implies that:

- U.S. dollar appreciates against the Euro or
- Euro depreciates against the U.S. dollar.

Purchasing Power Parity (PPP): PPP is based on law of one price, which states that in competitive markets, free of transportation costs and official barriers to trade, identical goods sold in different countries must sell for the same price when their prices are expressed in terms of the same currency. Hence, according to PPP, the nominal exchange rates would adjust so that identical goods (or baskets of goods) will have the same price in different markets. For example,

- If domestic price level ↑ by 10%, domestic, then domestic currency will ↓ by 10%.
- This implies that, changes in relative prices in two countries will change the exchange rate of their currencies i.e. the currency of a country with the highest price inflation should depreciate.

Assumptions of PPP:

- Homogenous goods and services
- No barriers to trade (tariffs, import duties and quotas.)
- No transportation costs.

Criticism of PPP: PPP does not hold for most goods and nominal exchange rates reflect persistent deviations from PPP because:

1) Goods and services are not identical across countries.
2) Baskets of goods and services produced and consumed are not identical.
3) Many goods and services are not traded internationally.
4) There are trade barriers and transaction costs (e.g., shipping costs and import taxes).
5) PPP ignores capital flows in determining nominal exchange rates.

Stanley Notes™ 2025

Hence, PPP is not a good predictor of future movements in nominal exchange rates.

Real exchange rates: The Real exchange rate is the relative price of domestic goods in terms of foreign goods (e.g., Japanese Big Macs per U.S. Big Mac). Real exchange rate can be used to compare the relative purchasing power between countries. It can be constructed for both the domestic currency relative to a single foreign currency or relative to a basket of foreign currencies.

The higher the real exchange rate,

- The more expensive the foreign goods (in real terms).
- The lower the relative purchasing power of an individual compared with the other country.

$$Foreign\ price\ level\ in\ domestic\ currency = S_{d/f} \times P_f$$

where,

$S_{d/f}$ = Spot exchange rate (quoted in terms of the number of units of domestic currency per one unit of foreign currency)
P_f = Foreign price level quoted in terms of the foreign currency.
P_d = Domestic price level in terms of the domestic currency.

$$Real\ exchange\ rate_{(D/f)} = (S_{d/f} \times P_f)/P_d = S_{d/f} \times (P_f/P_d)$$

Real exchange rate

$$Real\ Exchange\ Rate_{\frac{GBP}{EUR}} = S_{\frac{GBP}{EUR}} \times \left(\frac{CPI_{EUR}}{CPI_{UK}}\right)$$

Important to note:

- Real exchange rates are not quoted or traded in global FX markets.
- Real exchange rate is a poor predictor of future nominal exchange rate movements.

Example: Suppose,

- Nominal spot exchange rate (GBP/ EUR) increases by 10%.
- Euro-zone price level increases by 5%.
- U.K. price level increases by 2%.

The change in the real exchange rate = $\left(1 + \frac{\Delta S_{\frac{d}{f}}}{S_{\frac{d}{f}}}\right) \times \frac{\left(1+\frac{\Delta P_f}{P_f}\right)}{\left(1+\frac{\Delta P_d}{P_d}\right)}$ -1 = (1+ 10%) $\times \frac{1+5\%}{1+2\%} - 1 = 10\%$

- Increase in the real exchange rate for the U.K. based individual ≈ 13%.
- This implies that Euro zone goods have become expensive (in real terms) or U.K. based individual's real purchasing power relative to Euro zone goods has decreased by 13%.

Practice: Example 1 and Question-set from the CFA Institute's Curriculum.

Market Participants

A. Sell side: The sell side consists of the FX dealing banks that sell FX products to the buy side. They include

1) Large FX trading banks e.g. Citigroup, UBS, and Deutsche Bank.

- These banks have economies of scale, broad global client base, IT expertise that is needed to offer competitive pricing across a wide range of currencies and FX products.
- These banks account for a large and growing proportion of the daily FX turnover.
- Sell side banks are also known as interbank market.

2) Small FX trading banks that fall into the second and third tier of the FX market sell side e.g. regional or local banks.

- These banks have well-developed business relationships.
- However, they lack economies of scale, broad global client base, IT expertise etc.
- Therefore, these banks outsource FX services from large banks or depend on the largest FX market participants to obtain deep and competitive liquidity.

B. Buy side: They include clients of sell-side banks. The buy side can be further classified into following categories:

1) Corporate accounts: They refer to foreign exchange transactions (i.e. cross-border purchases and sales of goods and services) undertaken by corporations.

2) Real money accounts: These refer to investment funds that have restrictions on the use of leverage or financial derivatives. These accounts are managed by:

- Insurance companies
- Mutual funds
- Pension funds
- Endowments
- Exchange-traded funds (ETFs)

- Other institutional investors

3) **Leveraged accounts**: They are referred to as the **professional trading community accounts**. They represent a large and growing proportion of daily FX market turnover. These accounts are managed by:

- Hedge funds
- Proprietary trading shops
- Commodity trading advisers (CTAs)
- High-frequency algorithmic traders
- Proprietary trading desks at banks
- Any active trading account that accepts and manages FX risk for profit.

These accounts can be classified into different trading styles i.e.

- *Macro-hedge funds:* These funds are based on underlying economic fundamentals of a currency and take longer term FX positions.
- *High-frequency algorithmic trading:* It involves using technical trading strategies (i.e. moving averages or Fibonacci levels) and trading strategies with trading cycles and investment horizons measured in milliseconds.

4) **Retail accounts:** It refers to an account managed by an individual e.g. foreign tourist exchanging currency at an airport kiosk.

5) **Governments:** Public entities manage FX accounts for various purposes e.g.

- To maintain consulates in foreign countries
- To purchase military equipment
- To maintain overseas military bases.

In addition, many governments both at the federal and provincial/state levels issue debt in foreign currencies, resulting in FX flows.

6) **Central banks:** Central banks manage FX accounts to intervene in FX markets in order to influence exchange rate of the domestic currency and/or to manage their home country's FX reserves (i.e. like real money investment fund). Central banks intervene in FX markets when;

- Domestic currency is considered to be too weak by the central bank, or
- The FX market has become so erratic and dysfunctional, or
- Domestic currency is considered to be too strong by the central bank.

7) **Sovereign wealth funds (SWFs):** SWFs are used by countries with large current account surpluses. These surpluses are deposited into SWFs rather than into foreign exchange reserves managed by central banks. SWFs are government (public) entities. However, it is managed purely for investment purposes rather than public policy purposes.

- SWFs are similar to real money accounts.
- But, unlike real money accounts, SWFs are allowed to use leverage, derivatives, and aggressive trading strategies.

NOTE:

- Electronic trading technology has reduced the barriers to entry into FX markets and the costs of FX trading.
- Globally, U.S. dollar is the most widely held currency while the Euro is the second most widely held currency in central bank foreign exchange reserves.
- **Non-bank financial entities** include real money and leveraged accounts, SWFs, and central banks.
- **Non-financial customers** include corporations, retail accounts, and governments.

Market Size and Composition

The Bank for International Settlements (BIS): It is an umbrella organization for the world's central banks. According to the survey conducted by BIS,

- The proportion of average daily FX flow accounted for by **financial** clients is much larger than that for **nonfinancial** clients.
- The proportion of average daily FX flow accounted for by corporations and individuals buying and selling foreign goods and services is much smaller.
- The largest proportion of average daily trade in FX is accounted for by the FX swap market.
- The top 5 currency pairs in terms of their % share of average daily global FX turnover are:
  i. USD/EUR
  ii. JPY/USD
  iii. USD/GBP
  iv. USD/AUD
  v. CAD/USD
- The largest proportion of global FX trading occurs in London,
- The 2nd largest proportion of global FX trading occurs in New York.
- The 3rd largest proportion of global FX trading occurs in Tokyo.

Practice: Example 2 and Question-set from the CFA Institute's Curriculum.

Exchange Rate Quotations

Direct Quote: Direct Quote is a home (domestic) currency price for a unit of foreign currency i.e. FC: DC.

- DC is the price currency or quote currency.
- FC is the base currency.

For example, **$0.989986/€** is a direct quote of about $0.99 for one Euro in the U.S. it means that 1 Euro costs 0.99 USD.

Indirect Quote: Indirect Quote is a foreign currency price for a unit of home currency. For example, **¥122.481/$** is an indirect quote of 122.481 Yen for one US dollar.

- The base currency is the home currency.

Direct Quote = 1 / Indirect Quote

Two-sided Price: Two-sided Price is the price of a **base** currency quoted by a dealer. It involves bid and ask price.

Bid rate: The price at which the bank (dealer) is willing to buy the currency i.e. number of units of price currency that the client will receive by selling 1 unit of base currency to a dealer.

Ask or offer rate: The price at which the bank (dealer) is willing to sell the currency i.e. number of units of price currency that the client must sell to the dealer to buy 1 unit of base currency.

Bid is always < Offer.

- Dealers generate profits by buying at low rate and selling at higher rate. Hence, the wider the bid-ask spread, the higher the profit for a dealer.
- Electronic dealing systems and growing worldwide competition for business have resulted in tighter bid-ask spreads.

Example:

(US$/SFr)	**Bid**	**Ask**
Spot	0.3968	0.3978

- Typically, exchange rates are quoted to four decimal places; except for yen, for which exchange rate is quoted to two decimal places.

Percentage appreciation and depreciation of a currency against the other currency:

Suppose, the exchange rate for the euro i.e. USD/EUR increases from 1.2500 to 1.3000.

% change in exchange rate (un-annualized) = $\frac{1.3000}{1.2500} - 1$ = 4%.

- It shows that euro appreciated against USD by 4%.

To calculate depreciation of USD against Euro, convert the quote from USD/EUR to EUR/USD i.e. euro is now the price currency.

$$\frac{\left(\frac{1}{1.3000}\right)}{\left(\frac{1}{1.2500}\right)} - 1 = \frac{1.2500}{1.3000} - 1 = -3.85\%$$

- It shows that USD depreciated against Euro by 3.85%.

Practice: Example 3 from the CFA Institute's Curriculum.

3. EXCHANGE RATE REGIMES – I DEALS AND HISTORICAL PERSPECTIVE

Exchange rate Regime: The exchange rate regime is the way through which a country manages its currency with respect to foreign currencies and the foreign exchange market.

The Ideal Currency Regime

The ideal currency regime would have the following three properties.

1) The value of the currency would be fixed in relationship to other currencies.
   - It facilitates to eliminate currency-related uncertainty with respect to the prices of goods/services, real and financial assets.

2) All currencies can be freely exchanged for any purpose and in any amount (fully convertible)
   - It facilitates free flow of capital.

3) Each country would be able to undertake fully independent monetary policy to meet domestic objectives (i.e. growth and inflation targets).

Limitations:

The aforementioned three conditions are not mutually consistent and cannot be achieved simultaneously e.g., if first two conditions are met, there would only be one currency in the world.

As a result, country will not be able to undertake independent monetary policy. For example,

- If 1st condition is met (i.e. exchange rate are credibly fixed) and capital is perfectly mobile, then, if a central bank decreases default-free interest rate, → there would be outflow of capital to seek higher return, → central bank would start selling foreign currency & buying domestic currency to maintain fixed exchange rate, → foreign currency reserves will fall, → domestic supply ↓, → as a result, domestic interest rates ↑, resulting in offsetting effect on the initial expansionary monetary policy.
- Opposite would occur in case of contractionary monetary policy (higher interest rates).
- Generally, the more freely the exchange rate is allowed to float and the tightly the convertibility is controlled, the more effectively a central bank can meet domestic macroeconomic objectives.

Floating exchange rate regime:

Under a floating exchange rate system, if a central bank decreases (increases) domestic interest rate, the domestic currency would depreciate (appreciate) in value. Consequently, exports would rise (fall) and imports would fall (rise).

Advantage:

Under floating exchange rate regime, poor domestic monetary policy and trade barriers do not exist.

Limitations:

When exchange rates are highly volatile, they

- Decrease the efficiency of real economic activity.
- Decrease the efficiency of financial transactions.
- Result in inefficient allocation of financial capital.
- Increase the exchange rate risk.

In addition, selecting appropriate hedging strategies for foreign currency exposures depend on exchange rate volatility.

Limitations of Fixed exchange rates regime:

Under a fixed exchange rates system, central bank is not able to undertake independent monetary policy.

Historical Perspective on Currency Regimes

Gold standard:

A classical gold system refers to a system of fixed exchange rates in which the value of currencies was fixed relative to the value of gold and gold was used as the primary reserve asset. Hence, the official value of each currency was expressed in ounces of gold.

- This system was operated through "**price-specie-flow**" mechanism. The gold-specie-flow mechanism was the long-run mechanism that used to maintain the gold standard i.e.
  - When a country had BOP deficit (surplus), gold flowed out of (into) the country.
  - When gold flowed into (out of) the country, the domestic money supply increases (decreases), prices rise (fall) and exports fall (rise).
  - Thus, under a classical gold standard, expansion and contraction of monetary base directly depends on trade and capital flows.
- Since the value of each currency was fixed in terms of gold, the exchange rates were therefore fixed.
- The amount of money a country issued was directly backed by gold.
- Although the system was limited by the amount of gold, gold acted as an automatic adjustment.
- Gold reserves can be increased through new gold discoveries as well as more efficient methods of refining gold.

Benefits of using a common currency e.g. Euro:

Using a common currency

- Increases transparency of prices across countries.
- Increases market competition
- Facilitates more efficient allocation of resources.

Drawbacks of using a common currency e.g. Euro:

The member countries are not able to manage their exchange rate and undertake independent monetary policy.

Practice: Question-set from the CFA Institute's Curriculum.

A Taxonomy of Currency Regimes

Different exchange rate regimes are adopted by different countries for various reasons e.g.

- A fixed exchange rate regime can be adopted by a country with a persistent hyperinflation.
- A specific exchange rate regime can be adopted due to a political reason e.g. using common currency to enhance political union.

It should be noted that exchange rates are affected by various factors i.e.

- Private sector market forces

- Currency regimes i.e. legal and regulatory framework of an economy.
- Policies of a government e.g., fiscal, monetary, and intervention.

Exchange rate arrangements of IMF members include:

Arrangements with No Separate Legal Tender

There are two types of arrangements in which a country does not have its own legal tender.

a. **Dollarization:** Under dollarization, a country uses another country's currency as sole legal tender i.e. as its medium of exchange and unit of account. For example, the use of the US dollar as the official currency of the country.

Advantages:

- It eliminates the risk of sharp exchange rate fluctuations.
- It imposes fiscal discipline on the economy's central bank so that government debt is not financed by printing money.
- It increases economic & financial stability, which facilitates the growth of international trade and capital flows.
- It increases the credibility of the country. However, it does not affect the credit-worthiness of a country. This implies that interest rates on U.S. dollars in a dollarized economy are not necessarily equal to interest rates on dollar deposits in the U.S.

Disadvantages:

- The central bank is not able to undertake independent monetary policy.
- The central bank of the country no longer can serve as lender of last resort.
- The benefits (profit) accruing to a government from the ability to print its own money are lost. These benefits (profits) are referred to as **Seigniorage**. It is earned by the country whose currency is used.

b. **Currency Union:** Under currency union, a country belongs to a monetary or currency union and shares same currency with other union members.

Currency Board System (CBS)

Under currency board system, a country promises to exchange the domestic currency for a specified foreign currency at a fixed exchange rate.

This implies that a country is required to issue domestic currency only against foreign currency (i.e. domestic currency is backed by foreign currency).

- Like classical gold standard, money supply under CBS depends on trade and capital flows.
- Like classical gold standard, CBS is successful in countries:
  i. With flexible domestic prices and wages,
  ii. With relatively small non-traded sectors, and
  iii. Global reserve asset supply grows at a stable & steady growth rate consistent with long-run real growth with stable prices. This condition depends on monetary policy of the foreign currency.

Advantages:

- Unlike dollarization, under CBS, a central bank can earn (**seigniorage)** profit by paying little or no interest on its liability (i.e. monetary base) and can earn a market interest rate on its assets (i.e. foreign currency reserves).

Disadvantages:

- Monetary policy independence is lost.
- The central bank loses its ability to perform as lender of last resort. However, it can provide short-term liquidity to banks by lending foreign currency collateral.

NOTE:

Generally, Seigniorage is the profit which is earned when value of money issued > cost of producing it.

Fixed Parity (i.e. Conventional pegged (fixed) exchange rates):

Under fixed parity regime, a country pegs its currency (formal or de facto) at a fixed rate to a major currency or a basket of currencies where exchange rate fluctuates within a narrow margin i.e. a band of up to ± 1% around the parity level. The monetary authority is obligated to maintain the rate within these bands.

- The credibility of fixed parity depends on the ability & willingness of a country to maintain the exchange rate within these bands and the level of reserves maintained by a country.

Effect of excess private demand for domestic currency:

When private demand for domestic currency ↑, → foreign currency reserves grow rapidly, → as a result, domestic money supply ↑ and inflation accelerates.

Effect of deficient private demand for domestic currency:

When private demand for domestic currency ↓, → foreign currency reserves depletes, → as a result, domestic money supply ↓ and deflation accelerates.

Advantage: Fixed parity system provides macroeconomic discipline and stability to a country by maintaining prices of tradable goods and facilitates internationals trade.

Disadvantages:

- Central bank has limited ability to adopt independent monetary policy as long as parity is maintained.
- If there are weak institutional constraints (e.g. insufficient reserves to maintain the parity), the fixed parity system is subject to speculative attacks, resulting in immediate devaluation of the domestic currency.
- It may involve additional risk i.e. uncertainty regarding the country's ability or willingness to maintain the parity.

Differences between Fixed parity and CBS:

1. Unlike CBS, fixed parity does not involve any legal commitment to maintain the specified parity. Thus, a country has a flexibility to either adjust or abandon the parity e.g. devalue or revalue its currency or allow its currency to float.
2. Unlike CBS, a country has discretion to set any target level of foreign currency reserves. Thus, under fixed parity, the level of foreign currency reserves is not related to domestic monetary aggregates.
3. Unlike CBS, under fixed parity, a central bank can perform as lender of last resort.

Target Zone

It is a type of fixed parity regime where the value of the currency is maintained within fixed horizontal margins (bands) of fluctuation around a formal or de facto fixed peg that are wider than ± 1% around the parity e.g. ± 2%.

- Due to wider bands, it provides relatively higher discretion to monetary authority to undertake its policy than fixed parity.

Active and Passive Crawling Pegs

a. **Passive crawling peg:** The exchange rate is adjusted frequently (i.e. weekly or daily) to account for inflation or other factors.

- Frequent unannounced changes in exchange rate peg provide protection against speculative attacks.

b. **Active crawling peg:** The exchange rate is adjusted periodically in small amounts at a fixed, preannounced rate in response to changes in certain quantitative measures e.g. inflation.

- The exchange rate is only allowed to fluctuate within a narrow range.

Advantages:

- In case of high inflation, crawling peg helps to avoid overvaluation of real exchange rate.
- Pre-announced adjustments help to guide public expectations.

Fixed Parity with Crawling Bands

In this regime, exchange rate is maintained within certain fluctuation bands around a fixed parity that is adjusted periodically over time i.e. it involves pre-announced widening band around the central parity.

Advantage: Due to pre-announced widening band, it provides some degree of flexibility to monetary authority, resulting in an enhanced credibility.

Managed Float:

Under managed float regime, the value of a currency is determined by market forces but a monetary authority actively intervenes in the foreign exchange markets to influence the changes in the exchange rate if necessary.

However, the monetary authority does not pre-announce the desired path for the exchange rate to avoid any speculative attacks.

- Dirty float is the same as managed float; but, unlike managed float, it does not involve explicit intervention.

Advantage:

- It helps to reduce excessive exchange rate fluctuations.

Disadvantages:

- It requires a monetary authority to maintain high foreign reserves.
- It creates uncertainty due to lack of transparency regarding monetary authority intervention.
- The effects of intervention by monetary authority are typically short-lived and may result in decrease in stability in foreign exchange markets.

Independently floating exchange rates

Under this regime, exchange rate is determined by the market forces (i.e. demand & supply of currency). The country's central bank does not influence the value of the currency by intervening in the foreign exchange market.

Advantages:

- The monetary authority can exercise independent monetary policy to meet its domestic objectives e.g. price stability, full employment etc.
- The monetary authority can act as a lender of last resort.

Disadvantages: It introduces exchange rates volatility, which leads to:

- Decrease in the efficiency of real economic activity.
- Decrease in the efficiency of financial transactions.
- Inefficient allocation of financial capital.
- Increase the exchange rate risk.

Practice: Question-set from the CFA Institute's Curriculum.

Exchange Rates and the Trade Balance: Introduction

When imports > exports, a country has trade deficit. Thus, it needs to borrow from foreigners or sell assets to foreigners to finance the trade deficit.

When imports < exports, a country has trade surplus. Thus, it needs to invest the excess either by lending to foreigners or by buying assets from foreigners.

This implies that a trade deficit (surplus) **must be exactly matched** by an offsetting capital account surplus (deficit).

Relationship between the trade balance and expenditure/ saving decisions: It can be expressed as follows:

$$X - M = (S - I) + (T - G)$$

where,

X = exports
M = imports
S = private savings
I = investment in plant and equipment
T = taxes net of transfers
G = government expenditure

This exhibits that:

- Trade surplus (deficit) occurs when:
  i. A fiscal surplus i.e. T > G (deficit i.e. T < G) exists. Fiscal surplus represents government savings.
  ii. Private savings > (<) investment.
  iii. Or both

It must be noted that this relationship does not provide any information regarding the type of financial assets that will be exchanged or the currency in which they will be denominated.

- In case of expectations of significant change in an exchange rate, investors start to sell the currency that is expected to depreciate and buy the currency that is expected to appreciate. As a result, capital flows out of the country and capital deficit occurs. It must be offset by a simultaneous shift in the trade balance or by changes in asset prices and exchange rates.
- Generally, the adjustment usually takes place within the financial markets (i.e. through financial investment decisions and asset prices) because expenditure/ saving decisions and prices of goods change much more slowly. This implies that
  - In the short-to-intermediate term, the major determinant of changes in exchange rate is capital flows (both potential and actual).
  - Whereas in the long-term, the important determinant of changes in exchange rate is trade flows.

Impact of changes in exchange rates on the trade balance: The impact of changes in exchange rates on the trade balance can be analyzed through two different approaches.

1) **Elasticities approach:** It focuses on the effect of changing the relative price of domestic and foreign goods. Thus, it exhibits a microeconomic view of the relationship between exchange rates and trade balance.
2) **Absorption approach:** It focuses on the impact of exchange rates on aggregate expenditure/saving decisions.

Practice: Question-set from the CFA Institute's Curriculum.

4. CAPITAL RESTRICTIONS

Capital restrictions are used to limit or redirect capital flows in an economy. Such restrictions include:

- Taxes or Price controls e.g., special taxes on returns to international investment, taxes on certain types of transactions, or mandatory reserve requirements*.
- Quantity controls e.g. ceilings or special authorization requirements for new or existing borrowing from foreign creditors.
- Outright prohibitions on international trade in assets.
- Administrative controls e.g. foreign firms need approval from government agency regarding transactions for certain types of assets.

***Mandatory reserve requirements:** Under mandatory reserve requirements, foreign parties are required to deposit some % of the inflow with the central bank for a minimum period at zero interest rate if they want to deposit money in a domestic bank account.

Inward Capital restrictions include:

- Restrictions imposed on foreigners to invest in the domestic country.
- Limits on inward investment by foreigners i.e. amount that they can invest in the domestic country and/or type of industries in which capital can be invested.

Restricted capital outflows refer to limits imposed on the purchase of foreign assets or loans abroad. Outward Capital restrictions include:

- Restrictions placed on foreigners on repatriation of capital, interest, profits, royalty payments, and license fees.
- Restrictions imposed on citizens to invest abroad particularly in foreign exchange, scarce economies.
- Deadlines placed on citizens regarding repatriation of income earned from any investments abroad.

Reasons for governments to restrict inward and outward flow of capital:

- To meet objectives regarding employment or regional development.
- To meet strategic or defense-related objectives.
- To exercise control over a country's external balance.
- To exercise a degree of monetary policy independence.
- To raise revenues for the government by keeping capital in the domestic economy. It facilitates taxation of wealth and generates interest income.
- To maintain a low level of interest rates to reduce the government's borrowing costs on its liabilities.

Benefits of Free flow of Financial Capital:

- Free movement of financial capital allows capital to be invested efficiently i.e. where it will earn the highest return.
- Free Capital inflows allow countries to invest in productive capacity at a rate greater than the domestic savings rate.
- Free Capital inflows enable countries to achieve a higher rate of growth.
- Longer-term investments by foreign firms provide spillover benefits to local firms (e.g. new technology, skills, and advanced production and management practices), create a network of local suppliers, and increase efficiency of domestic firms through increased foreign competition.

Drawbacks of Free flow of Financial Capital:

- During macroeconomic crisis, free movement of financial capital may result in capital flight out of the country.
- Due to increased foreign competition, domestic industry may be hurt and are forced to exit the market.

Capital restrictions and fixed exchange rate targets:

Capital restrictions and fixed exchange rate targets are viewed as complementary instruments because under perfect capital mobility, governments can achieve their domestic and external policy objectives simultaneously using both capital restrictions and fixed exchange rate rather than using only standard monetary and fiscal policy tools.

If a government follows tight exchange rate peg system, then capital restrictions help in two ways i.e.

i. They make it easier to maintain the tight exchange rate peg.
ii. They protect domestic interest rates against external market forces. Thus, also helps in managing the domestic banking and real estate sectors.
iii. They allow countries to exercise a degree of monetary policy independence that is difficult to

Copyright © FinQuiz.com. All rights reserved.

achieve under a fixed exchange rate regime with free capital flows.

NOTE:

In order to have effective restrictions on capital inflows, they should have broad coverage and should be implemented forcefully.

Drawbacks of Capital Restrictions:

- Implementing capital restrictions may involve significant administration costs.
- Capital restrictions may affect necessary domestic policy adjustments.
- Capital restrictions give negative signals regarding the economy, resulting in high costs and difficulty to access foreign funds.
- Capital restrictions may lead to decline in trade, employment and living standards.

Practice: Example 5 from the CFA Institute's Curriculum.

Practice: CFA Institute's end of Chapter Practice Problems and FinQuiz Questions.

1. INTRODUCTION

Currency exchange rates are critical for international trade and finance. Cross-rates allow traders to expand opportunities by pricing currencies not directly traded. Arbitrage relationships help understand the interdependencies of key market inputs.

Global entities trade currencies for a variety of reasons, and it is crucial to understand the market forces affecting spot and forward rates.

2. CROSS-RATE CALCULATIONS

Suppose,

Exchange rate for CAD/USD = 1.0460
Exchange rate for USD/EUR = 1.2880

The exchange rate for CAD/EUR is determined as follows:

$$\frac{\text{CAD}}{\text{USD}} \times \frac{\text{USD}}{\text{EUR}} = \frac{\text{CAD}}{\text{EUR}}$$

1.0460 × 1.2880 = 1.3472 CAD per EUR

Now Suppose,

Exchange rate for CAD/USD = 1.0460
Exchange rate for JPY/USD = 85.50

The exchange rate for JPY/CAD is determined as follows:

$$\frac{\text{CAD}}{\text{USD}} \times \frac{\text{JPY}}{\text{USD}} = \frac{1}{\frac{\text{CAD}}{\text{USD}}} \times \frac{\text{JPY}}{\text{USD}} = \frac{\text{USD}}{\text{CAD}} \times \frac{\text{JPY}}{\text{USD}} = \frac{\text{JPY}}{\text{CAD}}$$

(1/ 1.0460) × 85.50 = 81.74 JPY per CAD

NOTE:

- Bid Rate (CAD per USD) = 1 / Ask Rate (USD per CAD)
- Ask Rate (CAD per USD) = 1 / Bid Rate (USD per CAD)

Triangular arbitrage:

Market participants can receive both a cross-rate quote as well as the component underlying exchange rate quotes. Hence, these cross-rate quotes **must be consistent** with the above equation. If they are not consistent with the above equation, then arbitrage opportunities exist.

Suppose, a misguided dealer quotes JPY / CAD rate of 82.00. Hence, profit can be earned by:

- Buying CAD1 at the lower price of JPY81.74.
- Selling CAD1 at JPY82.00.
- A riskless arbitrage profit that can be earned by a trader = JPY0.26 per CAD1.

This arbitrage is known as triangular arbitrage because it involves three currencies.

Practice: Example 1 from the CFA Institute's Curriculum.

3. FORWARD CALCULATIONS

Typically, forward exchange rates are quoted in terms of points (called **pips**).

Points on a forward rate quote = forward exchange rate quote – spot exchange rate quote

i. These points are scaled to relate them to the last decimal in the spot quote.
ii. Points are typically quoted to one (or more) decimal places. Thus, the forward rate will typically be quoted to five or more decimal places; except yen, which is typically quoted to two decimal places for spot rates and forward points are scaled up by two decimal places by multiplying the forward point by 100.
- Point is positive when the forward rate > spot rate.
  - It implies that the base currency is trading at a forward premium and price currency is trading at a forward discount.
- Point is negative when the forward rate < spot rate.
  - It implies that the base currency is trading at a forward discount and price currency is trading at a forward premium.

Example:

Spot exchange rate USD/ EUR =1.2875

Stanley Notes™ 2025

One year forward rate USD/ EUR = 1.28485

One year forward point = 1.28485 – 1.2875 = –0.00265

- It is scaled up by four decimal places by multiplying it by 10,000 i.e. - 0.00265 × 10,000 = -26.5 points.

Swap points: Forward rates quotes are shown as the number of forward points at each maturity. These forward points are referred to as swap points.

Term to maturity and forward points are directly related (all else equal) i.e. given the interest rate differential, the longer the term to maturity, the greater the absolute number of forward points.

Interest rate differential and forward points are directly related (all else equal) i.e. given the term to maturity, the wider the interest rate differential, the greater the absolute number of forward points.

Converting forward points into forward quotes:

To convert the forward points into forward rate quote, forward points are scaled down to the fourth decimal place in the following manner:

Forward rate = Spot exchange rate + $\frac{\textbf{Forward points}}{\textbf{10,000}}$

$$\boldsymbol{Forward\ premium/discount\ (in\ \%)} = \frac{\textbf{spot exchange rate} + (\textbf{forward points}/\textbf{10,000})}{\textbf{spot exchange rate}} - \mathbf{1}$$

To convert spot rate into a forward quote when points are represented as %,

Spot exchange rate × (1 + % premium or discount)

Relationship between spot rates, forward rates and interest rates:

An investor has two alternatives available i.e.

a) Invest for one period at the domestic risk-free rate i.e. i_d;
   - This amount will grow to $(1 + i_d)$ at the end on of investment horizon.
b) Convert 1 unit of domestic currency into foreign currency using the spot rate = $S_{f/d}$. (direct quote)
   - Invest this amount for one period at foreign risk-free rate i.e. i_f.
   - The amount invested will grow to $S_{f/d}(1 + i_f)$ at the end on of investment horizon.
   - Then, convert this amount to domestic currency using the forward rate i.e. for each unit of foreign currency, investor would obtain $1/F_{f/d}$ units of domestic currency.
   - Hence, converting at the forward rate has eliminated FX risk.

Both of these alternatives are risk-free and have same risk characteristics. Since risk characteristics are same, they must have same return; otherwise, riskless arbitrage opportunity exists e.g.

- Investment that generates lower return can be short sold.
- Amount can be invested in an investment that generates higher return.

Arbitrage relationship is stated as follows:

$$(1 + i_d) = S_{f/d}(1 + i_f)\left(\frac{1}{F_{f/d}}\right)$$

In case of indirect quote, Arbitrage relationship is:

$$(1 + i_d) = (1/S_{f/d})(1 + i_f)F_{f/d}$$

$$F_{f/d} = S_{f/d}\left(\frac{1 + i_f}{1 + i_d}\right)$$

Forward rate as a % of spot rate can be stated as follows:

$$\frac{F_{f/d}}{S_{f/d}} = \left(\frac{1 + i_f}{1 + i_d}\right)$$

Given an f /d quoting convention (direct quote),

- The forward rate will be higher than (be at a premium to) the spot rate if foreign interest rates are > domestic interest rates.
- Currency with the higher (lower) interest rate will always trade at a discount (premium) in the forward market.

Example:

- Spot exchange rate ($S_{f/d}$) = 1.6535
- Domestic 12-month risk-free rate = 3.50%
- Foreign 12-month risk-free rate = 5.00%

The 12-month forward rate ($F_{f/d}$) must then be equal to:

$$1.6535 \times \frac{1.0500}{1.0350} = 1.6775$$

Now, suppose, 12-month forward rate ($F_{f/d}$) quoted by a dealer = 1.6900. Hence, a riskless arbitrage opportunity exists.

- Return on domestic-only investment = 3.50%.
- Return on hedged foreign investment (with a quoted forward rate) is calculated as follows:

$$S_{f/d}(1 + i_f)\left(\frac{1}{F_{f/d}}\right)$$

Return on hedged foreign investment =
$1.6535\ (1.05) \times \frac{1}{1.6900}$
= 1.0273 – 1 = 2.73%

Hence, this implies that an investor can generate riskless arbitrage profit (without any upfront capital invested) by:

- Borrowing at high foreign risk-free rate.
- Selling the foreign currency at spot exchange rate.
- Hedging the currency risk at the misquoted forward rate and
- Investing the funds at the lower domestic risk-free rate.

Riskless arbitrage profit = 3.50% – 2.73% = 0.77%

It should be noted that despite of borrowing at a higher interest rates, investor earned profit because the foreign currency is underpriced in the forward market.

Expected % change in the spot rate is stated as follows:

$$\frac{\hat{S}_{t+1}}{S_t} - 1 = \%\Delta\hat{S}_{t+1} = \left(\frac{i_f - i_d}{1 + i_d}\right)$$

- This shows that expected % change in the spot rate is proportional to the interest rate differential.
- Forward rates are unbiased predictors of future spot rates.
- However, forward rates are poor predictors of future spot rates because relationship between forward rates and expected change in spot rates is counter-intuitive e.g. (all else constant), if domestic interest rate ↑, it is expected to result in domestic currency appreciation. But, it may also indicate slower expected domestic currency appreciation.
- Factors that affect the level and shape of the yield curve in either domestic or foreign currency also affect the relationship between spot and forward exchange rates.

Forward points can be stated as follows:

$$\boldsymbol{F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{i_f - i_d}{1 + i_d\tau}\right)\tau}$$

This equation shows that forward points are proportional to:

- Spot exchange rate
- Interest rate differential
- Only approximately proportional to forward contract horizon.

Example:

Determining 30-day forward exchange rate:

- 30-day domestic risk-free rate = 2%.
- 30-day foreign risk-free rate = 3%.
- Spot exchange rate (direct quote) = 1.6555
- The risk-free assets used in this arbitrage relationship are typically bank deposits quoted using the London Interbank Offered Rate (LIBOR) for the currencies involved.
- The day count convention for LIBOR deposits = Actual days/360.

30-day forward rate is:

$$F_{f/d} = S_{f/d}\left(\frac{1 + i_f\tau}{1 + i_d\tau}\right) = 1.6555\left[\frac{1 + 0.0300\left(\frac{30}{360}\right)}{1 + 0.0200\left(\frac{30}{360}\right)}\right] = 1.6569$$

Hence,

Forward rates premium = 1.6569 – 1.6555 = 14 pips.

Or

$$F_{f/d} - S_{f/d} = S_{f/d}\left(\frac{i_f - i_d}{1 + i_f\tau}\right)\tau$$

$$= 1.6555\left[\frac{0.0300 - 0.0200}{1 + 0.0200\left(\frac{30}{360}\right)}\right]\left(\frac{30}{360}\right) = 0.0014$$

Important to understand:

- The forward points are directly proportional to the term of the forward contract i.e. the longer the term of the forward contract (i.e. 180 days, 270 days etc.), the greater the forward points.
- The forward points are directly proportional to the spread between foreign and domestic interest rates i.e. the greater the interest rate differential, the greater the forward points.

Practice: Example 2 from the CFA Institute's Curriculum.

Practice: End of Chapter Questions from the CFA Institute's Curriculum and from the FinQuiz Question-bank.