1.   **INTRODUCTION**

Common stock represents an ownership interest in a business. There are *four* broad steps to apply **Discounted Cash Flow (DCF)** analysis to equity valuation:

1) Selecting an appropriate definition of CFs.
2) Forecasting CFs.
3) Choosing an appropriate discount rate methodology.
4) Estimating the discount rate.

Present Value Models

1.   **Valuation Based on the Present Value of Future Cash Flows**

The value of an asset depends on the benefits or returns, which are expected to be received from it. These returns are known as expected future cash flows. There are *two elements* of DCF valuation:

1) Estimating the CFs.
2) Discounting the CFs by taking into account the time value of money.

Cash flows are known with **certainty** in case of government bonds, as they are default-free. Thus, risk free rate can be used to discount these CFs.

Unlike risk-free government bonds, future cash flows for equity investments are not known with certainty and are not risk free. There are two approaches to deal with such risky CFs.

1)   *Expected* cash flows are discounted.
2)   **Discount rate** is **adjusted** to reflect the risk inherent in CFs.

Asset's value is PV of its expected future CFs i.e.

$$V_0 = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t}$$

where,
V_0 = value of an asset at t = 0
n = number of CFs over the life of the asset
CF_t = expected CF at time t
r = required rate of return (discount rate)

Practice: Example 1 from the CFA Institute's Curriculum

2.   **Stream of Expected Future Cash Flows**

Three most commonly used definitions of Cash flows are:

a) Dividends
b) Free cash flows
c) Residual income

a)   **Dividend Discount Model**

- It uses dividends to represent CFs.
- Dividends are **less volatile** than earnings; therefore, DDM values are less sensitive to short-run fluctuations.
- DDM values reflect long term intrinsic value.
- Fast growing companies take advantage of profitable growth opportunities by **reinvesting** all earnings instead of paying dividends.
- A mature/established profitable company has fewer attractive investment opportunities; therefore, it **pays dividends** and is hesitant to reduce level of dividends.

DDM is most appropriate to use when:

- The company is dividend paying.
- Dividends represent clear and consistent relationship with the company's earnings/profitability i.e. if earnings rise, dividends also rise.
- The investor plans to purchase just a small ownership share (**minority** shareholder) that does not have the ability to either influence or control the timings & amount of dividends (i.e. Dividend Policy).

International differences in Dividend Policy:

- European and Asian small-cap companies usually pay dividends unlike U.S. companies.

- Developed markets **do not** prefer to pay cash dividends. They **prefer** to distribute cash in the form of share repurchases.

> **Practice: Example 2 from the CFA Institute's Curriculum**

b) Free Cash Flows Model

For a "**going concern**" firm, all of the cash generated from operations is not "freely available" for distribution among capital providers (debt & equity); rather some of the cash is needed to meet firm's working capital and fixed capital requirements.

i. *Free Cash Flow to the Firm (FCFF):* It is the cash generated from operations which is available to distribute among firm' suppliers of capital (debt & equity) after paying for operating expenses (e.g., taxes) and operating investments (both WC and Fixed capital investments).

ii. *Free Cash Flow to Equity (FCFE):* It is the cash available after paying for operating expenses (e.g., taxes), operating investments (both WC and Fixed capital investments) and debt payments.

Important:

- FCFF is a *pre-debt* free cash flow concept.
- FCFE is *post-debt* free cash flow concept.
- FCFF is *preferable* to use when*:*
  a) Company has volatile capital structure.
  b) Company is highly leveraged i.e. large % of debt in capital structure.
  c) Company has negative FCFE.

Free cash flow models are most appropriate to use when:

- The company is **non-dividend** paying.
- Dividends are **not** related to earnings/profitability of the company.
- Free cash flows appear to be a reasonable **representative** of company's earnings and profitability.

- The investor has a **controlling** share in the company i.e., has influence & control on company's policies.

Problem in applying Free Cash Flow Approach:

- Rapidly growing companies have higher capital expenditures and thus, have negative expected free cash flows far into the future. Discounting the negative cash flows will lead to a negative value, which is not meaningful.

c) Residual Income (RI)

It is the earnings for the period after taking into account the investors' dollar required return.

Residual Income = NI – (cost of equity × Beginning BV of common equity)

where, Cost of equity is the required return or opportunity cost of common shareholders.

- RI represents economic gain to shareholders (common) or returns earned in excess of opportunity costs.
- RI matches profits to the time period in which they are earned i.e. (accrual basis).

According to RI model,

Value of stock = BVPS at t = 0 + PV of expected future residual earnings

where,
BVPS *= common shareholders' equity / number of common shares outstanding*

Important: RI model is valid only when *Clean Surplus Accounting* exits i.e.

$BV_t = BV_{t-1} + NI_t - Dividends_t$

RI model is most appropriate to use when:

- The company is **not** paying dividends.
- The company's expected free cash flows are **negative** far into the future.

Problems in applying RI model:

- RI model **requires** detailed knowledge of accrual accounting.

- RI model **cannot** be used in case of higher degree of distortion and poor quality of accounting disclosure

2. THE DIVIDEND DISCOUNT MODEL

a) The Expression for a Single Holding Period

When an investor wishes to buy a stock and hold it for one year then

Value of Stock = PV of expected Dividend + PV of expected Selling Price at the end of year one.

$$V_0 = \frac{D_1}{(1+r)^1} + \frac{P_1}{(1+r)^1}$$

where,
V_0 = value of stock at t =0
D_1 = expected dividend for year 1
P_1 = expected selling price at t= 1
r = required rate of return

Practice: Example 3 from the CFA Institute's Curriculum

b) The Expression for Multiple Holding Periods

Value of stock for 2 years holding period is
$$V_0 = \frac{D_1}{(1+r)^1} + \frac{D_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}$$

The general expression for n-holding periods is:

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$$

When the holding period is extended into the indefinite future then:

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

Practice: Example 4 from the CFA Institute's Curriculum

Growth Patterns used in DDM:

1) Gordon growth model i.e. constant growth forever.
2) Two distinct stages of growth i.e. Two stage model & H-model.
3) Three distinct stages of growth i.e. Three stage model

3. THE GORDON GROWTH MODEL (GGM)

GGM assumes that dividends grow indefinitely at a constant rate. GGM equation is as follows:

$$V_0 = \frac{D_0 \times (1+g)}{r-g} \text{ or } V_0 = \frac{D_1}{r-g}$$

where,
g = expected constant growth rate in dividends
D_1 = expected dividend payable at t =1
r = required rate of return on equity

Assumptions & Characteristics of GGM:

- g remains constant indefinitely in the future.
- r > g
- Constant **proportional** relationship exists b/w earnings & dividends i.e. both earnings & dividends grow at g and payout ratio is constant.
- When prices are efficient (price = value), price is expected to grow at g.
- Dividend yield & capital gains yield stay **constant** through time.

- *GGM is a single stage DDM.*
- Growth rate is measured by growth in GDP.
- **Nominal Growth rate** = Real growth rate in GDP + Expected long-term rate of Inflation.
- For companies with earnings growth rate > economy's nominal growth rate, it is recommended to use **multistage DDM** instead of GGM.
- GGM can also be used to value broad equity market indices.
- GGM can also be used to value Preferred stock (fixed rate perpetual preferred stock) i.e.

$$V_0 = \frac{D}{r}$$

   where,
   r = capitalization rate, g = 0
- GGM can also be used to estimate equity risk premium i.e.
   GGM equity risk premium = one-year forecasted dividend yield on market index + consensus long-term earnings growth rate – long-term government bond yield

GGM is most appropriate to use when:

- Company is dividend paying.

- Dividends are **related** to company's earnings/profitability.
- Companies whose earnings are growing at a rate similar to economy's nominal growth rate or less than economy's nominal growth. (Earnings growth rate greater than the nominal GDP growth rate cannot exist for an indefinite period).
- When company exists in **developed** markets.

Limitations of GGM:

- The output of GGM is very **sensitive** to small changes in the inputs i.e. assumed growth rate (g) and required rate (r).
- GGM **cannot** be easily applied to non-dividend paying stocks.
- Growth patterns are **difficult** to predict for some firms.

Practice: Example 5 to 9 from the CFA Institute's Curriculum

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4. SHARE REPURCHASES AND THE IMPLIED DIVIDEND GROWTH RATE

Share Repurchases

Companies have **two options** regarding distribution of free cash flow to shareholders. In the form of

   i. share repurchases (buy backs)
   ii. dividends.

Share Repurchases v/s Cash Dividends

- Number of shares outstanding **decreases** when shares are repurchased. Thus, relative ownership of selling shareholders is reduced as compared to non-selling shareholders.
- Firms paying dividends are **committed** to their dividend paying policy and are hesitant to reduce cash dividends or stop paying cash dividends. While,

firms have **no obligation** to maintain their share repurchase practices.
- Share repurchases are generally difficult to **predict** than cash dividends (both in timing & monetary terms).
- Share repurchases have **neutral effect** on the wealth of existing shareholders when the shares are repurchased at market prices.

The Implied Dividend Growth Rate

One of the reasons of estimated value of a stock being different from its actual market value is the difference in growth rate assumptions. Thus, we can obtain the *implied growth rate* with the help of following inputs:

   a) Actual market price
   b) Expected dividend
   c) Required rate of return

5. THE GORDON GROWTH MODEL AND THE PRICE-TO-EARNING RATIO

The Present Value of Growth Opportunities

The actual value of a share is the sum of **two** components:

1. Value of a company without reinvestment of earnings i.e. no-growth value per share.
2. Present value of growth opportunities (PVGO).

$$V_0 = \frac{E_1}{r} + PVGO$$

where,
PVGO = Sum of PV of expected profitable opportunities of reinvesting the earnings.
E_1/r = no-growth value per share

- When rate of return from reinvestment **<** required rate of return, then in spite of increasing EPS, reinvestment should not be done. Earnings should rather be distributed to shareholders in the form of cash dividends.
- Reinvestment increases shareholders' value only when it generates returns > opportunity cost (r) i.e. Projects with positive NPV only.
- When $P_0 = V_0$ then.

$$PVGO = P_0 - \frac{E_1}{r}$$

Reasons for having Negative Value of PVGO:

- Instead of creating value, company's investment policy is destroying shareholders' value.
- Estimated no-growth value per share *(E_1/r) is too high*.
- Required return on equity is *too low*.

Determinants of PVGO:

- Presence of good business opportunities.

- Availability of real options to a company e.g. option available when to start a project, adjust the scale of project or option to abandon future projects.

Equation of Value of stock can be restated as

$$\frac{V_0}{E_1} \ or \ \frac{P_0}{E_1} \ or \ \frac{P}{E} = \left[\frac{1}{r}\right] + \left[\frac{PVGO}{E_1}\right]$$

where,
1/r = value of P/E for no-growth company.
PVGO/E_1 = component of P/E value that represents growth opportunities.

Gordon Growth Model and the Price-to-Earnings Ratio

Leading and trailing justified P/E multiples can be derived from the Gordon growth model.

Justified P/E expression has **two** uses:

i. Analyst can figure out whether the stock is under/over or fairly valued by comparing the actual P/E ratio with the justified P/E ratio.
ii. Analyst can evaluate whether the growth rate used to estimate P/E is reasonable or not.

$$Leading \ P/E \ ratio = \frac{P_0}{E_1} = \frac{D_1/E_1}{(r-g)} = \frac{(1-b)}{(r-g)}$$

$$Trailing \ P/E \ ratio = \frac{P_0}{E_0} = \frac{D_0(1+g)/E_0}{(r-g)}$$
$$= \frac{(1-b)(1+g)}{(r-g)}$$

where,
P_0/E_1 = *today's market price per share / forecasted next 12 months EPS (or next fiscal's year's earnings per share)*

P_0/E_0 = today's market price per share / trailing 12 months EPS

b = retention rate

1- b = dividend payout ratio

r = required rate of return

g = growth rate

- The higher the anticipated dividend growth rate, the higher the stock price (all else equal).

Practice: Example 11 from the CFA Institute's Curriculum

The GGM can be used to derive **required rate of return (r)** i.e.

$$r = \left[\frac{D_0(1+g)}{P_0}\right] + g = \left[\frac{D_1}{P_0}\right] + g$$

where,

D_1/P_0 = dividend yield

g = capital gains (or appreciation) yield

Practice: Example 12 from the CFA Institute's Curriculum

6. MULTISTAGE DIVIDEND DISCOUNT MODELS

Assumption of stable dividend growth rate for an indefinite period into the future (i.e. in GGM) is not appropriate to use in many firms. Practically, growth is assumed to have the following **three** stages.

1) Growth Phase

- **Firm enjoys** rapidly expanding markets.
- **Profit margin**: High
- **Growth rate in earnings**: Supernormal.
- **Capital requirements**: High
- **FCFE:** Negative
- **ROE > r**
- **Dividend payout ratio**: Low or zero.
- **Preferred Model for valuation**: The three-stage DDM

Justified P/E is not a preferred valuation method for high-growth companies because it assumes a constant growth rate in perpetuity.

The three-stage DDM is most appropriate to analyze firms with high growth rate because of its flexibility.

H-model may not be appropriate, because a linear decline from the high growth rate to the constant growth rate cannot be assumed and the dividend payout ratio is fixed.

2) Transition Phase

- **Pressure:** Increasing competition & market saturation put downward pressure on prices.
- **Growth rate in earnings:** May be above average but is decreasing toward the growth rate of the overall economy.
- **Profit margin:** May be above average but is decreasing toward the growth rate of the overall economy.
- **Capital requirements:** Low or decreasing.
- **FCFE:** Positive or increasing.
- ROE starts falling towards r. i.e. **ROE → r**
- **Dividend payout ratio**: Move from zero to a positive number or Increases
- **Preferred Model for valuation**: The two-stage DDM.

The two-stage DDM is well suited to firms that have high growth and are expected to maintain it for a specific period i.e., transition phase. The model is not useful in analyzing a firm that is in an industry with low barriers to entry.

3) Mature Phase

- **Earnings growth rate:** Stabilize at long-term level.
- **Profit margins:** Stabilize at long-term level.
- **Capital requirements:** Stabilize at long-term level.

- **FCFE:** Stabilize at long-term level.
- **ROE = r**
- **Dividend payout ratio**: Stabilize at long-term level.
- **Preferred Model for valuation**: Gordon Growth Model (GGM).

Types of Multistage DDMs:

1) The two-stage DDM
2) The H-Model (type of two-stage)
3) The three-stage DDM.

Two-Stage Dividend Discount Model

There are two versions of two-stage DDM. Both versions assume constant growth at a mature growth rate in stage-two.

a) First version of Two-stage Model (General Two-stage Model): Stage 1 represents abnormal growth e.g., 20% and stage 2 (transition to maturity) growth rate falls abruptly to sustainable lower level e.g. 5%.

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{V_n}{(1+r)^n}$$

where,
V_n = estimate of P_n

$$V_n = \frac{D_0 \times (1+g_S)^n (1+g_L)}{(r - g_L)}$$

$$V_0 = \sum_{t=1}^{n} \left[\frac{D_0(1+g_S)^t}{(1+r)^t} \right] + \left[\frac{D_0 \times (1+g_S)^n \times (1+g_L)}{(1+r)^n (r-g_L)} \right]$$

Assumptions of the model: First "n" dividends grow at supernormal growth rate i.e. g_S. Supernormal growth can be achieved through possession of a patent, first-mover advantage etc.

After time "n", growth rate changes to normal long-term rate due to the increased level of competition and growth in the overall economy i.e. g_L.

Limitations of the Model : Abrupt movement from initial supernormal growth period to the final sustainable lower growth period is an unrealistic assumption.

b) Second version of Two-stage Model: It is also known as H-Model. Here, growth rate after supernormal growth period does not fall abruptly; rather it is assumed to decline linearly to a sustainable lower level.

> **Practice: Example 13 & 14 from the CFA Institute's Curriculum**

Valuing a Non-dividend Paying Stock

> **Practice: Example 15 from the CFA Institute's Curriculum**

It is difficult to correctly estimate the timing and amount of dividends initiated by a non-dividend paying firm, therefore, it is preferred to use free cash flow or residual income models to value such stocks.

7. MULTISTAGE DIVIDEND DISCOUNT MODELS

The H-Model

It is a type of two-stage model in which initially growth is at a high rate and then it linearly declines to normal rate.

$$V_0 = \frac{D_0 \times (1+g_L)}{(r - g_L)} + \frac{D_0 \times H \times (g_S - g_L)}{(r - g_L)}$$

or

$$V_0 = \frac{[D_0 \times (1+g_L)] + [D_0 \times H \times (g_S - g_L)]}{(r - g_L)}$$

where,
V_0 =*value per share at t= 0*
D_0 = *current dividend*
g_L= *normal long-term dividend growth rate after year 2H*
g_S= *initial short-term dividend growth rate*
H = half-life in years of the high-growth period i.e. high growth period = 2H years
r = required rate of return on equity

- $[D_0 \times (1+g_L)] / (r - g_L)$ represents value of firm when it grows at g_L forever.
- $[D_0 \times H \times (g_S - g_L)] / (r - g_L)$ represents extra value of firm when it grows initially at higher growth rate i.e. $g_S > g_L$.
- *Larger the H (longer supernormal growth period), higher the share value, all else equal.*
- Since H-Model is just an approximation model, it is better to use more exact model when **H is high** and/or difference between g_S and g_L is large.

Practice: Example 16 from the CFA Institute's Curriculum

Three-stage Dividend Discount Model

There are two versions of three-stage DDM.

a) *First version*: The firm is assumed to have three distinct stages of growth and second stage (middle stage) growth

rate is constant i.e. stable growth rate in each of the three stages.

b) *Second version*: Second stage growth rate is not constant, rather it is assumed to decline linearly to the mature growth rate. Thus, second & third stages are treated as H-Model.

- *In three-stage model, the middle stage is known as transition stage.*

Practice: Example 17 & 18 from the CFA Institute's Curriculum

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8 & 9

GENERAL MODELING AND ESTIMATING A REQUIRED RETURN USING ANY DDM

+

THE FINANCIAL DETERMINANTS OF GROWTH RATES

Spreadsheet (General) Modeling

- Spreadsheets can be used to model complex and complicated growth patterns in DDMs. Spreadsheets reduce the likelihood of computational inaccuracies and allow analysts to more easily modify models to reflect many scenarios.

Practice: Example 19 from the CFA Institute's Curriculum

Estimating Growth Rate: There are two approaches to estimate "g".

1) Using Sustainable Growth Rate

2) Using Macroeconomic & Industry Growth Rate Projections.

Estimating the **sustainable growth rate*** with the help of following formula.

$$g = b \times ROE$$

where,
g = dividend growth rate
b = earnings retention rate
ROE = return on equity

**Sustainable growth rate:* Growth rate that can be sustained for a given level of ROE when capital structure is constant, and no additional common stock is issued. *(when debt is growing at rate g, capital structure is constant)*

or

$$g = \frac{NI - Dividends}{NI} \times \frac{NI}{Sales} \times \frac{Sales}{Total\ Assets} \times \frac{Total\ Assets}{Shareholders'\ Equity}$$

g = PRAT i.e.

g = profit margin (P) × retention rate (R) × asset turnover (A) × financial leverage (T)

NOTE: ROE is calculated using beginning of period shareholders' equity.

- Higher the ROE, higher the dividend growth rate, all else constant.
- *Dividend Displacement of earnings:* Higher the earnings retention ratio, higher the growth rate in dividends, all else constant.

ROE is estimated as follows:

a) DuPont decomposition of ROE.
b) Assuming ROE = r
c) Assuming ROE = Median industry ROE.

$$ROE = \frac{NI}{Shareholders'equity}$$

$$ROE = \frac{NI}{Total\ Assets} \times \frac{Total\ Assets}{Shareholder's\ equity}$$

$$ROE = \frac{NI}{Sales} \times \frac{Sales}{Total\ Assets} \times \frac{Total\ Assets}{Shareholders'equity}$$

ROE = Net profit margin × Asset Turnover × Leverage

Practice: Example 20 from the CFA Institute's Curriculum

Estimating Required Return Using Any DDM

- For **H-Model**, the expected rate of return is derived as

$$r = \left[\left\{ \frac{D_0}{P_0} \right\} \times \{ (1 + g_L) + H \times (g_S - g_L) \} \right] + g_L$$

- For **two-stage model**, trial & error method is used to estimate r.

Practice: Example 21 from the CFA Institute's Curriculum

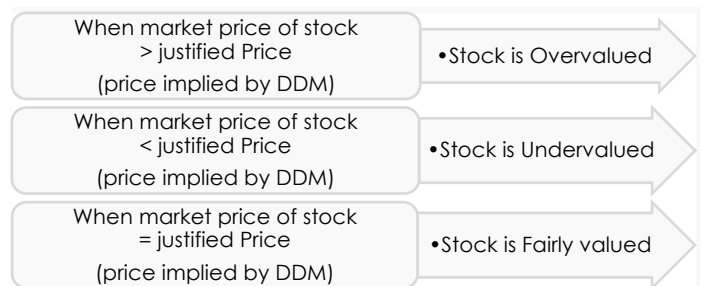## Multi-stage DDM: Concluding Remarks

Limitation of Multistage models:

- Large percentage of total value depends on terminal value; terminal value is very sensitive to minor changes in growth rate and required rate assumptions.
- Technological innovations can highly affect the assumptions of the model regarding life-cycle.

Terminal Stock Value: Terminal value represents a large percentage of total value in DDMs. There are *two* ways to estimate it:

a) Using Gordon Growth Model.
b) Applying a multiple (e.g. P/E) to a forecasted fundamental e.g. BVPS, EPS etc. as of the terminal date.

How to evaluate the value of stock using DDM estimates of Value:

When market price of stock > justified Price (price implied by DDM)	• Stock is Overvalued
When market price of stock < justified Price (price implied by DDM)	• Stock is Undervalued
When market price of stock = justified Price (price implied by DDM)	• Stock is Fairly valued

Practice: Example 22 to 25 from the CFA Institute's Curriculum

Practice: CFA Institute's Curriculum End of Chapter Questions & FinQuiz Question-bank (Item-sets + Questions.