FinQuiz Formula Sheet CFA Program Level I

Reading 1: Time Value of Money

1. Interest Rate (i)
   - i = Rf + Inf P + Default Risk P + Liquidity P + Maturity P
   - Nominal Rf i rate = Real Rf i Rate + Inf P
   - i rate as a growth rate = g = $\left(\frac{FV_N}{PV}\right)^{\frac{1}{N}}$-1

2. PV and FV of CF =
   - PV = $\frac{FV}{(1+r)^N}$
   - PV of Perpetuity = $\frac{PMT}{r}$
   - PV (for more than one Compounding per year) = PV= $FV_N\left(1+\frac{r_s}{m}\right)^{-m\times N}$ $where\ r_s = stated\ ann\ i-rate$
   - $FV_N = PV(1+r)^N$
   - FV (for more than one Compounding per year) = $FV_N = \left(1+\frac{r_s}{m}\right)^{m\times N}$
   - FV (for Continuous Compounding) = $FV_N = PVe^{r_s\times N}$
   - Solving for N = $\frac{LN\left(\frac{FV}{PV}\right)}{LN(1+r)}$ (where LN = natural log)

3. Stated & Effective Rates
   - Periodic i Rate = $\frac{Stated\ Ann\ i\ Rate}{No\ of\ Compounding\ Periods\ in\ One\ Year}$
   - Effective (or Equivalent) Ann Rate (EAR = EFF%) = $(1+Periodic\ i\ Rate)^m - 1$
   - EAR (with Continuous Compounding) = EAR = $e^{r_s} - 1$

4. PV & FV of Ordinary Annuity
   - $PV_{OA} = \sum_{t=1}^{n}\frac{PMT}{(1+r)^t} = PMT\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$
   - $FV_{OA} = \sum_{t=1}^{n}\left(PMT_t(1+r)\right)^{N-t} = PMT\left[\frac{(1+r)^N-1}{r}\right]$
   - Size of Annuity Payment = PMT = $\frac{PV}{PV\ of\ Annuity\ Factor}$
   - PV of Annuity Factor = $\frac{1-\frac{1}{\left[1+\left(\frac{rs}{m}\right)\right]^{m\times N}}}{\frac{rs}{m}}$

5. PV & FV of Annuity Due
   - $PV_{AD} = PMT\left[\frac{1-\frac{1}{(1+r)^N}}{r}\right]$ + PMT at t = PV_{OA} + PMT
   - $FV_{AD} = PMT\left[\frac{(1+r)^N-1}{r}\right](1+r)$ = $FV_{OA}\times(1+r)$

Reading 2: Organizing, Visualizing, and Describing Data

1. Range = Max. value – Min value
2. Bin width = $\frac{Range}{k}$
3. Relative Frequency = $\frac{Absolute\ Frequency}{Total\ No\ of\ Obvs}$
4. Cumulative Absolute Frequency = Add up the Absolute Frequencies
5. Cumulative Relative Frequency = Add up the Relative Frequencies
6. Arithmetic Mean = $\frac{Sum\ of\ obvs\ in\ database}{No.of\ obvs\ in\ the\ database}$
7. Median = Middle No (when observations are arranged in ascending/descending order)
   - For Even no of obvs locate median at $\frac{n}{2}$
   - For Odd no. of obvs locate median at $\frac{n+1}{2}$
8. Mode = obvs that occurs most frequently in the distribution
9. Weighted Mean = $\overline{X_w} = \sum_{i=1}^{n} w_iX_i$ = (w_1X_1+ w_2X_2+….+ w_nX_n)
10. Geometric Mean = GM = $\sqrt[n]{X_1X_2\ldots X_n}$ with $X_i\geq 0$ for i = 1,2,…n.

11. Harmonic Mean = H.M = $\overline{X_H} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{X_i}\right)}$

12. Population Mean = $\mu = \frac{\sum_i^n X_i}{N}$ with $X_i > 0$ for i = 1,2,.,.,n.

13. Sample Mean = $\bar{X} = \frac{\sum_i^n X_i}{n}$ where n = number of observation in the sample

14. Measures of Location:
    - Quartiles = $\frac{Distribution}{4}$
    - Quintiles = $\frac{Distribution}{5}$
    - Deciles = $\frac{Distribution}{10}$,
    - Percentiles = $L_y = (n+1)\frac{y}{100}$

15. Mean Absolute Deviation = MAD = $\frac{\sum_{i=1}^{n}|X_t - \bar{X}|}{n}$

16. Sample Var = $s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$

17. Sample S.D = s = $\sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$

18. Semi-var = $\sum_{For\ all\ X_i \le \bar{X}} \frac{(X_i - \bar{X})^2}{n-1}$

19. Semi-deviation (Semi S.D) = $\sqrt{semivariance} = \sqrt{\sum_{For\ all\ X_i \le \bar{X}} \frac{(X_i - \bar{X})^2}{n-1}}$

20. Target Semi-var = $\sum_{For\ all\ X_i \le B} \frac{(X_i - B)^2}{n-1}$ where B = Target Value

21. Target Semi-Deviation = $\sqrt{target\ semivariance} = \sqrt{\sum_{For\ all\ X_i \le B} \frac{(X_i - B)^2}{n-1}}$

22. Coefficient of Variation = CV = $\left(\frac{S}{\bar{X}}\right)$ where s= sample S.D and $\bar{X}$ = sample mean

23. Sharpe Ratio = $\frac{Mean\ Portfolio\ R - Mean\ Rf\ R}{S.D\ of\ Portfolio\ R}$

24. Excess Kurtosis = Kurtosis – 3

25. Geometric Mean Return $\approx Arithmetic\ Mean\ Return - \frac{Variance\ of\ R}{2}$

Reading 3: Probability Concepts

1. Empirical Prob of an event E = P(E) = $\frac{Prob\ of\ event\ E}{Total\ Prob}$

2. Odds for event E = $\frac{Prob\ of\ E}{1 - Prob\ of\ E}$

3. Odds against event E = $\frac{1 - Prob\ of\ E}{Prob\ of\ E}$

4. Conditional Prob of A given that B has occurred = P(A|B) = $\frac{P(AB)}{P(B)} \rightarrow P(B) \neq 0$.

5. Multiplication Rule (Joint probability that both events will happen):

   P(A and B) = P(AB) = P(A|B) × P(B)
   P(B and A) = P(BA) = P(B|A) × P(A)

6. Addition Rule (Prob that event A or B will occur):

   P(A or B) = P(A) + P(B) – P(AB)
   P(A or B) = P(A) + P(B) (when events are mutually exclusive because P(AB) = 0)

7. Independent Events:
    - Two events are independent if: P(B|A) = P(B) or if P(A|B) = P(A)
    - Multiplication Rule for two independent events = P(A & B) = P(AB) = P(A)× P(B)
    - Multiplication Rule for three independent events = P(A and B and C) = P(ABC) = P(A) × P(B) × P(C)

8. Complement Rule (for an event S) = P(S) + $P(S^C)$ = 1 (where S^C is the event not S)

9. Total Probability Rule:
   P(A) = P(AS) + $P(AS^C)$ = P(A|S)×P(S) + $P(A|S^C)\times P(S^C)$
   P(A) = $P(AS_1)$ + $P(AS_2)$ +….+ $P(AS_n)$ = $P(A|S_1)\times P(S_1)$ + $P(A|S_2)\times P(S_2)$… $P(A|S_n)\times P(S_n)$

(where S_1, S_2, …,S_n are mutually exclusive and exhaustive scenarios)

10. Expected R = $E(w_iR_i) = w_iE(R_i)$

11. Cov (R_i R_j) = $\sum_{i=1}^{n}\big(p(R_i - ER_i)\big)\big(R_j - ER_j\big)$
    Cov (R_i R_j) = Cov (R_j R_i)
    Cov (R, R) = σ^2 (R)

12. Portfolio Var = σ^2 (R_p) = $\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j Cov\big(R_i R_j\big)$
    σ^2 (R_p) = $w_1^2\sigma^2(R_1) + w_2^2\sigma^2(R_2) + w_3^2\sigma^2(R_3) + 2w_1w_2Cov(R_1,R_2) + 2w_1w_3Cov(R_1,R_3) + 2w_2w_3Cov(R_2,R_3)$

13. Standard Deviation (S.D) = $\sqrt{w_1^2R_i + w_2^2R_2 + w_3^2R_3}$

14. Correlation (b/w two random variables R_i, R_j) = $\rho\big(R_iR_j\big) = \frac{Cov\,(R_iR_j)}{\sigma R_i \times \sigma R_j}$

15. Bayes' Formula = $P(Event|New\ Information) = \frac{P(New\ Information|Event)}{P(New\ Information)} \times P(Prior\ prob.of\ Event)$

16. Multiplication Rule of Counting = n factorial = $n!$ = n (n-1)(n-2)(n-3)…1.

17. Multinomial Formula (General formula for labeling problem) = $\frac{n!}{n_1!n_2!\ldots n_k!}$

18. Combination Formula (Binomial Formula) = ${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$

    where n = total no. of objects and r = no. of objects selected.

19. Permutation = ${}^nP_r = \frac{n!}{(n-r)!}$

Reading 4: Common Probability Distributions

1. Probability Function (for a binomial random variable) p(x) = p(X=x) = $\binom{n}{x}p^x(1-p)^{n-x} = = \frac{n!}{(n-x)!x!p^x}(1-p)^{n-x}$ (for x = 0,1,2….n)
   - x = success out of n trials
   - n-x = failures out of n trials
   - p = probability of success
   - 1-p = probability of failure
   - n = no of trials.

2. Probability Density Function (pdf) = f(x) = $\begin{cases}\frac{1}{b-a}\ for\ a \le x \le b = \\ 0\end{cases}$
   F(x) = $\frac{x-a}{b-a}\ for\ a < x < b$

3. Normal Density Funct = $f(x) = \frac{1}{\sigma\sqrt{2\pi}}exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$ for $-\infty < x < +\infty$

4. Estimations by using Normal Distribution:
   - Approximately 50% of all obsv. fall in the interval $\mu \pm \frac{2}{3}\sigma$
   - Approx. 68% of all obvs fall in the interval $\mu \pm \sigma$
   - Approx. 95% of all obvs fall in the interval $\mu \pm 2\sigma$
   - Approx. 99% of all obvs fall in the interval $\mu \pm 3\sigma$
   - More precise intervals for 95% of the obvs are $\mu \pm 1.96\sigma$ and for 99% of the observations are $\mu \pm 2.58\sigma$.

5. Z-Score (how many S.Ds away from the mean the point x lies) $z = standard\ normal\ random\ variable = \frac{X-\mu}{\sigma}$ (when X is normally distributed)

6. Roy's Safety-Frist Criterion = SF Ratio = $\frac{[E(R_P)-R_L]}{\sigma_P}$

7. Sharpe Ratio = = $\frac{[E(R_P)-R_f]}{\sigma_P}$

8. Value at Risk = VAR = Minimum $ loss expected over a specified period at a specified prob level.

9. Mean (μ_L) of a lognormal random variable = exp ($\mu + 0.50\sigma^2$)