## FinQuiz Formula Sheet CFA Program Level I

## Quantitative Methods

## Learning Module 1:

Rates and Returns

1. Interest Rate $r$
$r=$ Real risk-free rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

Nominal risk-free rate $=$ Real risk-free rate + Inflation premium
2. Holding Period Return (HPR)
$R=\frac{\left(P_{1}+P_{0}\right)+I_{1}}{P_{0}}-\mathbf{1}$
where
$P_{0}=$ price at the beginning of period
$P_{1}=$ price at the end of period
I =income
3. Arithmetic mean (AM)
$\bar{R}_{i}=\frac{R_{i 1}+R_{i 2}+\cdots+R_{i . T-1}+R_{i T}}{T}$
$=\frac{1}{T} \sum_{t=1}^{T} R_{i t}$
4. Geometric Mean Return
$\overline{\mathrm{R}}_{\mathrm{Gi}}=\sqrt[\mathrm{T}]{\left(1+\mathrm{R}_{\mathrm{i} 1}\right) \ldots \times\left(1+\mathrm{R}_{\mathrm{i} T}\right)}-1$
where,
$R_{i t}=$ return in period $t$
$T=$ total number of periods
5. Harmonic Mean
$\overline{\mathrm{X}}_{\mathrm{H}}=\mathrm{n} / \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\frac{1}{\mathrm{X}_{\mathrm{i}}}\right)$
with $X_{i}>0$ for $i=1,2, \ldots, n$.
6. Money-weighted rate of return (MWR)
$\operatorname{IRR}=\sum_{t=0}^{T} \frac{C F_{t}}{(1+I R R)^{t}}=0$
where,
IRR = internal rate of return
$T=$ number of periods
$C F_{t}=$ cash flow at time $t$
7. Time-weighted Returns (TWR)

$$
\begin{aligned}
& r_{t w r}=[(1+r t, 1) \times(1+r t, 2) \times \ldots \times \\
& (1+r t, n)]^{1 / N}-1
\end{aligned}
$$

8. Non-annual Compounding

PV (for more than one Compounding per year)
$\mathrm{PV}=\mathrm{FV} \mathrm{V}_{\mathrm{N}}\left(1+\frac{\mathrm{r}_{\mathrm{s}}}{\mathrm{m}}\right)^{-\mathrm{m} \times \mathrm{N}}$
where $r_{s}=$ stated ann $i-$ rate
9. Annualized Return
$r_{\text {annual }}=\left(1+r_{\text {period }}\right)^{c}-1$
where,
$c=$ number of periods in a year
$r_{\text {weekly }}=\left(1+r_{\text {daily }}\right)^{5}-1$;
$r_{\text {weekly }}=\left(1+r_{\text {annualy }}\right)^{1 / 32}-1$
10. Continuously Compounded Return CCR $C C R$ associated with a HPR ( $t$ to $t+1$ )
$r_{t, t+1}=\ln (1+$ holding period return $)$ or
$r_{t, t+1}=\ln ($ price relative $)=\ln \left(P_{t+1} / P_{t}\right)=\ln$
(1 $+\mathrm{R}_{\mathrm{t}, \mathrm{t}+1}$ )

CCR associated with a HPR (0 to T)
$\mathrm{Ro}_{0, \mathrm{~T}}=\ln \left(\mathrm{P}_{\mathrm{T}} / \mathrm{P}_{\mathrm{o}}\right)$ or
$r_{0, T}=r_{T-1, T}+r_{T-2, T-1}+\cdots+r_{0,1}$
11. Gross Return

Gross return = Return - trading expenses-Other expense directly related to the generation of returns
12. Net Return

Net Return = Gross Return - all
managerial and administrative expenses
13. After-Tax Nominal Return

After-tax nominal return = Total return any allowance for taxes on dividends, interest \& realized gains
14. Real Returns
$(1+r)=\left(1+r_{r F}\right) \times(1+\pi) \times(1+R P)$
$\left(1+r_{\text {real }}\right)=\left(1+r_{r F}\right) \times(1+R P)$ or
$\left(1+r_{\text {real }}\right)=(1+r) \div(1+\pi)$
where,
$r=$ Nominal return
$r_{r F}=$ Real risk-free return
$\pi=$ Inflation
$R P=$ Risk premium

## Learning Module 2:

The Time Value of Money in Finance

1. Present Value (PV) and Future Value
(FV) Relation
$F V_{N}=P V(1+r)^{N}$
$\mathrm{FV}_{\mathrm{N}}=\mathrm{PVe}^{\mathrm{r}_{\mathrm{s}} \times \mathrm{N}}$ (for continuous
compounding)
$P V=F V_{t}(1+r)^{-t}$ or $P V=\frac{F V_{t}}{(1+r)^{t}}$
$P V=F V e^{-r t}$ (for continuous
compounding)

PV for Fixed Income
2. Discount Instrument:
$\mathrm{PV}=\frac{F V_{t}}{(1+r)^{t}}$
3. Coupon Instrument:
$\mathrm{P}=\frac{\mathrm{PMT}_{1}}{(1+\mathrm{r})^{1}}+\frac{\mathrm{PMT}_{2}}{(1+\mathrm{r})^{2}} \ldots \frac{\mathrm{PMT}_{\mathrm{N}}+\mathrm{FV}_{\mathrm{N}}}{(1+\mathrm{r})^{1}}$
4. Perpetual Bond
$P V=P M T / r$
5. Annuity Instruments
$A=\frac{r(P V)}{1-(1+r)^{-t}}$
$A=$ periodic cash flow.
$r=$ market interest rate per period.
$P V=$ initial value/principal of the loan or
bond.
$t=$ total no. of payment periods.

PV for Equity
6. Constant Dividends
$P V_{t}=\frac{D_{t}}{r}$
7. Constant Dividend Growth Rate
$D_{t+1}=D_{t}(1+g)$
$P V_{t}=\frac{D_{t}(1+g)}{(r-g)}$
assuming $r-g>0$
8. Changing Dividend Growth Rate
$P V_{t}=\sum_{i=1}^{n} \frac{D_{t}\left(1+g_{s}\right)^{i}}{(1+r)^{i}}+\frac{E\left(S_{t+n}\right)}{(1+r)^{n}}$
where $E\left(S_{t+n}\right)=$ stock value in $n$ period
$E\left(S_{t+n}\right)=\frac{D_{t+n+1}}{r-g_{l}}$
9. Implied Return for Fixed-Income

Implied return: $r=\left(\frac{\mathrm{FV}_{\mathrm{t}}}{\mathrm{PV}}\right)^{1 / \mathrm{t}}-1$
$\mathrm{PV}($ Coupon Bond $)=\sum_{i=1}^{N} \frac{P M T_{i}}{(1+r)_{i}}$
10. Implied Return and Implied Growth for Equity
Implied Return: $r=\frac{D_{t}(1+g)}{P V_{t}}+g$
Implied Growth: $g=r-\frac{D_{t+1}}{P V_{t}}$
where $D_{t}(1+g)=D_{t+1}$
11. Price-to-Earnings Ratio ( $P / E$ ):
$\frac{P V_{t}}{E}=\frac{D_{t}}{E_{t}} \times \frac{(1+g)}{r-g}$
where
$P V_{t}=\frac{D_{t}(1+g)}{r-g}$
12. Forward $P / E$ Ratio
$\frac{\mathrm{PV}_{\mathrm{t}}}{\mathrm{E}_{t+1}}=\frac{\frac{\mathrm{D}_{\mathrm{t}+1}}{\mathrm{E}_{\mathrm{t}+1}}}{r-g}$
13. Cash Flow Additivity

Two-Year Bond Future Value
$F V_{2 \text { yrs }}=1\left(1+r_{2}\right)^{2}$
14. Forward Exchange Rates
$F=S_{0} \times \frac{\left(1+r_{d}\right)}{\left(1+r_{f}\right)}$
where
$d=$ domestic currency
$f=$ foreign currency

Learning Module 3
Statistical Measures of Asset Returns

Measures of Central Tendency

1. Arithmetic Mean: AM
$A M=\frac{\text { Sum of obvs in database }}{\text { No.of obvs in the database }}$
2. Sample Mean $\bar{X}$
$\overline{\mathrm{X}}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{\mathrm{i}}}{\mathrm{n}}$
where,
$X_{i}=i^{\text {th }}$ observation
$N=$ no. of observations in the sample
3. Median = Middle Value

- For Even no of obvs locate median at $\frac{\mathrm{n}}{2}$
- For Odd no. of obvs locate median at mean of $\frac{n}{2}$ and $\frac{(n+1)}{2}$

4. Mode

Observation that occurs most frequently in the distribution
5. Weighted Mean: $X_{w}$
$\overline{X_{w}}=\sum_{i=1}^{n} w_{i} X_{i}=\left(\mathrm{w}_{1} \mathrm{X}_{1}+\mathrm{w}_{2} \mathrm{X}_{2}+\ldots .+\right.$
$W_{n} X_{n}$ )
where,
$X_{1}, X_{2}, \ldots, X_{n}=$ observed values
$w_{1}, w_{2}, \ldots, w_{3}=$ Corresponding weights,
sum to 1 .
6. Geometric Mean: GM
$\mathrm{GM}=\sqrt[n]{X_{1} X_{2} \ldots X_{n}}$ with $\mathrm{X}_{\mathrm{i}} \geq 0$ for $\mathrm{i}=$ 1,2,...n.
or
In $\mathrm{G}=\frac{1}{\mathrm{n}} \operatorname{In}\left(\mathrm{X}_{1} \mathrm{X}_{2} \mathrm{X}_{3} \ldots \mathrm{X}_{\mathrm{n}}\right)$
or
$\operatorname{In} \mathrm{G}=\frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \operatorname{In} \mathrm{X}_{\mathrm{n}}}{\mathrm{n}}$
$G=e^{\ln G}$
7. Harmonic Mean: H.M
H.M $=\overline{X_{H}}=\frac{n}{\sum_{i=1}^{n}\left(\frac{1}{X_{i}}\right)}$
with $X_{i}>0$ for $i=1,2, \ldots, n$

Measures of Location
8. Four Measures called Quantiles (collectively)

- Quartiles $=\frac{\text { Distribution }}{4}$
- Quintiles $=\frac{\text { Distribution }}{5}$
- Deciles $=\frac{\text { Distribution }}{10}$,
- Percentiles $=\mathrm{L}_{y}=(n+1) \frac{y}{100}$
- Interquartile range (IQR) = Q3 - Q1


## Measures of Location

9. Range $=$ Max. value - Min value
10. Mean Absolute Deviation: MAD

MAD $=\frac{\sum_{i=1}^{n}\left|X_{t}-\bar{X}\right|}{n}$
where,
$\bar{X}=$ Sample mean
$n=$ No. of observations in the sample
11. Sample Var: $\mathrm{s}^{2}$

$$
\mathbf{S}^{2}=\frac{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}{n-1}
$$

12. Sample Standard Deviation: S.D Sample S.D $=\mathrm{s}=\sqrt{\frac{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}{n-1}}$
13. Geometric vs. Arithmetic:
$\mathrm{GM} \approx A M-\frac{\text { Variance of } R}{2}$
14. Semi-deviation (Semi S.D) Semi S.D $=\sqrt{\text { semivariance }}=$
$\sqrt{\sum_{\text {For all } X_{i} \leq \bar{X}} \frac{\left(X_{i}-\bar{X}\right)^{2}}{n-1}}$
15. Target Semi Var

Target Semi-var $=\sum_{\text {For all } X_{i} \leq B} \frac{\left(X_{i}-B\right)^{2}}{n-1}$
where $B=$ Target Value
16. Target Semi-Deviation

Target S. $D=\sqrt{\text { target semivariance }}$
$=\sqrt{\sum_{\text {For all } X_{i} \leq B} \frac{\left(X_{i}-B\right)^{2}}{n-1}}$
17. Coefficient of Variation CV
$\mathrm{CV}=\left(\frac{S}{\bar{X}}\right)$
where s= sample S.D and $\bar{X}=$ sample mean
18. Excess Kurtosis $=$ Kurtosis -3

## Correlation Between Two Variables

19. Sample Covariance
$s_{X Y}=\frac{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)\left(Y_{i}-\bar{Y}\right)}{n-1}$
where,
n = sample size
$X_{i}=i t h$ observation on variable $X$
$\bar{X}=$ mean of the variable $X$ observations
$Y_{i}=$ ith observation on variable $Y$
$\bar{Y}=$ mean of the variable $Y$ observations
20. Correlation coefficient: $r$
$r_{X Y}=\frac{\text { covariance of } X \text { and } Y}{\binom{\text { sample S.D }}{\text { of } X}\binom{\text { sample S. } D}{\text { of } Y}}$
$r=\frac{\operatorname{cov}(\mathrm{x}, \mathrm{y})}{\sqrt{\operatorname{var}(\mathrm{x})} \sqrt{\operatorname{var}(\mathrm{y})}}$

## Learning Module 4

Probability Trees and
Conditional Expectations

1. Expected Value of Random Variable $\mathrm{E}(\mathrm{X})$ $E\left(w_{i} X_{i}\right)=$ Probability-weighted average of the possible outcomes
2. Variance of a random variable $\sigma^{2}(X)$
$\sigma^{2}(X)=E\left\{[X-E(X)]^{2}\right\}$
3. Standard Deviation S.D
S.D $=\sqrt{\text { Variance }}$
4. Conditional Expected Value: $\mathrm{E}(\mathrm{X} \mid \mathrm{S})$
of a random variable $X$ given a scenario
S.
$E(X \mid S)=P\left(X_{1} \mid S\right) X_{1}+P\left(X_{2} \mid S\right) X_{2}$
$\left.\ldots+P\left(X_{n} \mid S\right) X_{n}\right)$
5. Total Probability Rule
$E(X)=E(X \mid S) P(S)+E\left(X \mid S^{c}\right) P\left(S^{c}\right)$
$E(X)=E\left(X \mid S_{1}\right) P\left(S_{1}\right)+E\left(X \mid S_{2}\right)$
$P\left(S_{2}\right)+\ldots+E\left(X \mid S_{n}\right) P\left(S_{n}\right)$
where,
$E\left(X \mid S_{i}\right)=$ Expected value of $X$ given
Scenario $i$
$P\left(\mathrm{~S}_{i}\right)=$ Probability of Scenario $i$
$S_{1}, S_{2} \ldots S_{n}$ are mutually exclusive and exhaustive scenarios.
6. Bayes' formula
$P($ Event $\mid$ New Information $)$
$=\frac{P(\text { New Information } \mid \text { Event })}{P(\text { New Information })}$
$\times P($ Prior prob. of Event $)$

## Learning Module 5

## Portfolio Mathematics

1. Expected Value of Weighted Sum of random Variables
$E\left(w_{i} R_{i}\right)=w_{i} E\left(R_{i}\right)$
where,
$w i=$ weight of variable $i$
$R i=$ random variable $i$
2. Expected Return on the Portfolio
$E\left(R_{p}\right)=E\left(w_{1} R_{1}+w_{2} R_{2}+\ldots+w_{n} R_{n}\right)$
$=w_{1} E\left(R_{1}\right)+w_{2} E\left(R_{2}\right)+\ldots+w_{n} E\left(R_{n}\right)$
3. Covariance between $R_{i}$ and $R_{j}$
$\operatorname{Cov}\left(\mathrm{R}_{\mathrm{i}}, \mathrm{R}_{\mathrm{f}}\right)=\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\mathrm{p}\left(\mathrm{R}_{\mathrm{i}}-E R_{\mathrm{i}}\right)\left(\mathrm{R}_{\mathrm{j}}-E R_{\mathrm{f}}\right)\right]$
4. Portfolio variance
$\sigma^{2}\left(R_{p}\right)=\sum_{i=1}^{n} \sum_{j=1}^{n} \omega_{i} \omega_{j} \operatorname{Cov}\left(\mathrm{R}_{\mathrm{i}}, \mathrm{R}_{\mathrm{j}}\right)$

For three assets
$\sigma^{2}\left(R_{p}\right)=w_{1}^{2} \sigma^{2}\left(R_{1}\right)+w_{2}^{2} \sigma^{2}\left(R_{2}\right)+$
$\mathrm{w}_{3}^{2} \sigma^{2}\left(\mathrm{R}_{3}\right)+2 \mathrm{w}_{1} \mathrm{w}_{2} \operatorname{Cov}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)+$
$2 \mathrm{w}_{1} \mathrm{w}_{3} \operatorname{Cov}\left(\mathrm{R}_{1}, \mathrm{R}_{3}\right)+2 \mathrm{w}_{2} \mathrm{w}_{3} \operatorname{Cov}\left(\mathrm{R}_{2}, \mathrm{R}_{3}\right)$
where,
$\sigma^{2}=$ Corresponding variance of each asset in the portfolio
5. Correlation: $\rho\left(R_{i} R_{j}\right)$
(b/w two random variables $R_{i}, R_{j}$ )
$\rho\left(R_{i} R_{j}\right)=\frac{\operatorname{Cov}\left(R_{i} R_{j}\right)}{\sigma R_{i} \times \sigma R_{j}}$
6. Safety-first Ratio: SFRatio

SFRatio $=\left[E\left(R_{P}\right)-R_{L}\right] / \sigma_{P}$
7. Sharpe Ratio:
$=\left[E(R p)-R_{f}\right] / \sigma_{p}$

Learning Module 6
Common Probability Distributions

For lognormal random variable

1. Mean: $\mu \mathrm{L}$

$$
\mu_{\mathrm{L}}=\exp \left(\mu+0.50 \sigma^{2}\right)
$$

2. Variance: $\sigma^{2}{ }^{2}$
$\sigma_{\mathrm{L}}{ }^{2}=\exp \left(2 \mu+\sigma^{2}\right) \times\left[\exp \left(\sigma^{2}\right)-1\right]$.
3. Log Normal Price
$\mathrm{S}_{\mathrm{T}}=\mathrm{S}_{0} \exp \left(\mathrm{r}_{\mathrm{o}, \mathrm{T}}\right)$
where,
exp $=e$ and $r_{0, t}=$ Continuously compounded return from 0 to $T$
4. Price Relative
= End price/Beg price
$=S_{t+1} / S_{t}=1+R_{t, t+1}$
where,
$R_{t, t+1}=$ holding period return on the stock from $t$ to $t+1$.
5. Continuously compounded return (associated with a holding period from $t$ to $t+1$ )
$r_{t, t+1}=\ln (1+$ holding period return $)$

## or

$r_{t}, t+1=\ln ($ price relative $)=\ln \left(S_{t+1} / S_{t}\right)=$
$\ln \left(1+R_{t, t+1}\right)$
6. Continuously compounded return
(associated with a holding period from 0
to $T$ )
$\mathrm{R}_{0, \mathrm{~T}}=\ln \left(\mathrm{S}_{\mathrm{T}} / \mathrm{S}_{0}\right)$ or $r_{0, T}=r_{T-1, T}+$
$r_{T-2, T-1}+\cdots+r_{0,1}$
where,
$r_{T-I, T}=$ One-period continuously
compounded returns
7. When one-period continuously
compounded returns are random variables.
$E\left(r_{0, T}\right)=E\left(r_{T-1, T}\right)+E\left(r_{T-2, T-1}\right)+\cdots$

$$
+E\left(r_{0,1}\right)=\mu T
$$

Variance $=\sigma^{2}\left(r_{0, T}\right)=\sigma^{2} T$
S.D. $=\sigma\left(r_{0, T}\right)=\sigma \sqrt{T}$
8. Annualized volatility
= sample S.D. of one period
continuously compounded returns $\times$
$\sqrt{T}$

## Learning Module 7

Estimation and Inference

For Sample Mean

1. Var of the distribution $=\frac{\sigma^{2}}{\mathrm{n}}$
2. S.D of the distribution $=\sqrt{\frac{\sigma^{2}}{\mathrm{n}}}$
3. Standard Error of the sample mean:

- When the population S.D $(\sigma)$ is
known $=\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}$
- When the population S.D $\sigma$ ) is unknown $=s_{\bar{X}}=\frac{s}{\sqrt{n}}$
where $s=$ sample S.D
estimate of $\mathrm{s}=\sqrt{\text { sample variance }}=$ $\sqrt{s^{2}}$
$s^{2}=\frac{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}{n-1}$

Learning Module 8
Hypothesis Testing

1. Standard Error of Sample Mean $\sigma_{\bar{X}}$ When Population S.D/variance is known $\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}$

- Test statistic is $\mathbf{Z}$-distributed
$z=\frac{\bar{X}-\mu_{0}}{\frac{\sigma}{\sqrt{n}}}$

2. Power of Test
= 1 - Prob of Type II Error
3. Test Statistic for a Test of Difference between Two Population Means
Normally Distributed Populations,
Variances Unknown but Assumed Equal) based on Independent samples
$t=\frac{\left(\bar{X}_{1}-\bar{X}_{2}\right)-\left(\mu_{1}-\mu_{2}\right)}{\sqrt{\left(\frac{S_{p}^{2}}{n_{1}}+\frac{S_{p}^{2}}{n_{2}}\right)}}$
where,
$\mathrm{S}_{p}{ }^{2}=$ Pooled estimator of the common variance
$S_{p}^{2}=\frac{\left(n_{1}-1\right) S_{1}^{2}+\left(n_{2}-1\right) S_{2}^{2}}{n_{1}+n_{2}-2}$
where $d f=n_{1}+n_{2}-2$.
4. Test Statistic for a test of mean differences
Normally distributed populations,
unknown population variances

- $t=\frac{\bar{d}-\mu_{d 0}}{s \bar{d}}$
- sample mean difference $=\bar{d}=$ $\frac{1}{n} \sum_{i=1}^{n} d_{i}$
- sample variance $=S_{d}^{2}=\frac{\sum_{i=0}^{n}\left(d_{1}-\bar{d}\right)^{2}}{n-1}$
- $\quad$ sample S.D $=\sqrt{S_{d}^{2}}$
- sample error of the sample mean difference $=s \bar{d}=\frac{s_{d}}{\sqrt{n}}$

5. Chi Square Test Statistic

For test concerning the value of a normal population variance
$X^{2}=\frac{(n-1) S^{2}}{\sigma_{0}^{2}}$
where $(n-1)=d f$ and $S^{2}=$
sample variance $=\frac{\sum_{i=0}^{n}\left(X_{i}-\bar{X}\right)^{2}}{n-1}$
6. Chi Square Confidence Interval for variance
Lower limit $=\mathrm{L}=\frac{(n-1) S^{2}}{X_{a / 2}^{2}}$
Upper limit $=U==\frac{(n-1) S^{2}}{X_{1-a / 2}^{2}}$
7. Test Statistic for a Test of Mean Differences
Normally Distributed Populations,
Unknown Population Variances
$\mathrm{t}=\frac{\overline{\mathrm{d}}-\mu_{\mathrm{d} 0}}{\mathrm{~S}_{\overline{\mathrm{d}}}}$
where,

- $\quad$ Sample mean difference $=\bar{d}=$ $\frac{1}{n} \sum_{i=1}^{n} d_{i}$
- Sample variance $=S_{d}^{2}=\frac{\sum_{i=1}^{n}\left(d_{i}-\bar{d}\right)^{2}}{n-1}$
- Sample S.D. $=\sqrt{S^{2}{ }_{d}}$
- $n=$ number of pairs of observations
- Standard error of sample
mean difference $=s \bar{d}=\frac{S_{d}}{\sqrt{n}}$

8. F-test

Test concerning differences between
variances of two normally distributed populations.
$\mathrm{F}=\frac{S_{1}^{2}}{S_{2}^{2}}$
where
$S_{1}^{2}=1$ st sample var with $n_{1}$ obs $S_{1}^{2}=$
2nd sample var with $n_{2}$ obs
$d f_{1}=n_{1}-1$ numerator $d f$
$d f_{2}=n_{2}-1$ denominator $d f$
9. Relation between Chi Square and Fdistribution
$F=\frac{X_{1}^{2} / m}{X_{2}^{2} / n}$
where

- $X_{1}^{2}$ is one chi square random variable with one $m d f$.
- $X_{2}^{2}$ is another chi square random variable with one $n d f$.


## Learning Module 9

## Hypothesis Testing

Parametric Test of a Correlation

1. Consider two variables $X \& Y$
$r_{X Y}=\frac{s_{X Y}}{s_{X} s_{Y}}$
$s_{X Y}=$ sample covariance between X \& Y . $s_{X} \& s_{Y}=$ S.D of $X$ and $Y$ respectively
2. Sample Correlation: $r$
$t=\frac{r \sqrt{n-2}}{\sqrt{1-r^{2}}}$
3. Spearman Rank Correlation: $r_{s}$
$r_{s}=1-\frac{6 \sum_{i=1}^{n} d_{1}^{2}}{n\left(n^{2}-1\right)}$

- For small samples use table to find rejection points.
- For large sample size ( $n>30$ ) use $t-$ test as below:

$$
t=\frac{(n-2)^{1 / 2} r_{s}}{\left(1-r_{s}^{2}\right)^{1 / 2}}
$$

4. Chi-Square Statistic: $\chi^{2}{ }_{s}$

Test of Independence
$\chi^{2}=\sum_{\mathrm{i}=1}^{\mathrm{m}} \frac{\left(\mathrm{O}_{\mathrm{ij}}-\mathrm{E}_{\mathrm{ij}}\right)^{2}}{\mathrm{E}_{\mathrm{ij}}}$

## where

- $\Sigma$ sum of all cells,
- $O_{i j}$ is observed frequency
- $\quad E_{i j}$ is expected frequency
- $E_{i j}=$ The expected frequencies
- $E_{i j}=\frac{(\text { Total row } i) \times(\text { Total row } j)}{\text { Ow }}$
- $m=n o$. of cells, calculated by multiplying the no. of groups in the rows by the no. of groups in the columns.

Learning Module 10
Simple Linear Regression

1. Simple Linear Regression $Y_{i}$
$Y_{i}=b_{0}+b_{1} X_{i}+\varepsilon_{i}$,
where,
$Y=$ dependent variable
$X=$ independent variable
$b_{0}=$ intercept
$b_{1}=$ slope coefficient
$\varepsilon=$ error term $=Y_{i}-\hat{Y}_{i}$
$b_{0}$ and $b_{1}$ are called regression coefficients
2. Sum of Squares Error SSE
$(S S E)=\sum_{i=1}^{n}\left(y_{i}-\hat{y}\right)^{2}$
as $\hat{Y}_{i}=\hat{b}_{0}+\hat{b}_{1} X_{i}$ therefore
SSE $=\sum_{i=1}^{n}\left(Y_{i}-\left(\hat{b}_{0}+\hat{b}_{1} X_{i}\right)\right)^{2}$
3. Slope Coefficient
$\widehat{\mathrm{b}_{1}}=\frac{\operatorname{cov}(\mathrm{x}, \mathrm{y})}{\operatorname{var}(\mathrm{x})}=\frac{\sum\left(\mathrm{X}_{i}-\overline{\mathrm{X}}\right)\left(\mathrm{Y}_{i}-\overline{\mathrm{Y}}\right)}{\sum\left(X_{i}-\overline{\mathrm{X}}\right)^{2}}$
4. Intercept $\mathrm{b}_{0}$
$\widehat{b_{0}}=\bar{Y}-\widehat{\mathrm{b}_{1}} \bar{X}$
where
$\bar{Y}$ and $\bar{X}$ are mean values
5. Sample correlation: $r$
$r=\frac{\operatorname{Cov} \text { of } Y \text { and } X}{(S . D \text { of } Y)((S . D \text { of } X))}$
6. Covariance of X and $\mathrm{Y}: \operatorname{Cov}_{X Y}$
$\operatorname{Cov}_{X Y}=\frac{\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)\left(\left(X_{i}-\bar{X}\right)\right)}{n-1}$
7. Standard deviation of $\mathrm{Y}: \mathrm{S}_{\mathrm{Y}}$
$S_{Y}=\sqrt{\frac{\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2}}{n-1}}$
8. Squared residuals $\mathrm{E}\left(\varepsilon_{\mathrm{i}}^{2}\right)$
$E\left(\varepsilon_{i}^{2}\right)=\sigma_{i}^{2}, i=1, \ldots n$
9. Sum of Squared Regression: SSR
$S S R=\sum_{i=1}^{n}\left(\hat{y}_{i}-\bar{y}\right)^{2}$
10. Coefficient of Determination: $\mathrm{R}^{2}$
$R^{2}=\frac{\text { Sum of square regression }}{\text { Sum of square total }}$
$=\frac{\sum_{i=1}^{n}(\hat{Y}-\bar{Y})^{2}}{\sum_{i=1}^{n}\left(Y_{i}-\bar{Y}\right)^{2}}$
(for single independent variable $R^{2}=r^{2}$ )
11. Mean square regression: MSR

MSR $=\frac{\text { Sum of square regression }}{k}$
$=\frac{\sum_{i=1}^{n}(\hat{Y}-\bar{Y})^{2}}{1}$
12. Mean Square Error: MSE

MSR $=\frac{\text { sum of squares error }}{n-k-1}$
13. F-Statistic or F-Test
$\mathrm{F}=\frac{M S R}{M S E}=\frac{\left(\frac{\text { Sum of square regression }}{k}\right)}{\left(\frac{\text { Sum of squares error }}{n-k-1}\right)}$
(df numerator $=k=1$ )
(df denominator $=n-k-1=n-2$ )
14. ANOVA

| ANOVA | SS | MSS | F |
| :---: | :---: | :---: | :---: |
| Regression $\mathrm{df}=1$ | $\begin{aligned} & \text { SSR } \\ & =\sum_{\substack{\mathrm{i}=1 \\ -\overline{\mathrm{y}})^{2}}}\left(\hat{y}_{\mathrm{i}}\right. \end{aligned}$ | $\frac{\text { SSR }}{\mathrm{k}}$ | $\frac{\mathrm{SSR} / \mathrm{k}}{\mathrm{SSE} /(\mathrm{n}-\mathrm{k}-1)}$ |
| $\begin{aligned} & \text { Error } \\ & \text { df = n-2 } \end{aligned}$ | $\begin{aligned} & \text { SSE } \\ & =\sum_{\substack{\mathrm{i}=1 \\ -\hat{y})^{2}}}\left(y_{i}\right. \end{aligned}$ | $\frac{\mathrm{SSE}}{\mathrm{n}-\mathrm{k}-1}$ |  |
| $\begin{array}{\|l\|l} \hline \text { Total } \\ \text { df }=n-1 \end{array}$ | $\begin{aligned} & \text { SST } \\ & =\sum_{\substack{i=1 \\ -\bar{y}}}^{\substack{n}}\left(y_{i}\right. \end{aligned}$ |  |  |

15. Test statistic
$\mathrm{t}=\frac{\hat{b}_{1}-B_{1}}{s_{\widehat{b}_{1}}}$
16. Standard error of slope coefficient: $s_{\widehat{b}_{1}}$
$s_{\widehat{b}_{1}}=\frac{s_{e}}{\sqrt{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}}$
17. Standard error of the intercept: $\mathrm{s}_{\mathrm{b}_{1}}$
$s_{\widehat{b}_{1}}=\sqrt{\frac{1}{n}+\frac{\bar{X}^{2}}{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}}$
18. Forecasted value of dependent variable: $\widehat{Y f}$
$\hat{Y} f=\hat{b}_{0}+\hat{b}_{1} X_{f}$
19. Standard error of the intercept: $\mathrm{s}_{\mathrm{f}}$ $s_{f}=s_{e} \sqrt{1+\frac{1}{n}+\frac{\left(X_{f}-\bar{X}\right)^{2}}{\sum_{i=1}^{n}\left(X_{i}-\bar{X}\right)^{2}}}$
20. Log-lin Model: $\ln Y_{i}$
$\ln Y_{i}=b_{0}+b_{1} X_{i}$
21. Lin-log Model $\mathrm{Y}_{\mathrm{i}}$
$Y_{i}=b_{0}+\ln X_{i}$
22. Log-log Model $\ln Y_{\mathrm{i}}$ $\ln Y_{i}=b_{0}+b_{1} \ln X_{i}$

## ECONOMICS

## Learning Module 1

Firms and Market Structures

1. Break-Even Price
$P=A T C$
$P=A R=M R=A T C$
where $T R=T C$.
$P=$ price, $A T C=$ Avg, Total Cost, MR = Marginal Revenue, $A R=$ Avg. Revenue, $T R=$ Total revenue
2. Concentration Ratio: $C R$

CR = Sum of sales values of the largest
10 firms / Total market sales
3. Herfindahl-Hirschman index: HH $\mathrm{HHI}=\sum \mathrm{Xi}^{2}$
where,
$\mathrm{X}_{\mathrm{i}}{ }^{2}$ is squared market share of the $\mathrm{ith}^{\text {th }}$ firm.

HHI = 1 for monopoly.
$H H I \approx 0$ for a perfectly competitive industry.

Understanding Business Cycles

