

Reading 6: Overview of the Global Investment Professional Standards

1. Total return (when no external cash flows)

$$\text{Total return} = r_t = \frac{V_1 - V_0 - CF}{V_0 + (CF \times w_i)}$$

2. Time weighted return = $r_{twr} = (1 + r_{t,1}) \times (1 + r_{t,2}) \times \dots \times (1 + r_{t,n}) - 1$

3. Original Dietz Method = $R_{Dietz} =$

$$\frac{V_1 - V_0 - CF}{V_0 + (CF \times 0.5)}$$

4. Modified Dietz Method = $R_{ModDietz} =$

$$\frac{V_1 - V_0 - CF}{V_0 + (CF \times w_i)}$$

5. Time weighted return using Modified Dietz = $r_{ModDietz} =$

$$r_{ModDietz} = \frac{V_1 - V_0 - CF}{V_0 + \sum_{i=1}^n (CF \times w_i)}$$

where,

$$w_i = \frac{CD - D_i}{CD}$$

CD = total calendar days,

D_i = no. of calendar days from beginning of period to tie cash flow CF_i occurs.

$$V_1 = \sum_{i=1}^n [CF_i \times (1 + r)] + v_0(1 + r)$$

6. Sum of beginning assets and weighted external cash flows = $V_0 + \sum_{i=1}^n (CF \times w_i)$

7. composite return under the beginning of period value method =

$$r_c = \sum_{i=1}^n \left[r_{pi} \times \frac{V_{0,pi}}{\sum_{pi=1}^n V_{0,pi}} \right]$$

8. return for a portfolio under the beginning of period value and weighted cash flows (r_c) is $R_c =$

$$9. \sum_{i=1}^n \left(r_{pi} \times \frac{V_{pi}}{\sum V_{pi}} \right)$$

10. Standard Deviation of Composite (in which constituents are equally weighted) =

$$S_c = \sqrt{\frac{\sum_{i=1}^n (r_i - \bar{r}_c)^2}{n - 1}}$$

11. Asset weighted Standard Deviation of individual portfolio returns within a composite =

$$S_c = \sqrt{\sum_{i=1}^n [(r_i - \bar{r}_{proxy})^2 \times w_i]}$$

$$\text{where, } \bar{r}_{proxy} = \sum_{i=1}^n (w_i - r_i)$$

12. Position of a percentile y in an array with n entries sorted in descending order =

$$L_y = (n + 1) \frac{y}{100}$$

13. Annualized Internal Rate of Return from Value of $r =$

$$V_0 = \frac{CF_1}{(1 + r)^1} + \frac{CF_2}{(1 + r)^2} + \dots + \frac{V_N}{(1 + r)^N}$$

Reading 7: The Behavioral Finance Perspective

1. Expected utility (U) = Σ (U values of outcomes \times Respective Prob)

2. Subjective expected U of an individual = $\Sigma [u(x_i) \times \text{Prob}(x_i)]$

3. Bayes' formula = $P(A|B) = [P(B|A) / P(B)] \times P(A)$

4. Risk premium = Diff. b/w Certainty Equivalent and Expected Value

5. Perceived value of each outcome = $U = w(p_1) v(x_1) + w(p_2) v(x_2) + \dots + w(p_n) v(x_n)$

6. Abnormal return (R) = Actual R - Expected R

Reading 8: The Behavioral Biases of Individuals

Reading 9: Behavioral Finance & Investment Processes

- After-tax (AT) *Real* required return (RR) %

$$= \frac{\text{Client's required expenditures in Year } n}{\text{Net Investable Assets}} - \frac{\text{Projected needs in Year } n}{\text{Net Investable Assets}}$$
- AT *Nominal* RR % = $\frac{\text{Projected needs in Year } n}{\text{Net Investable Assets}} + \text{Current Annual (Ann) Inflation (Inf) \%} = \text{AT real RR\%} + \text{Current Ann Inf\%}$ Or

$$\text{AT Nominal RR\%} = (1 + \text{AT Real RR\%}) \times (1 + \text{Current Ann Inf \%}) - 1$$
- Total Investable assets = Current Portfolio - Current year cash outflows + Current year cash inflows
- Pre-tax income needed = AT income needed / (1 - tax rate)
- Pre-tax Nominal RR = (Pre-tax income needed / Total investable assets) + Inf%

If Portfolio returns are tax-deferred:

- Pre-tax projected expenditure \$ = AT projected expenditure \$ / (1 - tax rate)

- Pre-tax real RR % = Pre-tax projected expenditures \$ / Total investable assets
- Pre-tax nominal RR = $(1 + \text{Pre-tax real RR \%}) \times (1 + \text{Inflation rate\%}) - 1$

If Portfolio returns are NOT tax-deferred:

- AT real RR% = AT projected expenditures \$ / Total Investable assets
- AT nominal RR% = $(1 + \text{AT real RR\%}) \times (1 + \text{Inf\%}) - 1$
- Procedure of converting nominal, pre-tax figures into real, after-tax return:
 - Real AT R = [Expected total R - (Expected total R of Tax-exempt Invst × wt of Tax-exempt Invst)] × (1 - tax rate) + (Expected total R of Tax-exempt Invst × wt of Tax-exempt Invst) - Inf rate
Or
 - Real AT R = [(Taxable R of asset class 1 × wt of asset class 1) + (Taxable R of asset class 2 × wt of asset class 2) + ... + (Taxable return of asset class n × wt of asset class n)] × (1 - tax rate) + (Expected total R of Tax-exempt Invst × wt of Tax-exempt Invst) - Infrate

Reading 10: Capital market Expectations: Part 1

$$1. \quad i^* = r_{\text{neutral}} + \pi_e + 0.5 \times (\hat{Y}_e - \hat{Y}_{\text{trend}}) + 0.5 \times (\pi_e - \pi_{\text{target}})$$

where,

i^* = target nominal policy rate

r_{neutral} = real policy rate that would be targeted if GDP growth were on trend & inflation on target

$\pi_e, \pi_{\text{target}}$ = respectively the expected and target inflation rates

$\hat{Y}_e, \hat{Y}_{\text{trend}}$ = respectively the expected and trend real GDP growth rates

By readjusting the above equation:

Real inflation adjusted target rate =

$$i^* - \pi_e = r_{\text{neutral}} + 0.5 \times (\hat{Y}_e - \hat{Y}_{\text{trend}}) + 0.5 \times (\pi_e - \pi_{\text{target}})$$

- Net exports** = Net Private Savings + Government Surplus
 $(X-M) = (S-I) + (T-G)$
- Government Surplus = Taxes - Government spending

Reading 11: Capital market Expectations: Part 2

$$1. \quad E(R_e) \approx \frac{D}{P} + (\% \Delta S - \% \Delta E) + \Delta P/E$$

Where,

- $E(R_e)$ = Expected rate of return on equity

- D/P = Expected dividend yield
- %ΔS = Expected % change in number of shares outstanding

2. Under Basic CAPM model:

- $RP_i = \beta_{i,M} RP_M$
- $\beta_{i,M} = \text{Cov}(R_i, R_M) / \sigma_M^2 = \rho_{i,M} \sigma_i \left(\frac{\sigma_i}{\sigma_M} \right)$

Where,

$RP_i = [ER_i - R_F]$ risk premium on *ith* asset

$RP_M = [ER_M - R_F]$ risk premium on market portfolio

$\beta_{i,M}$ = *ith* asset sensitivity to market

portfolio = $\frac{\text{Cov}(R_i, R_M)}{\sigma_M^2} = \rho_{iM} \sigma_i \left(\frac{\sigma_i}{\sigma_M} \right)$

σ is standard deviation and ρ is correlation

Expected Return using Singer-Terhaar Model

Model's 1st component (full integration assumption):

$$3. \quad RP_i^G = \beta_{i,GM} RP_{GM} = \rho_{i,GM} \sigma_i \left(\frac{RP_{GM}}{\sigma_{GM}} \right)$$

Model's 2nd component (completely segmented market assumption):

$$4. \quad RP_i^S = 1 \times RP_{GM} = 1 \times \sigma_i \left(\frac{RP_{GM}}{\sigma_i} \right)$$

$$5. \quad RP_i = \phi RP_i^G + (1 - \phi) RP_i^S$$

$$6. \quad \text{Cap rate} = \frac{\text{Current year's NOI}}{\text{Property value}}, \text{ where NOI} = \text{net operating Income}$$

7. $E(R_{re})$ = Expected return on real estate

- **long run** (assuming constant growth rate for NOI) is:

$$E(R_{re}) = \text{Cap rate} + \text{NOI growth rate}$$

- for a **finite horizon** (to reflect expected rate of change in the cap rate) is:

$$E(R_{re}) = \text{Cap rate} + \text{NOI growth rate} - \% \Delta \text{ Cap rate}$$

8. Implication of capital mobility:

$$E(\% \Delta S_{d/f}) = (r^d - r^f) + (\text{Term}^d - \text{Term}^f) + (\text{Credit}^d - \text{Credit}^f) + (\text{Equity}^d - \text{Equity}^f) + (\text{Liquid}^d - \text{Liquid}^f)$$

$$9. \quad r_i = \alpha_i + \sum_{k=1}^K \beta_{ik} F_k + \varepsilon_i$$

r_i = return on *ith* asset

α_i = constant intercept

β_{ik} = asset's sensitivity to *kth* factor

F_k = *kth* common factor return

ε_i = error term

$$10. \quad \text{Variance on } i\text{th asset} = \sigma_{ij}^2 = \sum_{m=1}^K \sum_{n=1}^K \beta_{im} \beta_{jn} \rho_{mn} + v_i^2$$

where,

ρ_{mn} = covariance between the *mth* and *nth* factor

v_i^2 = variance of *ith* asset return

$$11. \quad \text{Covariance between } i\text{th and } j\text{th} = \sigma_{ij} =$$

$$\sum_{m=1}^K \sum_{n=1}^K \beta_{im} \beta_{jn} \rho_{mn}$$

$$12. \quad \text{Current return} = R_t = (1 - \lambda)r_t + \lambda R_{t-1}$$

where λ may range from 0 to 1

$$13. \quad \text{var}(r) = \left(\frac{1+\lambda}{1-\lambda} \right) \text{var}(R) > \text{var}(R)$$

14. ARCH Methodology

$$\sigma_t^2 = \gamma + \alpha \sigma_{t-1}^2 + \beta \eta_t^2$$

Rearranging the above equation:

$$\sigma_t^2 = \gamma + (\alpha + \beta) \sigma_{t-1}^2 + \beta (\eta_t^2 - \sigma_{t-1}^2)$$

Reading 12: Overview of Asset Allocation

$$1. \quad \text{Risky Asset Allocation} = w^* = \frac{1}{\lambda} \left[\frac{\mu - r_f}{\sigma^2} \right]$$

Reading 13: Principles of Asset Allocation

$$1. \quad U_m = E(R_m) - 0.005 \lambda \sigma_m^2$$

$$2. \quad w_i \times \text{Cov}(r_i, r_p) = \frac{1}{n} \sigma_p^2$$

$$3. \quad \text{Marginal contribution to risk (MCTR}_i) = (\text{Beta of Asset Class } i \text{ relative to Portfolio}) \times (\text{Portfolio Std. Dev.})$$